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TRANSIENTS IN POWER SYSTEMS

by

Harold A. Peterson

Professor of Electrical Engineering
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To one who was courageous

MY MOTHER

PREFACE

This book is written primarily for those individuals responsible for power system and apparatus design. It is written secondarily for those in engineering colleges who may wish to use it as a reference for advanced study in electric circuit phenomena. The book is intended therefore to serve as a combination reference and textbook.

The subject matter as the title suggests deals with "Transients in Power Systems." It is not the author's intention to emphasize the rigorous mathematical circuit-theory approach but rather to go beyond that to the point of discussing what the analysis discloses so that conclusions may be reached which will be of value to the power system and apparatus designer. It is hoped that such an approach will also serve to stimulate the thinking of graduate students interested in advanced circuit study in this or a closely related field.

Much has been learned about electric transient phenomena during the last several decades. The pioneering work of men like Steinmetz and Heaviside was most notable in serving as a means for achieving clearer understanding of transient phenomena. Such analysis can be applied to linear circuits without reservation. However, as a circuit becomes more complex, or as the switching sequence to which a circuit is subjected becomes more complex, the application of these methods of analysis becomes very laborious and time consuming. Furthermore, many of the transients of interest in power systems are not linear, i.e., the circuit is not a linear system. Consequently, any method of calculation is either extremely and prohibitively time consuming or is so riddled with simplifying assumptions that the finally calculated result is itself of questionable value.

During the past decade, new tools were developed and put to use in studying many of the circuit transient phenomena previously avoided simply because of the prohibitive amount of time required for arriving at the desired understanding with means then available. Both the differential analyzer and the transient analyzer were found useful in reducing the time and effort required to analyze these more complex circuit problems. Because of its simplicity and directness, the transient analyzer was found exceptionally useful in studying the behavior of a large variety of complex linear and non-linear circuits. It was found unusually powerful in obtaining solutions to problems involving distributed constants as well as non-linear impedance elements.

Use of such tools as these has resulted in the publication of much technical literature in the course of recent years. The *Transactions of the American Institute of Electrical Engineers,* in particular, served as a means of summarizing from time to time the results of new investigations. These results are scattered through the literature. Furthermore, much has been done that has not been published. Consequently, it is largely the object of this book to organize and consolidate the information contained in these numerous publications as well as much of that which has not been published so that the power-system and apparatus designer can more readily avail himself of the knowledge pertinent to his particular responsibility.

The completion of such a task is not the result of the effort of one individual alone but rather is the result of the cooperative efforts of a group of associates in the Central Station Engineering Divisions of the General Electric Company. In particular, to S. B. Crary I express my appreciation for his encouragement and enthusiasm. Without his encouragement, this book would not have been written. To Charles Concordia I owe much for his patient counsel concerning material to be included and its most advantageous arrangement. Also I wish to thank Miss Edith Clarke for her early advice concerning the method of using symmetrical components and for her consent to draw freely from her publications as references and to some extent as source material useful in the preparation of this book.

I also wish to thank D. M. Jones and Robert Treat for their encouragement in undertaking studies of power systems and circuits from the point of view of acquiring immediately useful results.

To others who have been helpful in obtaining data or in the preparation of this book I am also thankful. To mention a few: F. J. Maginniss, I. B. Johnson, W. J. Rudge, S. B. Farnham, P. H. Light, E. M. Hunter, H. S. Shott, T. W. Schroeder, L. E. Saline, L. K. Kirchmayer, and E. W. Boehne. Finally to Miss Rose Pileggi goes real appreciation for her invaluable work in typing the manuscript, supervising the drawing of figures, proofreading, and corresponding with the publishers.

HAROLD A. PETERSON

SCHENECTADY, NEW YORK
May, 1951

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1 OVERVOLTAGES DURING POWER SYSTEM FAULTS

1·1 INTRODUCTION

Faults occur in a power system because insulation at the point of fault is momentarily subjected to voltage stress in excess of its dielectric strength. In the past, lightning has probably been the most frequent cause of fault initiation and subsequent line and apparatus outages. Switching surges also have been frequent causes of overvoltages of sufficient magnitude to cause insulation to be dielectrically overstressd, resulting in system faults. Birds, snakes, kites, airplanes, and rifles in capable, though morally misdirected, hands have been known to initiate faults. Mechanical failure of insulation may and occasionally does result in a power system fault.

The list of causes of these faults is long and varied. However, it is not the purpose of this chapter to concern itself further with the causes, but rather to consider the consequences of the occurrence of a fault as far as the system itself is concerned. In considering these consequences, it will be preferable to think of the power system as an electrical circuit made up of linear impedance elements of resistance, inductance, and capacitance. The circuit is normally energized and carrying load until a fault suddenly occurs. The fault then corresponds to the closing of a switch (or switches, depending on the type of fault) in the electrical circuit. The closing of this switch (or switches) changes the circuit so that a new distribution of currents and voltages is brought about. This redistribution is accompanied in general by a transient period during which the resultant currents and voltages may momentarily be relatively high.

There are two components of voltage in such linear circuits due to the occurrence of a system fault (or due to closing the switch in the electrical network), viz., (1) fundamental-frequency voltages, and (2) natural-frequency voltages usually of short duration which are superimposed upon the fundamental-frequency voltages.

There is a third component also which will be listed here and discussed more completely in Chapter 2, viz., (3) harmonic voltages resulting from unbalanced currents flowing in rotating machines in which the reactances in the direct and quadrature axes are unequal.

In general, an overvoltage protective device must function on any transient overvoltage of sufficient magnitude and limit that voltage to a value lower than the value corresponding to the dielectric strength of the insulation being protected; then it must automatically interrupt the power-follow current * that flows through the protective device as a result of this sustained voltage. In the design and application of overvoltage protective devices such as arresters, the rating of the device is expressed in terms of the maximum rms fundamental-frequency voltage at which automatic interruption of power-follow current takes place. It is therefore of vital importance to know what these system overvoltages are in magnitude and duration so that arresters of proper design and rating may be selected for application in any given system. Since sustained overvoltages during faults exceeding the rating of the arrester will result in its failure, an important factor in the effectiveness of arrester protection is the proper design and operation of the power system, particularly with regard to system neutral grounding.

The fundamental-frequency overvoltages largely determine the arrester rating and the corresponding insulation protective levels. These in turn are important factors in determining the allowable apparatus insulation levels.

1·2 FUNDAMENTAL-FREQUENCY VOLTAGES

Fundamental-frequency voltages can be determined with fair accuracy. Calculations can, of course, be made very precise for a given set of assumptions. For the results shown in connection with this section, the following assumptions were made:

1. The fault is a solid fault, i.e., there is no arcing.
2. Sudden disconnection of load and overspeeding of connected generators are not considered.

---

* Power-follow current is the current predominantly of fundamental frequency which flows through an arrester after it has been sparked over by transient overvoltage. It is the current that continues to flow after lightning discharge currents, for example, have been passed through the arrester to ground.

3. Negative-sequence impedances are assumed equal to positive-sequence impedances.

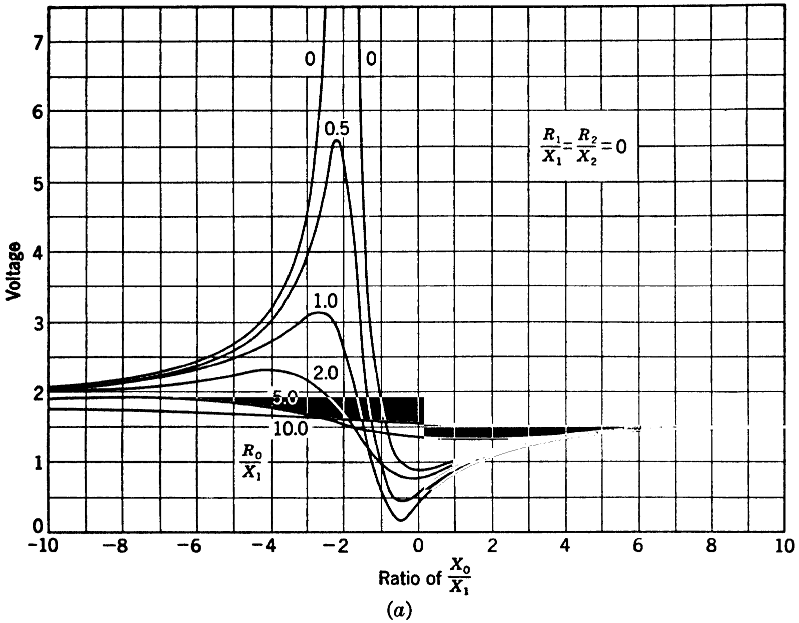
4. The magnitude of the positive-sequence impedance is assumed to be constant for the period in which the overvoltage is to be determined. As the effective impedance of a rotating machine changes with time, it is necessary to consider the impedance under conditions that may result in the greatest overvoltage. On an isolated-neutral system, for example, the greatest overvoltage may be obtained based on impedances corresponding to the synchronous condition rather than the subtransient or transient condition. Because these impedances vary widely after the fault occurs, it is desirable that results be presented in the form of curves so that the range of voltage can be determined.

5. The effects of saturation and corona are not included. Both would tend to reduce the magnitude of overvoltage. The error in neglecting these factors, in general, increases with increase in the magnitude of overvoltage. In the region of resonance, saturation may be an important factor in limiting the excessive fundamental-frequency overvoltages.

6. Overvoltages caused by unusual system conditions are neglected. For example, non-linear circuit instability may be produced as a result of the opening of fuses, or single-pole switches, thereby causing the magnetizing currents of potential or power transformers to flow through line capacitance with resulting overvoltages. Such phenomena will be discussed in Chapter 9.

Based on these assumptions and considerations, it is possible to calculate fundamental-frequency voltages on the unfaulted phases for a single-line-to-ground fault and for a double-line-to-ground fault. A three-phase fault and a line-to-line fault are of no interest, since no overvoltages are produced in view of the foregoing assumptions. The overvoltages calculated are those existing on the unfaulted phases at the point of fault in a three-phase system with phase rotation *a,b,c*. The single-line-to-ground fault is assumed to occur on phase *a*, so that voltages on phases *b* and *c* are of interest. If resistance in the system is negligible, then voltages on phases *b* and *c* are equal in magnitude, but differ in phase. If resistance is present, the general effect is to increase the voltage on phase *c* and decrease it on phase *b*.

The generalized curves for the single-line-to-ground fault are shown in Figs. 1·1 and 1·2. $Z_1 = R_1 + jX_1$ and $Z_0 = R_0 + jX_0$ are the positive- and zero-sequence fundamental-frequency impedances, respectively, viewed from the fault point. The equations from which these



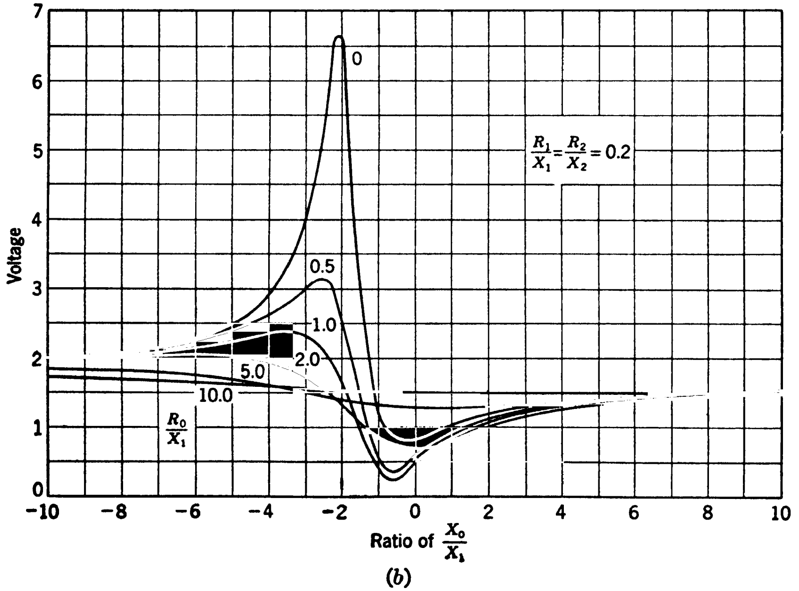


FIG. 1·1. **Phase** *b* **voltage-to-ground at point of fault with conductor-to-ground fault on phase** *a* **versus system impedances. Voltage is expressed as a ratio to the normal-system line-to-neutral voltage.**

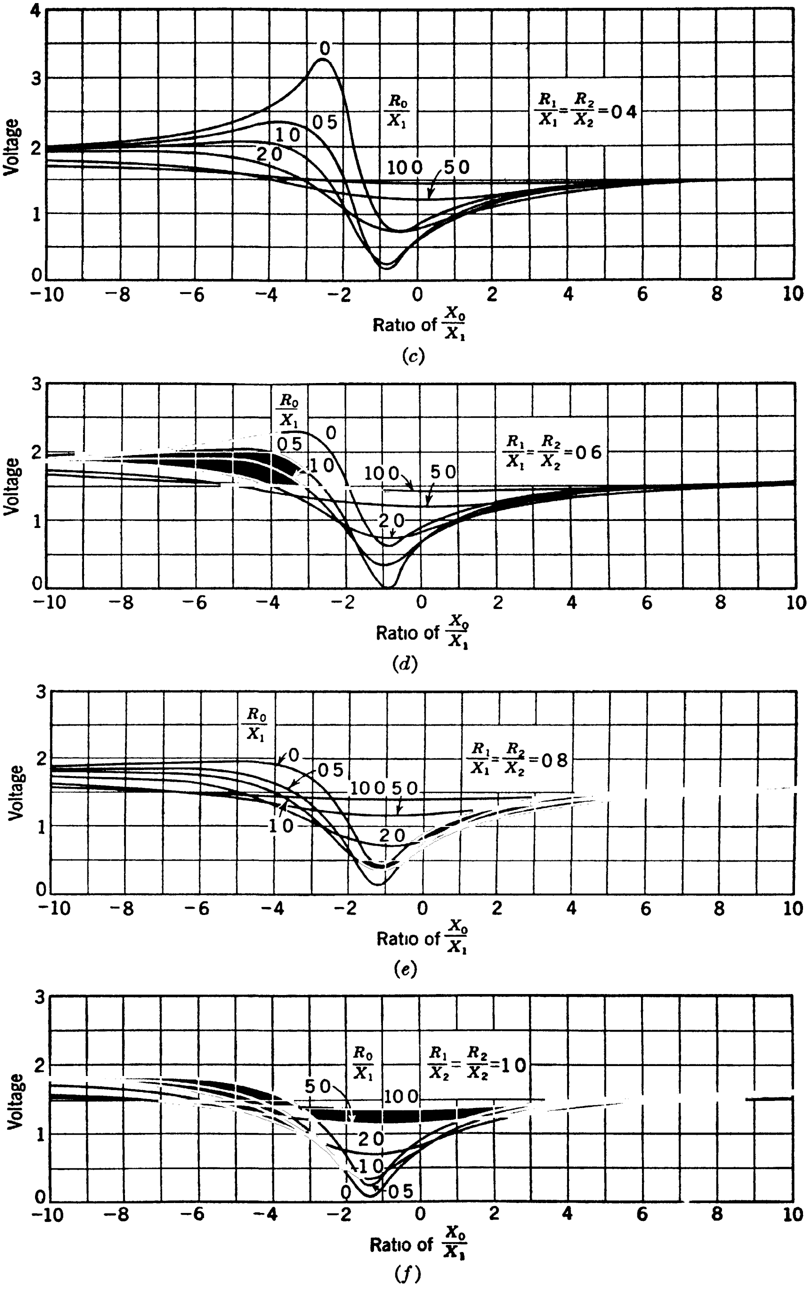


FIG. 1·1 (*Continued*).

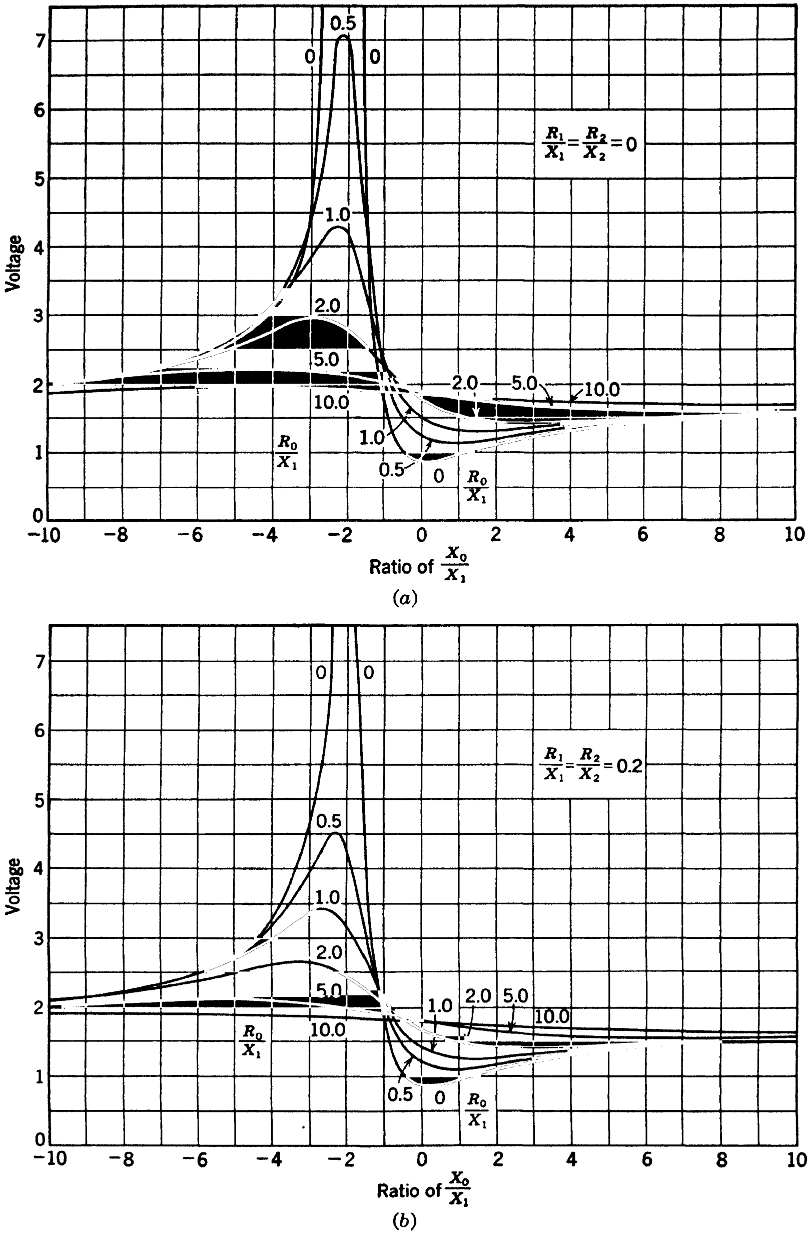
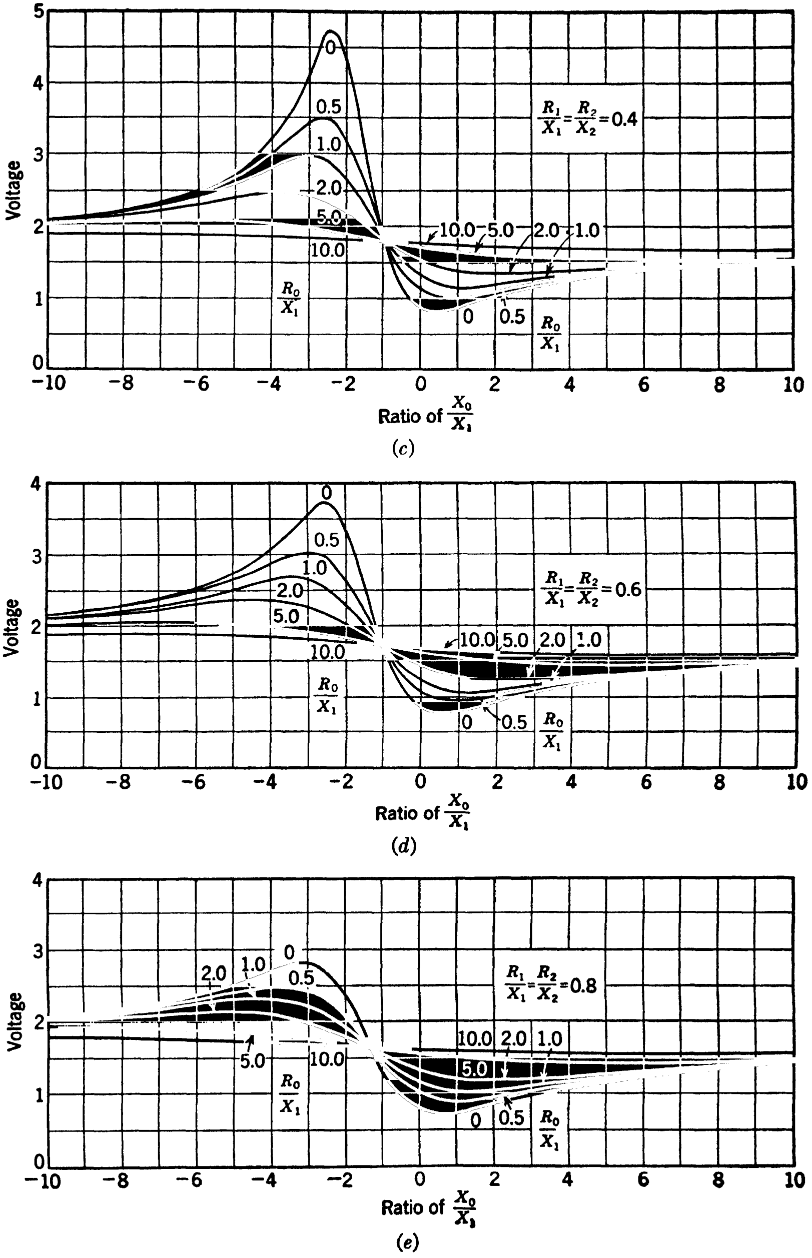


FIG. 1·2. Phase *c* voltage-to-ground at point of fault with conductor-to-ground fault on phase *a* versus system impedances. Voltage is expressed as a ratio to the normal-system line-to-neutral voltage.

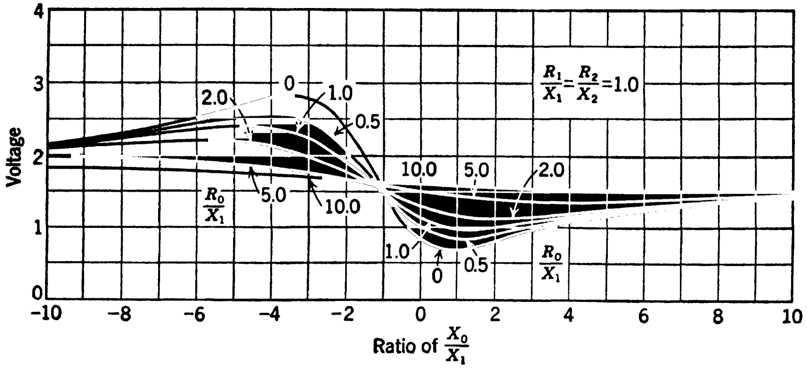


(c)

(d)

(e)

FIG. 1·2 (*Continued*).



(*f*)

FIG. 1·2 (*Continued*).

curves were calculated can easily be derived by means of symmetrical components. For a single-line-to-ground fault on phase *a*, the per-unit voltages on phases *b* and *c* are:

$$V_b = -\frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{Z_0 - Z_1}{2Z_1 + Z_0 + 3R_f} \qquad [1\cdot 1]$$

$$V_c = -\frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{Z_0 - Z_1}{2Z_1 + Z_0 + 3R_f} \qquad [1\cdot 2]$$

In these equations, R_f is the fault resistance. This was assumed to be zero in preparing the curves of Figs. 1·1 and 1·2. However, in many cases the effect of fault resistance is of particular significance. For example, in the region of negative values of X_0/X_1 of Fig. 1·2, if the voltage on phase *c* is plotted as a function of $3R_f/X_f$ with X_0/X_1 as a parameter, a family of curves such as shown in Fig. 1·3 is obtained. Thus for a range of values of $3R_f/X_1$ the fault resistance, which may be tower footing resistance, has the effect of increasing the fundamental-frequency voltage-to-ground on phase *c* above that which would exist with no fault resistance. The maximum increase is about 5 per cent. Similarly it is shown in Fig. 1·4 that the voltage on phase *b* *decreases* in this region of values of $3R_f/X_1$ with respect to the voltage that would otherwise exist if there were no fault resistance. Consequently there is little interest in the phase *b* voltage, whereas the phase *c* voltage is of significance from the standpoint of selecting arresters of proper rating in isolated-neutral systems. For completeness, Figs. 1·5 and 1·6 are included to show the maximum voltages on phases *b* and *c*, respectively, for any value of $3R_f/X_1$.

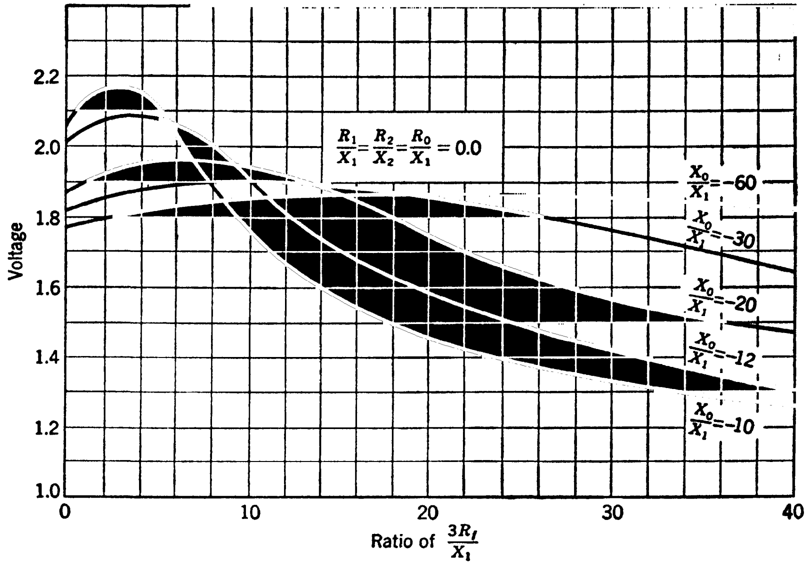


FIG. 1·3. Phase *c* voltage-to-ground at point of fault on phase *a* versus fault resistance. Voltage is expressed as a ratio to the normal-system line-to-neutral voltage.

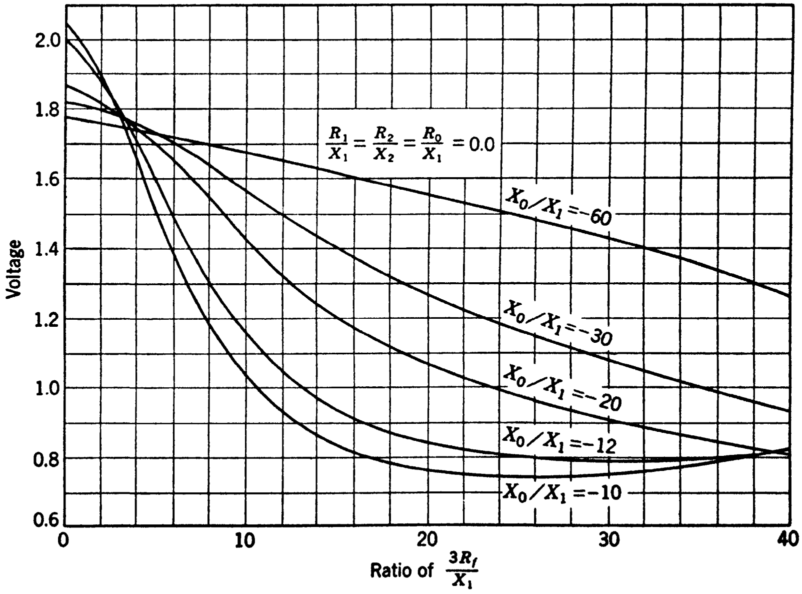


FIG. 1·4. Phase *b* voltage-to-ground at point of fault on phase *a* versus fault resistance. Voltage is expressed as a ratio to the normal-system line-to-neutral voltage.

Although not specifically shown on the foregoing curves, it is significant that if X_0/X_1 is infinite, then fault resistance does not cause an increase in the fundamental-frequency voltage-to-ground on either of the two unfaulted phases. This special case is encountered when the system is grounded through a ground-fault neutralizer.

The effect of fault resistance for a single-line-to-ground fault can be shown in another way, as in Fig. 1·7. Here the fault resistance is shown to have an effect of increasing voltages-to-ground in the

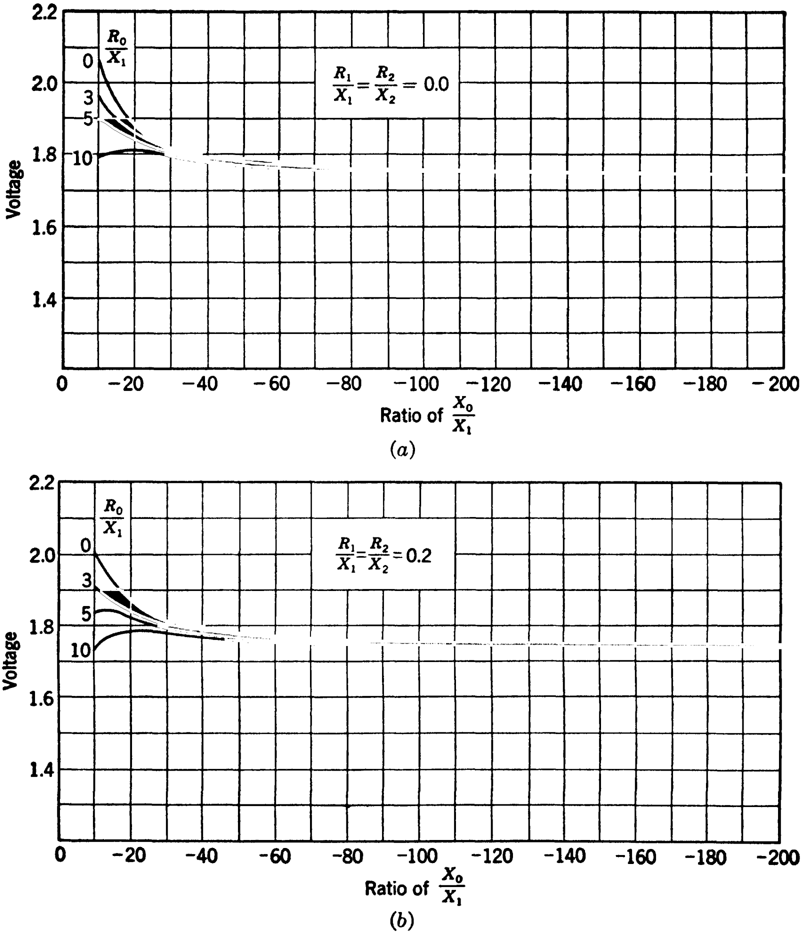
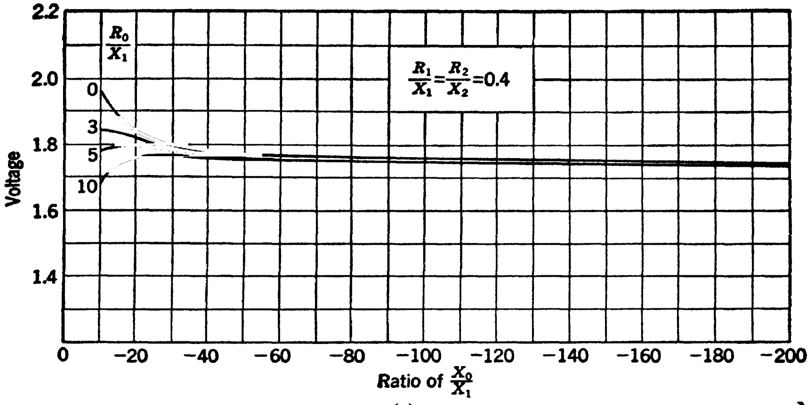
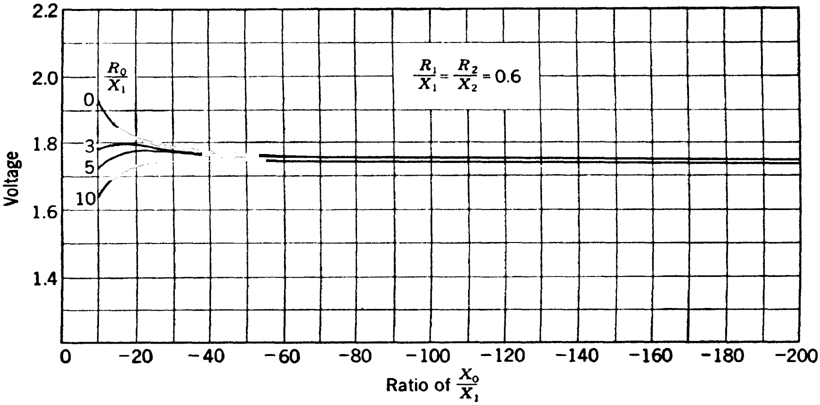


FIG. 1·5. Maximum phase b voltage-to-ground at point of fault on phase a for any fault resistance R_f versus system impedance ratios. Voltage is expressed as a ratio to the normal-system line-to-neutral voltage.



(*c*)

M



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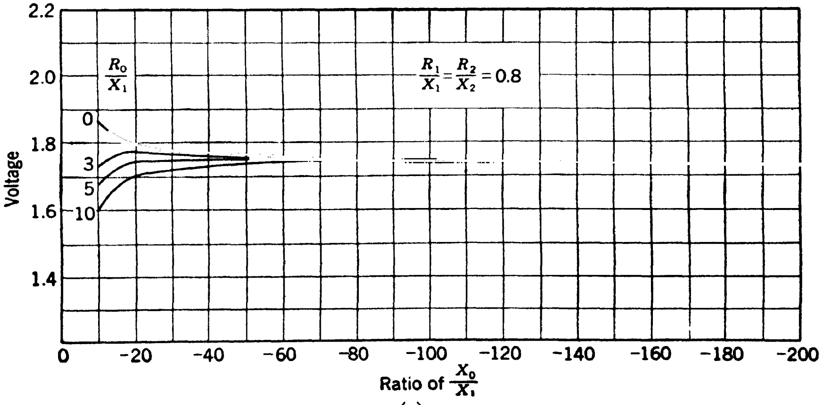

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FIG. 1·5 (*Continued*).

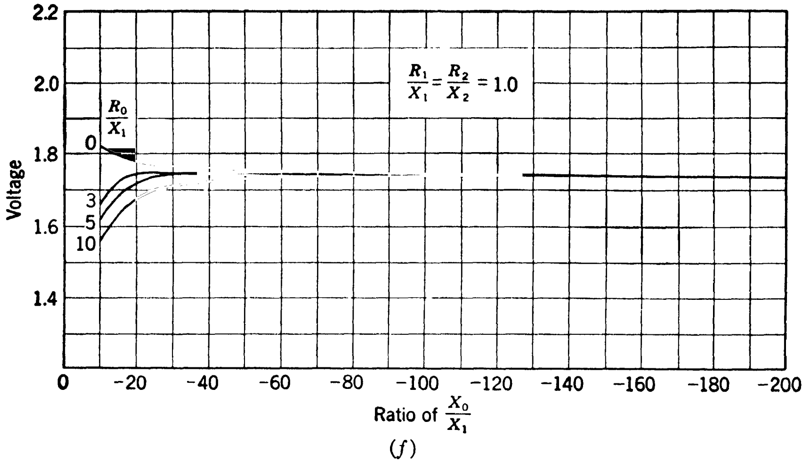


FIG. 1·5 (*Continued*).

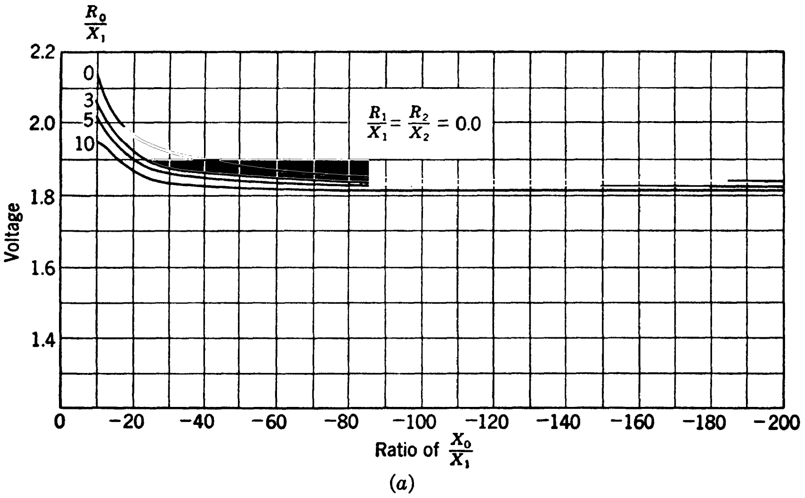
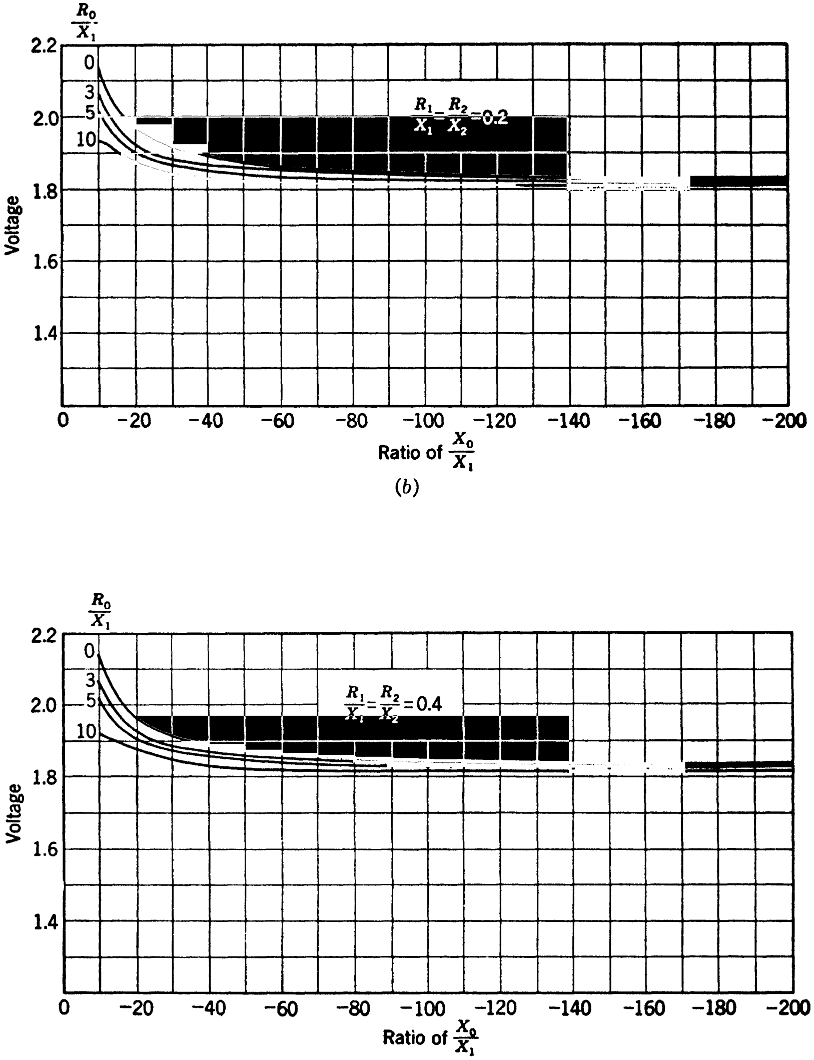


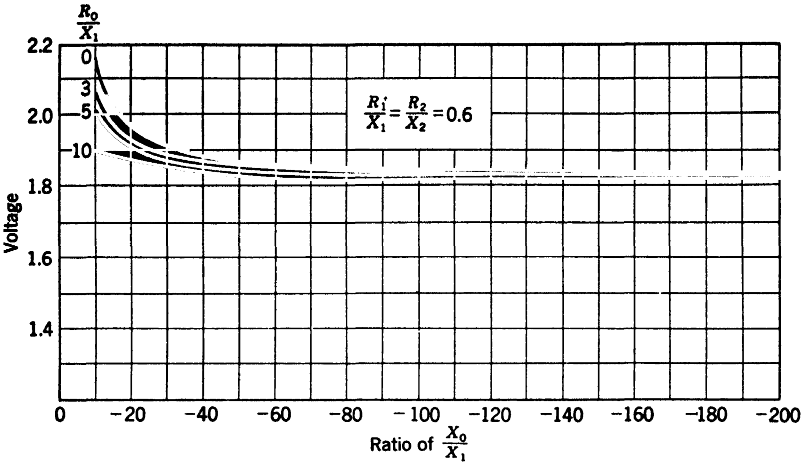
FIG. 1·6. Maximum phase *c* voltage-to-ground at point of fault on phase *a* for any fault resistance R_f versus system impedance ratios. Voltage is expressed as a ratio to the normal-system line-to-neutral voltage.



(b)

(c)

FIG. 1·6 (*Continued*).



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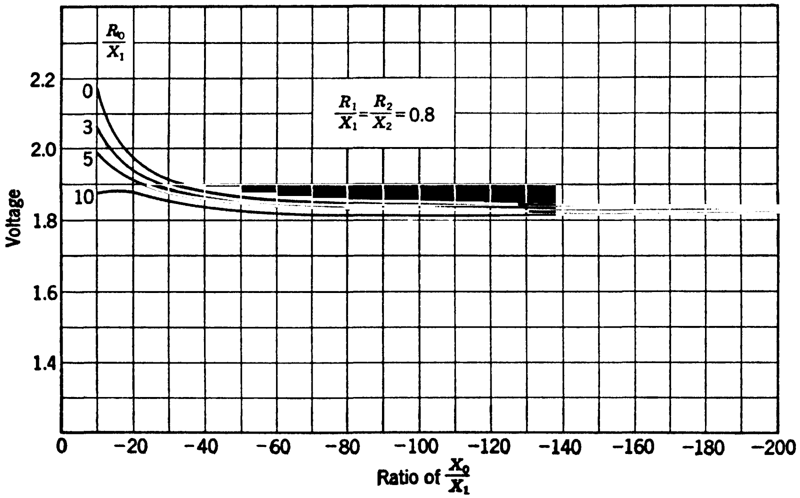

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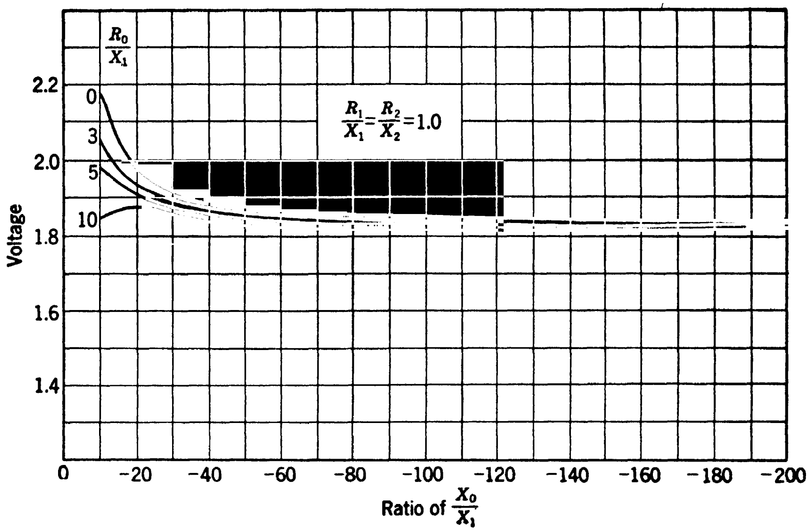

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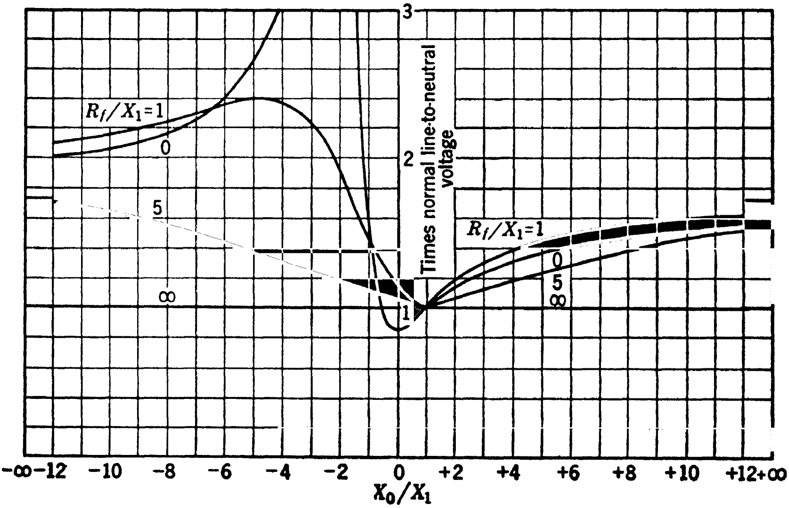

(a)

FIG. 1·7. Fundamental-frequency voltages. Line-to-ground fault. Maximum voltage-to-ground of unfaulted phases at fault. $Z_1 = Z_2 = 0 + jX_1$, $Z_0 = 0 + jX_0$.

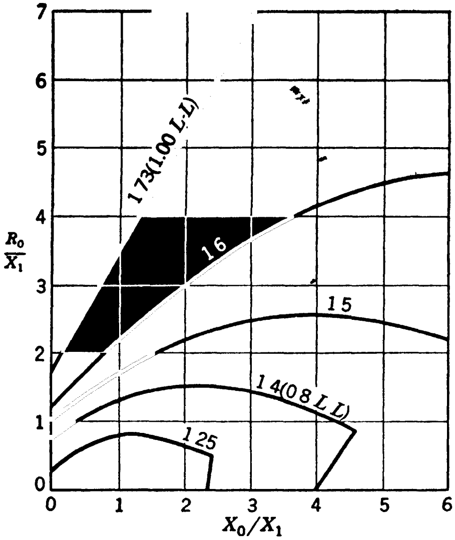

Fig. 1·8 Line-to-ground sustained voltage chart for grounded systems, in the region of small X_0/X_1 and R_0/X_1 Figures on curves are times normal line-to-neutral voltage. In parentheses are given times line-to-line voltage for two significant values Curves are for $X_2 = X_1$ and $R_1 = R_2 = 0$

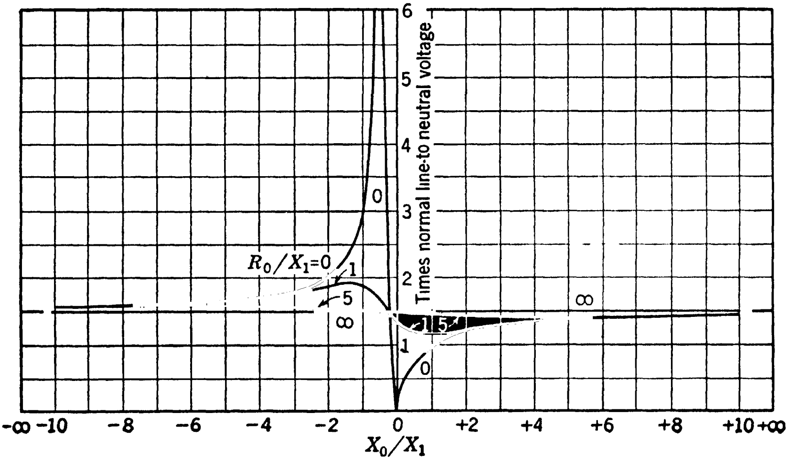

Fig. 1·9. Fundamental-frequency voltages Double-line-to-ground fault Voltage-to-ground of unfaulted phase at the fault $Z_1 = Z_2 = 0 + jX_1$; $Z_0 = R_0 + jX_0$; $R_f = 0$

region of positive values of X_0/X_1 also. This increase is of less general significance, however, because the voltages are considerably smaller in magnitude. The increase should be given due consideration in selecting lightning arresters of proper voltage rating.

The region in which both X_0/X_1 and R_0/X_1 are small is of considerable importance. This region has been plotted in Fig. 1·8 in a different way for the purpose of showing more clearly the effect of zero-sequence impedance upon the maximum sustained voltages to ground for any type of fault and for any value of fault resistance. These curves show the values of R_0/X_1 and X_0/X_1 for which a definite voltage will not be exceeded between any phase and ground, regardless of type of fault or the value of fault resistance.

In general, in practical systems the overvoltages during faults are lower for a double-line-to-ground fault than for a single-line-to-ground fault. This is illustrated in Fig. 1·9. Since these voltages are generally smaller in magnitude for the range of practical X_0/X_1 values, there is little reason for discussing the double-line-to-ground fault condition further as far as the purpose of this book is concerned.

1·3 NATURAL-FREQUENCY VOLTAGES

Natural-frequency voltages appear immediately after the sudden occurrence of a fault. They simply add to the fundamental-frequency voltages discussed in Section 1·2. Since resultant voltages are of greater interest from a practical point of view, it will be preferable to speak of the resultant of the fundamental-frequency and natural-frequency components simply as a transient voltage.

Curves for transient voltages following faults in terms of system impedances viewed from the fault cannot be drawn for the general case as was done for the fundamental-frequency voltages. In the actual system, the transient voltages are affected by the number, connection, and arrangement of the circuits. However, as a first approach which will give an indication of the maximum transient voltage to be expected, a system consisting of a synchronous generator, transformer bank, and transmission line open at the far end may be considered. If the fault is placed near the transformer terminals, and the open line is replaced by its lumped capacitance at the point of fault, the circuit lends itself to analysis quite readily. Such a system is shown in Fig. 1·10.

The voltages on phases *b* and *c* can be calculated for a single-line-to-ground fault on *a* using the following procedure. In Fig. 1·10 let the following nomenclature apply.

$z_1 = r_1 + jx_1$, $z_0 = r_0 + jx_0$ = positive- and zero-sequence fundamental-frequency impedances, respectively, from neutral to the fault point.

$z_n = r_n + jx_n$ = fundamental-frequency impedance between neutral and ground.

x_{c1}, x_{c0} = positive- and zero-sequence fundamental-frequency capacitive reactances, respectively, of transmission line.

$\theta = \theta_0 + t$ = angle between the direct axis of reference machine and axis of phase a.

ω_n = natural frequency in times fundamental frequency.

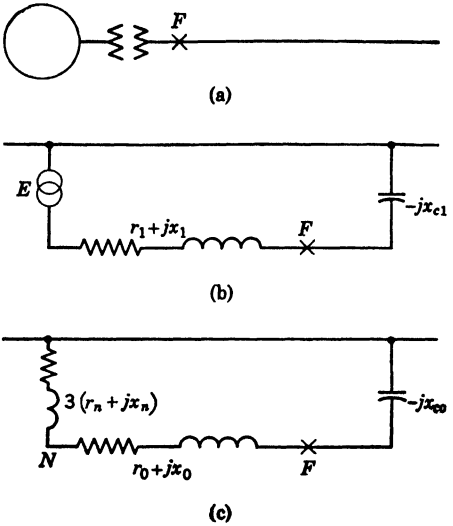

FIG. 1·10. (*a*) One-line diagram of power system consisting of synchronous machine, transformer bank, and transmission line. (*b*) Approximate positive-sequence fundamental-frequency impedance diagram. (*c*) Approximate zero-sequence fundamental-frequency impedance diagram.

The instantaneous phase voltages at any point in a balanced three-phase system in per-unit of crest voltages to neutral or to ground at that point are:

$$e_a = -\sin\theta$$

$$e_b = \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

$$e_c = \frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta$$

Let e and i with appropriate subscripts refer to instantaneous values of voltages-to-ground at the fault and currents flowing into the fault, respectively. In reference 9 are given the relations for transformation from three-phase quantities to symmetrical-component quantities for instantaneous currents and voltages, including the transient as well as the fundamental-frequency components. Using the generalized method of symmetrical components, the following equations (see reference 2) are obtained:

Line-to-Ground Fault (Phase a)

$$i_{a1} = i_{a2} = i_{a0} = \frac{-\sin\theta}{2Z_1(p) + Z_0(p) + 3R_f} \qquad [1\cdot3]$$

$$e_{a0} = -i_{a0}Z_0(p) = \frac{Z_0(p)\sin\theta}{2Z_1(p) + Z_0(p) + 3R_f} \qquad [1\cdot4]$$

$$e_b = \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta + \frac{Z_0(p) - Z_1(p)}{2Z_1(p) + Z_0(p) + 3R_f}\sin\theta \qquad [1\cdot5]$$

$$e_c = \frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta + \frac{Z_0(p) - Z_1(p)}{2Z_1(p) + Z_0(p) + 3R_f}\sin\theta \qquad [1\cdot6]$$

Double-Line-to-Ground Fault (Phases b and c)

$$i_{a0} = -(i_{a1} + i_{a2}) = \frac{\sin\theta}{Z_1(p) + 2Z_0(p) + 6R_f} \qquad [1\cdot7]$$

$$e_{a0} = -i_0Z_0(p) = \frac{-Z_0(p)\sin\theta}{Z_1(p) + 2Z_0(p) + 6R_f} \qquad [1\cdot8]$$

$$e_a = -\frac{3Z_0(p) + 6R_f}{Z_1(p) + 2Z_0(p) + 6R_f}\sin\theta \qquad [1\cdot9]$$

Line-to-Line Fault (Phases b and c)

$$e_a = -\sin\theta \qquad [1\cdot10]$$

$$e_b = e_c = \tfrac{1}{2}\sin\theta \qquad [1\cdot11]$$

To obtain the natural-frequency components of equations [1·3] to [1·9] it is necessary to replace $Z_1(p)$ and $Z_0(p)$ by their operational expressions in terms of system constants. For the system in Fig. 1·10,

$$Z_1(p) = \frac{(r_1 + px_1)\dfrac{x_{c1}}{p}}{r_1 + px_1 + \dfrac{x_{c1}}{p}} \qquad [1\cdot12]$$

$$Z_0(p) = \frac{[r_0 + 3r_n + p(x_0 + 3x_n)]\,\dfrac{x_{c0}}{p}}{r_0 + 3r_n + p(x_0 + 3x_n) + \dfrac{x_{c0}}{p}} \qquad [1\cdot 13]$$

If resistance is neglected, $Z_1(p)$, $Z_0(p)$, Z_1, and Z_0 become for Fig. 1·10,

$$Z_1(p) = \frac{px_1x_{c1}}{p^2x_1 + x_{c1}} \qquad [1\cdot 14]$$

$$Z_0(p) = \frac{p(x_0 + 3x_n)x_{c0}}{p^2(x_0 + 3x_n) + x_{c0}} \qquad [1\cdot 15]$$

$$Z_1 = \frac{jx_1x_{c1}}{x_{c1} - x_1} = jX_1 \qquad [1\cdot 16]$$

$$Z_0 = j\frac{(x_0 + 3x_n)x_{c0}}{x_{c0} - (x_0 + 3x_n)} = jX_0 \qquad [1\cdot 17]$$

Then for a line-to-ground fault at F on phase a in Fig. 1·10, the instantaneous zero-sequence voltage at the fault in per-unit of normal line-to-neutral peak voltage at the fault, obtained by substituting equations [1·14] and [1·15] in equation [1·4] and solving the operational equation, is

$$e_0 = \frac{Z_0}{2Z_1 + Z_0}\sin\theta$$
$$- \frac{2\left(\dfrac{x_0 + 3x_n}{x_{c0}} - \dfrac{x_1}{x_{c1}}\right)\left(\sin\theta_0\cos\omega_nt + \dfrac{1}{\omega_n}\cos\theta_0\sin\omega_nt\right)}{\left(\dfrac{2}{x_{c0}} + \dfrac{1}{x_{c1}}\right)\left[2x_1 + x_0 + 3x_n - x_1(x_0 + 3x_n)\left(\dfrac{2}{x_{c0}} + \dfrac{1}{x_{c1}}\right)\right]} \qquad [1\cdot 18]$$

In terms of X_0 and X_1

$$e_0 = \frac{X_0}{2X_1 + X_0}\sin\theta$$
$$- \frac{2\left(\dfrac{x_{c1}}{x_{c0}}X_0 - X_1\right)\left(\sin\theta_0\cos\omega_nt + \dfrac{1}{\omega_n}\cos\theta_0\sin\omega_nt\right)}{\left(1 + 2\dfrac{x_{c1}}{x_{c0}}\right)(2X_1 + X_0)} \qquad [1\cdot 19]$$

With $x_{c0} = x_{c1}$, equation [1·19] checks that given by Fallou.[8]

$$e_b = \frac{3}{2} e_0 + \frac{\sqrt{3}}{2} \cos \theta \tag{1·20}$$

$$e_c = \frac{3}{2} e_0 - \frac{\sqrt{3}}{2} \cos \theta \tag{1·21}$$

In these expressions

$$\omega_n = \sqrt{\frac{2x_1 + x_0 + 3x_n}{2x_1(x_0 + 3x_n)\left(\dfrac{1}{x_{c0}} + \dfrac{1}{2x_{c1}}\right)}} = \sqrt{\frac{\dfrac{X_0}{X_1}\left(\dfrac{x_{c1}}{x_1} + 2\dfrac{x_{c1}}{x_{c0}}\right) + 2\left(\dfrac{x_{c1}}{x_1} - 1\right)}{\dfrac{X_0}{X_1}\left(2\dfrac{x_{c1}}{x_{c0}} + 1\right)}} \tag{1·22}$$

Similarly for a double-line-to-ground fault at F on phases b and c, the instantaneous zero-sequence and phase a voltage-to-ground at the fault in per-unit of normal line-to-neutral peak voltage at the fault, are obtained by substituting equations [1·14] and [1·15] in equations [1·8] and [1·9] and solving the operational equations. Expressed in terms of the positive- and zero-sequence reactances viewed from the fault,

$$e_0 = -\frac{X_0}{X_1 + 2X_0} \sin \theta + \frac{\left(\dfrac{x_{c1}}{x_{c0}} X_0 - X_1\right)\left(\sin \theta_0 \cos \omega_n t + \dfrac{\cos \theta}{\omega_n} \sin \omega_n t\right)}{\left(2 + \dfrac{x_{c1}}{x_{c0}}\right)(X_1 + 2X_0)} \tag{1·23}$$

$$e_a = 3e_0 \tag{1·24}$$

In equation [1·23]

$$\omega_n = \sqrt{\frac{x_1 + 2(x_0 + 3x_n)}{2x_1(x_0 + 3x_n)\left(\dfrac{1}{2x_{c0}} + \dfrac{1}{x_{c1}}\right)}} \tag{1·25}$$

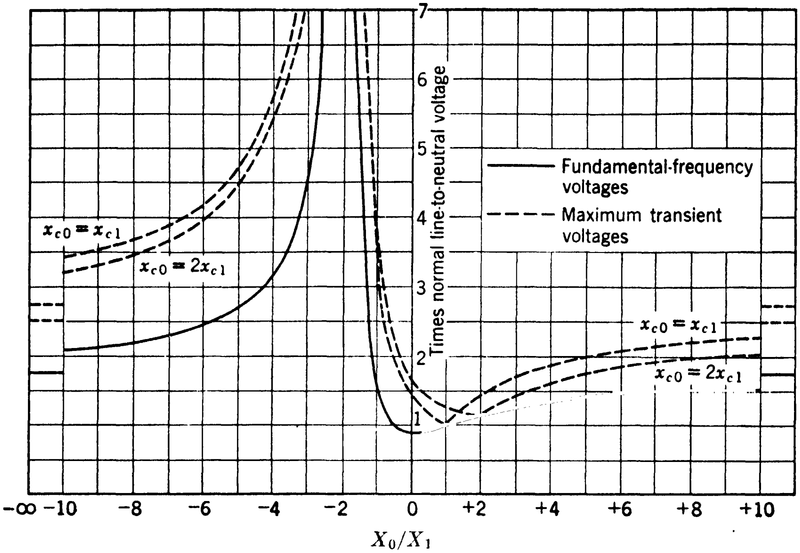

FIG. 1·11. Transient voltages with resistance neglected for system shown in Fig. 1·10. Line-to-ground fault. Maximum voltage-to-ground of unfaulted phases at the fault.

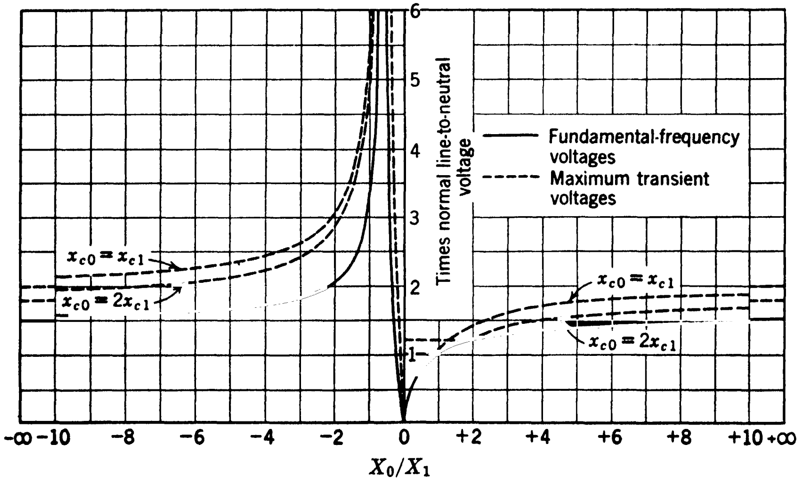

FIG. 1·12. Transient voltages with resistance neglected for system shown in Fig. 1·10. Double-line-to-ground fault. Maximum voltage-to-ground of unfaulted phase at the fault.

The equations [1·19], [1·20], [1·21], [1·23], and [1·24] were used to plot curves of transient voltage as a function of X_0/X_1 shown in Figs. 1·11 and 1·12, for single-line-to-ground and double-line-to-

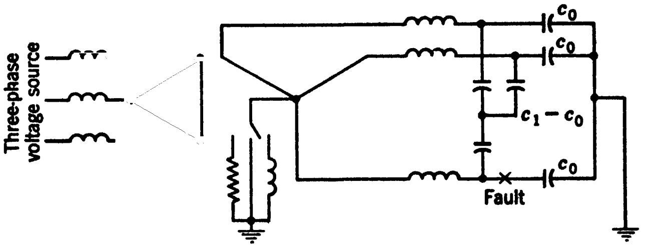

FIG. 1·13. Miniature equivalent system.

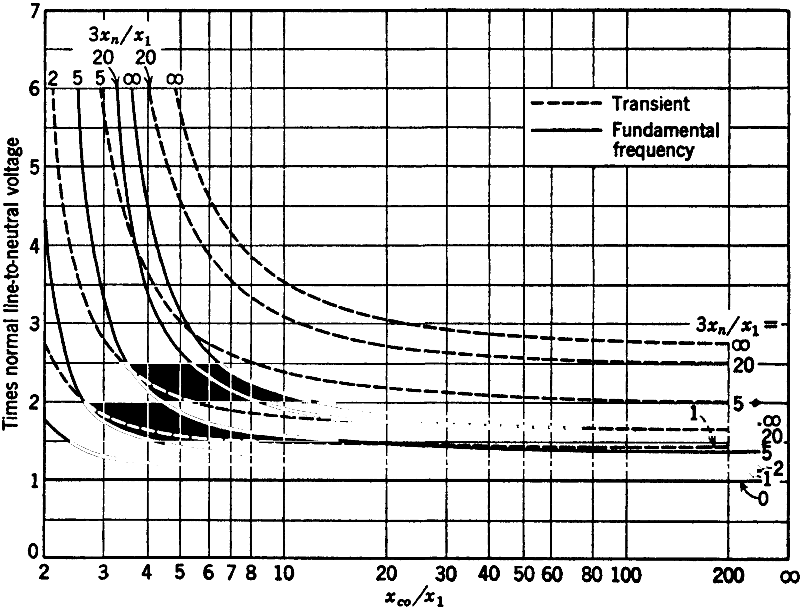

FIG. 1·14. Transient and fundamental-frequency voltages with resistance neglected for system shown in Fig. 1·10 with reactance grounded neutral. Line-to-ground fault. Maximum voltage-to-ground of unfaulted phases at the fault. $x_0 = x_1$, $x_{c0} = x_{c1}$.

ground faults, respectively. For convenience, the fundamental-frequency voltage is shown also. The maximum transient voltage is that obtained by adding the peak fundamental-frequency voltage to the peak natural-frequency voltage on the assumption that, since there is no damping in the circuit, sooner or later these two components can have their maximum value with the same polarity at the same instant. The maximum natural-frequency voltage is obtained when $\theta_0 = 90°$ for normally encountered values of ω_n; this value was used in calculating the curves of Figs. 1·11 and 1·12. These curves can be used to determine the maximum transient voltages with resistance neglected when the neutral is solidly grounded or grounded through any reactance, including a ground-fault neutralizer. For a solidly grounded neutral, $x_n = 0$. For a ground-fault neutralizer with resistance neglected, $3x_n + x_0 = x_{c0}$ and $X_0 = \infty$.

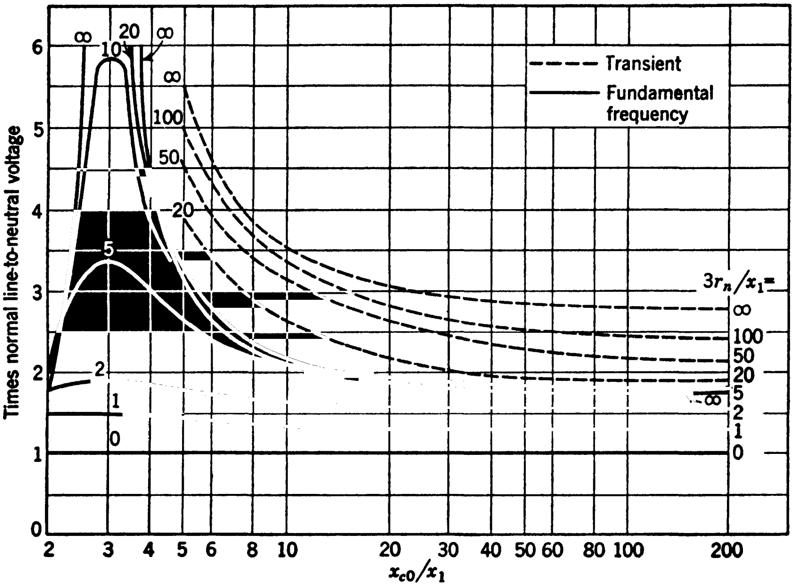

FIG. 1·15. Transient and fundamental-frequency voltages for system shown in Fig. 1·10 with resistance grounded neutral. Line-to-ground fault. Maximum voltage-to-ground of unfaulted phases at the fault. $x_0 = x_1$, $x_{c0} = x_{c1}$.

If resistance is included in the analysis, solutions are possible and curves could be plotted to show the resultant transient overvoltages. However, such calculations are extremely time consuming, and therefore other means of evaluating the transient voltages in systems where resistance is of importance were sought. To meet this situation a device referred to as a transient analyzer was developed. In principle it consists of elements of impedance (resistance, inductance, and capacitance) arranged so that they can be connected in almost any desired manner to represent electrically the system or circuit under investigation. By means of synchronous switches and a cathode-ray oscillograph it is possible to apply faults repeatedly and observe the resultant transient effects. A detailed description of this device will be postponed until the latter part of this chapter.

By means of the transient analyzer it is possible to set up circuits such as that shown in Fig. 1·13. Curves such as those shown in Figs. 1·14 and 1·15 can be obtained readily. The nomenclature used is the same as that used in connection with Fig. 1·10.

1·4 EFFECT OF METHOD OF GROUNDING

Comparison of Resistance and Reactance Grounding. A comparison of the effects of resistance and reactance grounding on the fundamental-frequency fault voltages following a line-to-ground fault can be obtained from Figs. 1·1 and 1·2 when the positive- and zero-sequence impedances viewed from the fault are known.

To compare the effects of reactance and resistance grounding on transient overvoltages following faults, the curves of Figs. 1·14 and 1·15 may be used. These curves are plotted in terms of the impedances indicated in Fig. 1·10 and show the magnitudes of the transient voltages which may be obtained following a line-to-ground fault when the neutral is grounded through reactance and through resistance, respectively, with $x_0 = x_1$ and $x_{c0} = x_{c1}$.

As will be noted, exceptionally high voltages are not obtained until the capacitive reactance x_{c0} becomes small compared with x_1. This condition corresponds to a system having a very large amount of line-charging capacity compared with the connected generation. Also it will be noted that, for the conditions assumed, transient voltages do not exceed twice the fundamental-frequency voltages with either reactance or resistance grounding.

The sustained voltages for resistance-grounded systems are generally higher than those for corresponding reactance-grounded systems. This is particularly true if the neutral grounding ohms are selected to give the same value of short-circuit current, as can be seen from an analysis of Figs. 1·14 and 1·15. For high ohmic values of neutral grounding impedance, the transient voltages may be higher for reactance-grounded than for resistance-grounded systems. For low ohmic values, the natural-frequency component of voltage for resistance-grounded systems decreases and becomes negligible for values of $3r_n/x_1 < 5$. Accordingly, a reactance-grounded system may not subject the protective equipment to as high sustained voltages as a resistance-grounded system, but from the standpoint of natural-frequency overvoltages, particularly those associated with the occurrence of a fault (and with switching phenomena as will be shown later), the resistance-grounded system may be more desirable.

In connection with the neutral grounding impedance, the voltages at the neutral are of importance. Therefore Figs. 1·16 and 1·17 are

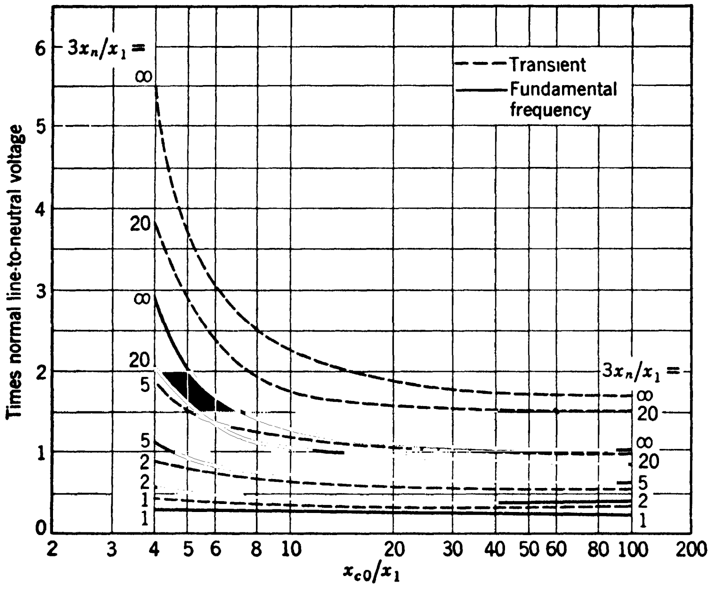

FIG. 1·16. Transient and fundamental-frequency voltages with resistance neglected for system shown in Fig. 1·10 with reactance grounded neutral. Line-to-ground fault. Maximum voltage-to-ground at the neutral. $x_0 = x_1$, $x_{c0} = x_{c1}$.

included to show the maximum transient voltage as well as the fundamental-frequency voltage at the neutral for the system in Fig. 1·10. It is clear that exceptionally high voltages are not obtained at the neutral except for systems that have a relatively large amount of connected line in proportion to the positive-sequence inductive reactance. The curves, in general, have the same shape as those in Figs. 1·14 and 1·15, as it is essentially the shift or rise in neutral voltage that causes the overvoltage on the open phases.

Since the magnitudes of overvoltages that are obtained are considerably affected by the method of grounding, the items in the following discussion are classified accordingly.

1. *Solidly Grounded System.* There are many degrees of *solid grounding,* and the term means very little as far as the overvoltages are concerned, as it depends on the number, location, and kilovolt-ampere capacity of the neutral grounding points. However, if the grounded-neutral system is considered to be one in which grounded-neutral rated arresters may be used (line-to-ground sustained voltages not to exceed 140 per cent of normal), it becomes apparent that the ratio of X_0/X_1 should be kept below about 3 and R_0/X_1 should be kept below about 1. See Figs. 1·1, 1·2, and 1·8. Accordingly, such a solidly grounded system would generally have a large percentage of all transformers or machines with neutrals connected directly to ground with no (or, at most, very little) impedance intentionally inserted in any neutral.

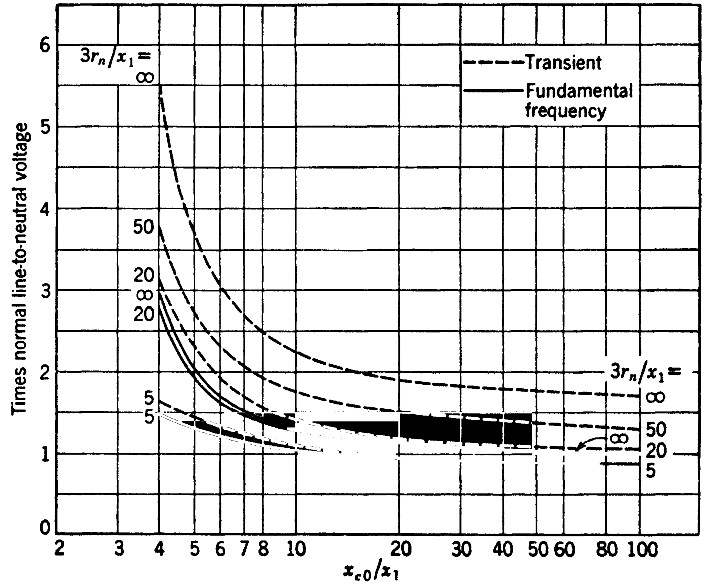

Fig. 1·17. Transient and fundamental-frequency voltages for system shown in Fig. 1·10 with resistance grounded neutral. Line-to-ground fault. Maximum voltage-to-ground at neutral. $x_0 = x_1$, $x_{c0} = x_{c1}$.

Systems operating in this classification, that is with X_0/X_1 less than 3 and R_0/X_1 less than 1, have a maximum transient line-to-ground voltage on the unfaulted phases not exceeding 2.0 times normal (see Figs. 1·11 and 1·12). The maximum neutral-to-ground transient voltage at any point in the system, for example, the neutral of an ungrounded bank, is about normal line-to-ground voltage due to the sudden occurrence of a solid fault. (Refer to Figs. 1·16 and 1·17.)

2. *Neutral Grounded through Reactance.* When a system is grounded through reactance less than that of a ground-fault neutralizer, the zero-sequence impedance viewed from the fault is inductive rather than capacitive and the zero-sequence resistance is relatively small; accordingly, the fundamental-frequency phase-to-ground voltages will not exceed normal line-to-line voltage, and the neutral-to-ground voltage will not exceed normal line-to-neutral voltage.

Following the sudden occurrence of a fault, systems with neutrals grounded through reactance will have maximum transient voltages-to-ground on the unfaulted phase not exceeding 2.73 times normal. The voltage-to-ground at the neutral will not exceed 1.67 times normal line-to-neutral voltage. See Figs. 1·11, 1·12, 1·16, and 1·17.

3. *Neutral Grounded through Resistance.* When a system is grounded through resistance, the zero-sequence impedance viewed from the fault may be inductive or capacitive, depending on the number and location of the neutral grounding points and the amount of connected line or cable. With low-resistance grounds, X_0 will ordinarily be positive and the fundamental-frequency phase-to-ground voltages will, in general, not exceed normal line-to-line voltage, and the neutral-to-ground voltages will not exceed normal line-to-neutral voltage. With high-resistance grounds, X_0 may be negative. In that event, phase-to-ground voltages may be greater than normal line-to-line voltages, and neutral-to-ground voltages greater than normal-line-to-neutral voltages. See Figs. 1·1 and 1·2.

If low-resistance grounding is used, the natural-frequency voltages are practically eliminated and the maximum voltages are essentially the fundamental-frequency voltages which, however, are generally higher than the fundamental-frequency voltages obtained with corresponding values of neutral-grounding reactance.

4. *Systems Grounded through Ground-Fault Neutralizer.* For systems grounded through ground-fault neutralizers, with resistance neglected, X_0 is infinite; with resistance included, R_0 is very large, while X_0 is also very large and by tuning can be negative or positive. Based on either assumption, the fundamental-frequency voltages on the unfaulted phases at the fault following a line-to-ground fault are essentially line-to-line voltages. They are not increased by the presence of fault resistance. The maximum transient voltages-to-ground of the unfaulted phases are less than 2.73 times normal, and of the neu-

tral-to-ground less than 1.67 times normal line-to-neutral voltage. See Figs. 1·11 and 1·14.

5. *Isolated Neutral.* In an isolated-neutral system X_0 is negative and of the order of magnitude of the zero-sequence capacitive reactance while R_0/X_1 is relatively small. From Figs. 1·1 and 1·2, the fundamental-frequency voltages obtained may be in excess of normal line-to-line voltage; and in some cases, particularly when the system is large in extent, they may be considerably in excess, so that a very undesirable condition is created when faults occur. The fact that such voltages may be obtained makes it highly desirable that under no condition of operation shall a system lose its grounding points, if by so doing it is likely to be in the region of resonance or near resonance. Otherwise, damage to the protective equipment or flashover of major equipment may result.

1·5 THE TRANSIENT ANALYZER

The major parts of the equipment that made up the original transient analyzer are shown in Fig. 1·18.

Briefly, the device consists of the following component parts:

1. Tapped inductive reactors for simulating transformer, generator, or other lumped reactances.
2. Capacitors of suitable sizes for simulating the capacitance of transmission lines, cables, or capacitor banks.
3. Resistors of suitable ohmic range to simulate resistance loads, losses in transmission lines, or losses of other kinds.
4. Artificial transmission-line units, for simulating transmission-line behavior under transient conditions.
5. Single-phase 1:1 turn ratio transformers having special saturation characteristics to simulate those of large power transformers.
6. A three-phase 60-cycle, 110-volt sine-wave variable-speed generator for energizing the miniature system.
7. Various recurring synchronous switches (both electronic and mechanical) for performing the desired switching operations in the miniature systems.
8. A cathode-ray oscillograph for observing the transient voltages and currents resulting from an imposed fault or switching condition.
9. A means for photographing the trace appearing on the cathode-ray oscillograph screen when a permanent record is desired.
10. Two ground-fault neutralizer coils designed to have per-unit saturation characteristics closely duplicating those of large full-voltage units.

11. Miniature Thyrite arresters. These are small wafers of Thyrite material about $\frac{5}{8}$ inch in diameter which can be stacked in a variety of combinations so that it is possible to simulate an arrester of almost any voltage rating.

Some of these component parts require no further description. Certain items in the preceding list have been selected or designed to have characteristics that make them particularly suitable for this type of work. Only those items will be selected for further consideration.

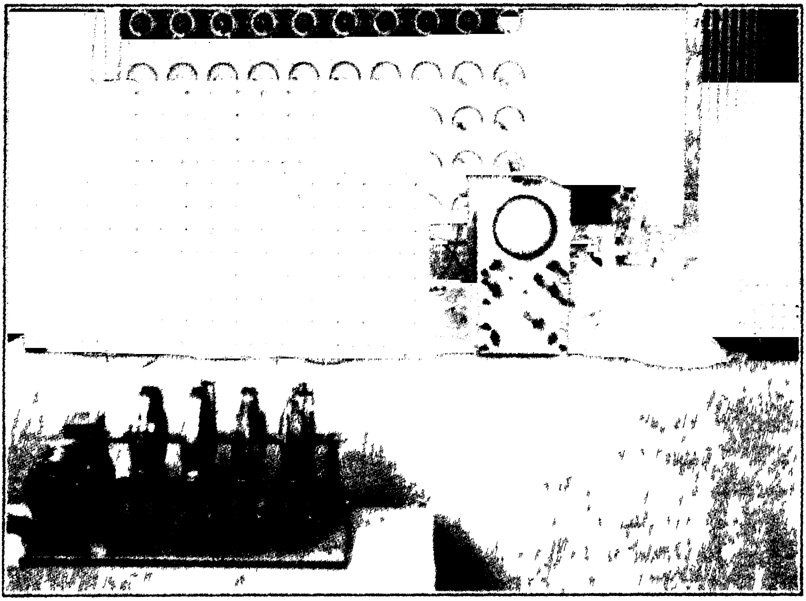

FIG 1·18. The electric circuit transient analyzer.

Each of the artificial transmission-line units was designed to represent ten miles of typical three-phase overhead line. Sixty such units are included in the device, making it possible to simulate 600 miles of actual three-phase transmission line. The equivalent circuit used in each line section is shown in Fig. 1·19.

The three-phase generator is large enough so that its terminal voltage remains essentially constant regardless of the transient conditions imposed in the miniature system which it energizes.

The recurring switching devices used are of several types, the type being used depending on the problem under investigation. For certain problems, the electronic switches have definite advantages in flexibility and precision, particularly when circuit interruption at a current zero is desired. This advantage becomes more apparent when high natural frequencies are encountered.

A synchronous motor-driven mechanical switch having four separate drums each having two contactors is used advantageously in many cases. Each contactor is equipped with a vernier control which changes the instant of switching 36 electrical degrees for each revolution of the control knob. The drum rotates at a speed of 360 rpm when the motor is energized from a 60-cycle voltage source. Generally this is used directly to perform a required switching operation, but occasionally it has been found desirable to use this switch to perform switching operations in the grid circuits of thyratron tubes which in turn would simulate the desired switching operations in the miniature system. The synchronous switch drums can be driven at other speeds simply by changing the gear ratio between driving motor and the drum shaft.

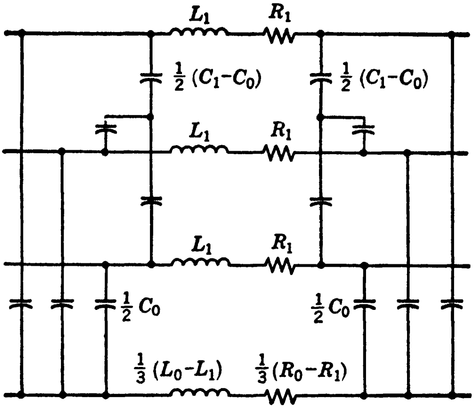

FIG. 1·19. Artificial line section connections.

The cathode-ray oscillograph used in the analyzer is of standard design. Provision has been made for eliminating certain portions of the trace from the screen, leaving only the portion desired for observation. In addition, a selsyn is used in the sweep synchronizing circuit to control the position of the trace on the screen. A sufficient amount of amplification is built into the oscillograph, so that, in addition to providing a means for directly measuring voltages, generally a ½-ohm resistor affords adequate voltage drop for measuring currents as well.

Items of equipment requiring further description will be discussed as the need arises in later chapters.

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2 EFFECT OF SALIENCY ON OVERVOLTAGES DURING FAULTS

2·1 INTRODUCTION

In the preceding chapter it was shown that, in general, unbalanced faults on a power system cause overvoltages on the unfaulted phases. These overvoltages were evaluated for systems that were made up of static impedance elements. In these networks, voltages of only fundamental frequency were *generated* even under unbalanced fault conditions. For systems that have no water-wheel generators in them, or, more specifically, that have no water-wheel generators without adequate amortisseur windings, the analysis as presented in Chapter 1 is sufficient. However, if water-wheel generators without amortisseur windings are connected in the system under study, then the overvoltages during unbalanced faults may be increased considerably because of the saliency effect which results in additional generated voltage components of higher harmonic order than the fundamental. The overvoltage increase may be abnormally high if the load on the system is predominantly capacitive.

Briefly stated, there are two reasons for this additional increase in overvoltage during unbalanced faults on a salient-pole machine:

1. The difference in permeance of the magnetic circuit in the direct and quadrature axes.

2. The capacitance of the line (or equivalent) at the terminals of the machine.

The first of these is the basic cause that produces, under unbalanced fault conditions, distorted (i.e., fundamental-frequency plus higher harmonic) voltages on the unfaulted phases which may have high peak values even with no terminal impedance present, capacitive or otherwise. The second may lead to resonance or partial resonance between the reactance of the machine and the terminal capacitance which will produce still greater distortion of the voltages on the unfaulted phases. The conditions for resonance will depend not only on the circuit constants but also on the type of fault as will be shown subsequently.

The qualitative effects have been known for some time, and quantitative analysis has disclosed much since about 1935. Mandl[1] has made an extended study of overvoltages caused by single-phase short circuits with terminal capacitance and has shown the existence of resonant points and the possibility of obtaining high voltages on the faulted phases of machines with electrically unsymmetrical rotors. Wagner[2] has made an experimental study of the problem. Doherty and Nickle[3] have shown that the magnitude of open-phase voltage during single-phase short circuit of an unloaded machine is a function principally of the ratio of quadrature-axis reactance to direct-axis reactance; Park and Skeats[4] similarly have shown how this ratio affects recovery voltage. Still another investigation[5] of double-line-to-ground faults has given expressions for the open-phase voltage for that case.

2·2 A SPECIFIC EXAMPLE—A DIFFERENTIAL-ANALYZER STUDY

In order to present clearly the consequences of the saliency effect on overvoltages during faults, a specific system subjected to a line-to-line fault will be discussed. The system is that which was studied in reference 6, and most of the material in this chapter is taken from this reference. The system is shown in Fig. 2·1. Actually it may be thought of as a system consisting of a generator, a transformer, and an open transmission line. A line-to-line fault was applied by closing the switch at the terminals of the transformer. This type of fault was selected for study since it may result in some of the highest overvoltages when the saliency effect is included. This is in contrast with what was pointed out in Chapter 1, that, neglecting the saliency effect and with $Z_1 = Z_2$, there is no overvoltage on the unfaulted phase during a line-to-line fault. In faults not involving ground the presence of overvoltages depends primarily on the fact that the synchronous machines supplying the fault current are not electrically symmetrical-rotor machines.

The system of Fig. 2·1 was studied by means of a differential analyzer. The detailed equations, nomenclature, and connection diagrams for this method of solution will be given in Appendix I. An approximate method of calculation of such overvoltages for any type of fault together with equivalent circuits will be given in this chapter. For the first part of this chapter it will be preferable to summarize the results and to tabulate the machine and system constants used to obtain these results. With this arrangement it will be possible for the reader to make use of these results without following through the derivation of the mathematical relationships involved. For the benefit of those who may be interested in the basic theory, Appendix I will include the derivation of the pertinent equations.

The procedure in the analysis was to select certain machine constants and then, for a given value of terminal capacitance, make a run on the differential analyzer for some arbitrary initial conditions. It was soon established that the highest overvoltages, as one might predict, were obtained for the short circuit occurring at the instant of maximum flux linkage in the shorted phases except near the third-harmonic resonant point where there was no appreciable difference in overvoltage with angle of fault application. Consequently, after establishing that fact, all overvoltages were determined for short circuits occurring at the instant of maximum flux linkage.

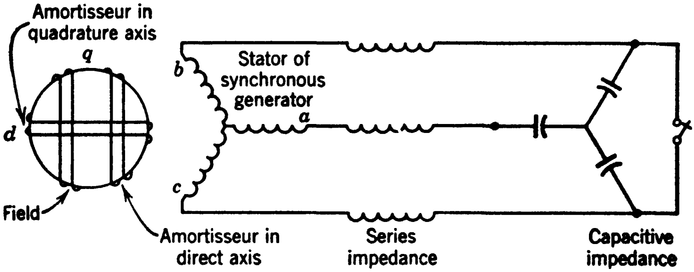

Amortisseur in quadrature axis
q
d
Field
Amortisseur in direct axis
b
Stator of synchronous generator
a
c
Series impedance
Capacitive impedance

FIG. 2·1. System studied using differential analyzer.

For each set of machine constants, the amount of terminal capacitance was varied in successive runs to cover the range in general of about $3x_2 < x_c < 30x_2$ where x_2 is the average of the direct- and quadrature-axis subtransient reactances including the external reactance.

The machine constants for a group of cases studied are shown in Table 2·1. Refer to Appendix I for nomenclature.

The machine constants were taken as average values for salient-pole (water-wheel) generators, except for the constants of the additional rotor circuits, which were varied over a wide range. The constants (see Nomenclature, Appendix I, and Fig. 2·2 for definition of

TABLE 2·1

Case	x_d''	x_q''	$\frac{x_d'' + x_q''}{2}\ (= x_2)$	x_q''/x_d''
1	0.439	0.456	0.447	1.04
2	0.456	0.490	0.473	1.07
3	0.478	0.544	0.511	1.14
4	0.489	0.567	0.528	1.16
5	0.439	0.516	0.477	1.17
6	0.456	0.553	0.504	1.21
7	0.478	0.616	0.547	1.29
8	0.489	0.650	0.569	1.33
9	0.556	0.900	0.728	1.62

symbols) used in this study were the following in per-unit of the machine rating.

$$x_l = 0.20$$

$$x_{ad}' = \frac{x_{ad}x_f}{x_{ad} + x_f} = 0.18, \qquad x_d' = 0.38$$

$$x_{aq} = 0.52, \qquad x_q = 0.72$$

$$r_{kd} = 0.02$$

$$r_{kq} = 0.02 \text{ for Cases 1–4}$$

$$r_{kq} = 0.04 \text{ for Cases 5–8}$$

Case	x_{kd}	x_{kq}
1	0.09	0.09
2	0.14	0.14
3	0.22	0.24
4	0.29	0.29
5	0.09	0.18
6	0.14	0.26
7	0.22	0.43
8	0.29	0.56
9	∞	∞

Cases 1 to 4 are for amortisseur windings with substantially symmetric leakage reactances; Cases 5 to 8 apply to amortisseurs with quadrature-axis leakage reactance substantially twice that of the direct axis. Case 9 represents a machine with no amortisseur. In all these cases the series impedance (Fig. 2·1) consisted of a reactance of 0.18 per unit and a resistance of 0.05 per unit except as otherwise indicated. Also in Cases 2 and 6, the amortisseur resistances were varied.

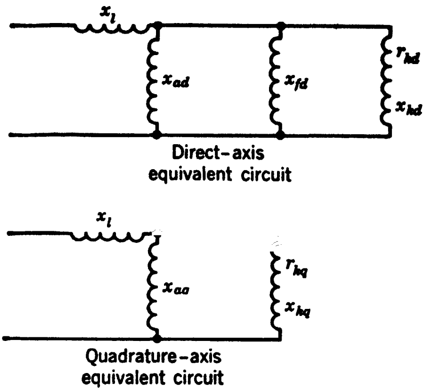


FIG. 2·2. Direct- and quadrature-axis equivalent circuits of synchronous machine.

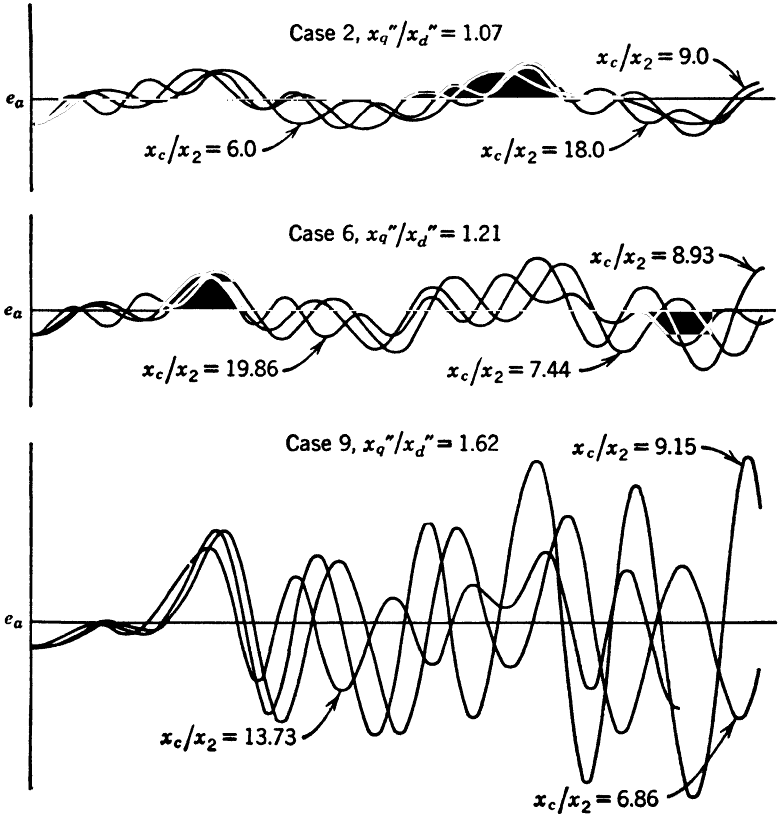


FIG. 2·3. Voltage-time curves determined by differential analyzer.

Differential analyzer solutions for several cases are shown in Fig. 2·3. Here the unfaulted phase voltage e_a is shown as a function of time for several cases studied.

Figures 2·4 to 2·6 show additional results obtained. On these curves are recorded the maximum voltage reached in the first four or five cycles following short circuit, as read from curves of instantaneous voltage versus time from analyzer solutions such as are shown in Fig. 2·3.

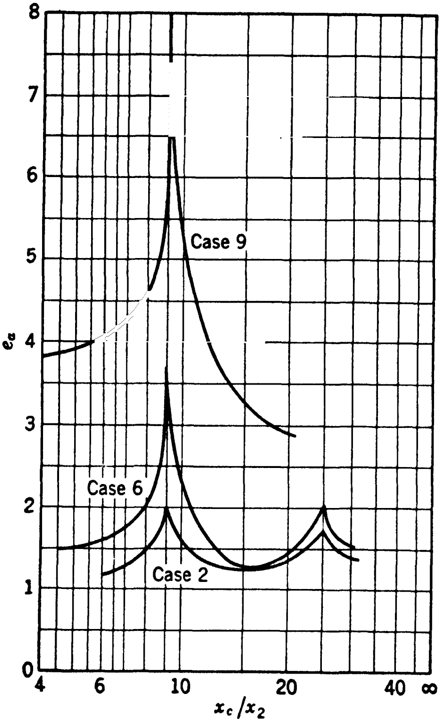


FIG. 2·4. Maximum overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur winding.

It is seen that as a function of the ratio of line capacitive reactance to average generator subtransient reactance (or negative-sequence reactance) the overvoltage curves have the same characteristic shape for all machines. Voltage peaks are observed at $x_c/x_2 = 9.0$ and $x_c/x_2 = 25$, which correspond to series resonance of the line capacitance with the negative-sequence reactance for the third and fifth harmonics, respectively.

Figure 2·4 shows how the addition of the amortisseur winding of Case 6 reduces the maximum voltage to about half of the value obtained with no amortisseur winding, and also the further reduction obtainable by the use of the amortisseur of Case 2. This latter is particularly noticeable at the resonant point $x_c/x_2 = 9.0$. The overvoltages obtained when line capacitance is not considered are indicated at the extreme right of the figure. Line capacitance is seen in general to increase the voltages obtained, especially when it is great enough to approach the point of third-harmonic resonance.

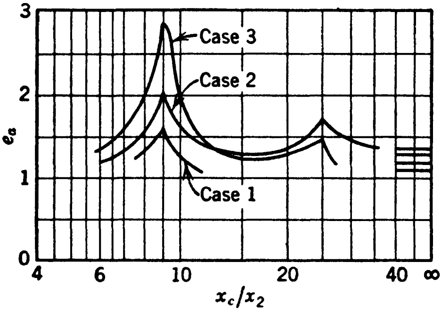


FIG. 2·5. Maximum overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur reactance.

Figures 2·5 and 2·6 show the effect of variation of the amortisseur leakage reactances. Figure 2·5 covers the range of amortisseurs with symmetrical-leakage reactances (Cases 1–4); Fig. 2·6 covers the range of unsymmetrical amortisseurs (Cases 5–8). As would be expected, the low-reactance windings are the most effective. As an indication of the relative effectiveness of the several amortisseur windings Table 2·2 is given.

TABLE 2·2

Case	e_0	e_c	$\frac{e_c - 1}{e_0 - 1}$
1	1.08	1.6	7.5
2	1.15	2.0	6.66
3	1.28	2.85	6.6
4	1.32	....	....
5	1.35	3.15	6.15
6	1.42	3.75	6.55
7	1.58	5.15	7.15
8	1.66	6.1	7.73
9	2.24	8.0	5.65

In this table, the e_0 column contains the maximum open-phase voltages obtained by the formula

$$e_0 = 2\frac{x_q''}{x_d''} - 1 \qquad [2\cdot 1]$$

in which the subtransient reactances include the external inductive reactance. This is the expression for open-phase voltage given by

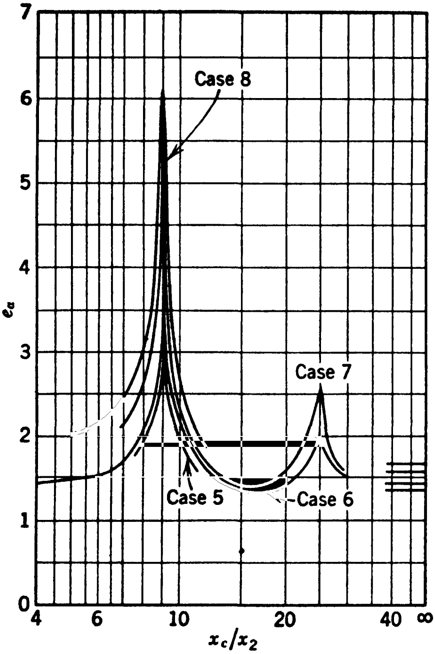


FIG. 2·6. Maximum overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur reactance.

Doherty and Nickle, neglecting capacitance and assuming constant amortisseur flux linkage and short circuit at the instant of maximum flux linkage in the shorted phases. The e_c column contains the maximum voltages found by means of the differential analyzer under the worst condition of connected external capacitance $x_c/x_2 = 9.0$. The last column gives the ratio of maximum excess voltage $(e_c - 1)$ with capacitance to the excess voltage $(e_0 - 1)$ for an open-circuited machine. This ratio is seen to be very nearly a constant quantity since the armature time constant was not varied very much (r_a = constant

$= 0.05$) and the field resistance was constantly zero. Since the maximum voltage e_c occurs several cycles after short circuit it will be somewhat reduced if allowance is made for the decay of field flux linkage, whereas, on the other hand, the voltage e_0, which occurs in the first cycle, will not be affected appreciably.

In general, the overvoltages depend principally on the reactances x_d'', x_q'', and x_c, the effect of the resistances r_{kd}, r_{kq}, and r_a being of secondary importance except in the case of r_a at or near one of the resonant points. This is particularly noticeable at $x_c/x_2 = 9.0$. Figure 2·7 shows how, at a particular value of line capacitance and for machines of Cases 2 and 6, the overvoltage is affected by changes of the amortisseur resistances in the ratio of 32 to 1. For the cases investigated, amortisseur resistance evidently has very little effect.

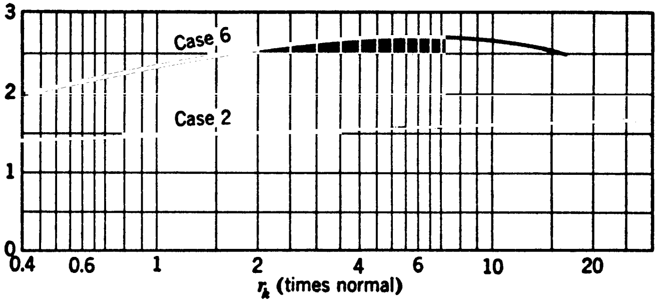


FIG. 2·7. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur resistance. Case 6, $x_c/x_2 = 9.9$, $x_q''/x_d'' = 1.21$; Case 2, $x_c/x_2 = 10.6$, $x_q''/x_d'' = 1.07$.

The effect of line resistance r_a in reducing the magnitude of overvoltage is seen in Fig. 2·8 for the machine of Case 9 with a rather large line capacitance. This is not a resonant case. At a resonant point the line resistance has much more effect, the voltage in some cases being almost inversely proportional to resistance.

It should be pointed out in connection with numerous curves like those shown in Fig. 2·3, the voltages in the first few cycles are only slightly if at all higher than the values during later cycles after the transients caused by the amortisseur and stator resistances have died away (if the decay of field flux linkages is neglected). In fact, in the region where the highest overvoltages are likely to occur, i.e., near $x_c/x_2 = 9.0$, the voltage requires several cycles to build up to its peak value, which occurs after armature transients have vanished.

Furthermore, inspection of numerous voltage-time curves such as those shown in Fig. 2·3 reveals that the voltage consists principally of a term of fundamental frequency and one having the frequency $\sqrt{x_c/x_2}$ in per-unit.

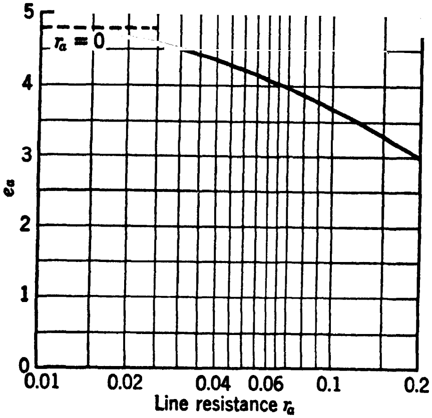


FIG. 2·8. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of line resistance. Case 9, $x_c/x_2 = 6.86$.

2·3 CALCULATION OF OVERVOLTAGES

In view of the foregoing discussion, one may conclude that for machines with amortisseur windings a good approximation to the voltages may be reached neglecting rotor resistances and also neglecting armature transients. If this is done a relatively simple method of calculation is available as will be shown. The results of calculations based upon this method show approximate agreement with those found with the differential analyzer. Figure 2·9 presents a comparison of calculated and analyzer results for Case 6.

For the synchronous machine with no amortisseur windings (Case 9) the armature transients had to be considered in order to obtain a reasonable check. Figure 2·10 shows a curve of voltage versus time determined by the differential analyzer compared with a step-by-step solution of the equations given later for $r_a = 0$, $x_c = 5$, and no amortisseur. The complete solution including armature transients has also been found analytically as explained later, and points calculated by the formula developed are shown in Fig. 2·10.

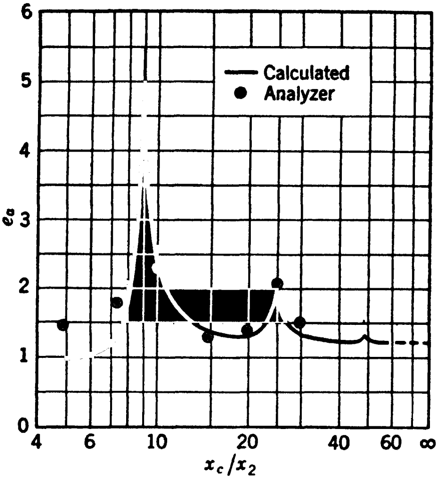


FIG. 2·9. Comparison of calculated steady-state overvoltages with differential-analyzer results. Case 6, $x_q''/x_d'' = 1.21$.

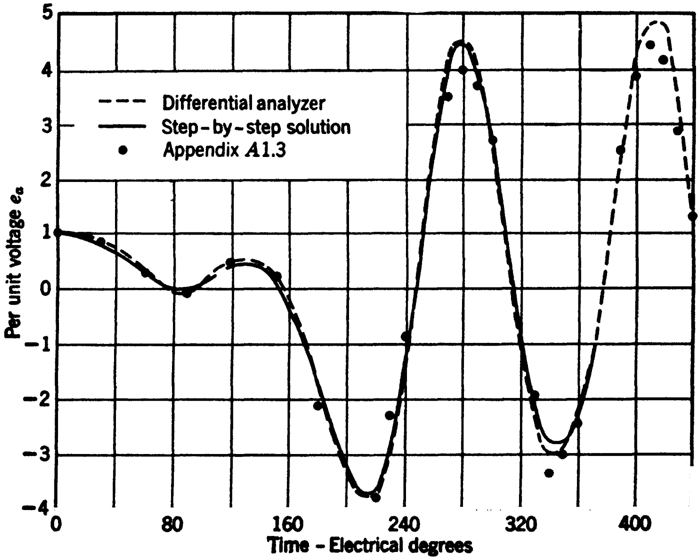


FIG. 2·10. Overvoltage following line-to-line fault on salient-pole synchronous generator.

In Appendix I, equations A[1·16] and A[1·17] show that the voltage is a function of the three ratios y/x, r/x, and x_c/x. The ratio y/x is simply related to the ratio x_q''/x_d'' by the formula,

$$\left(1 + \frac{y}{x}\right)\left(1 + \frac{x_q''}{x_d''}\right) = 2 \qquad [2\cdot 2]$$

while x is very nearly the negative-sequence reactance x_2. The voltage produced is thus a function of x_q''/x_d'', the armature short-circuit time constant x_2/r, and the ratio of x_c/x_2. Moreover, if only the maximum possible overvoltage obtainable is of interest, x_c/x_2 will always be very nearly 9.0, and so the voltage is a function only of x_q''/x_d'' and x_2/r. This, however, neglects all rotor decrement factors.

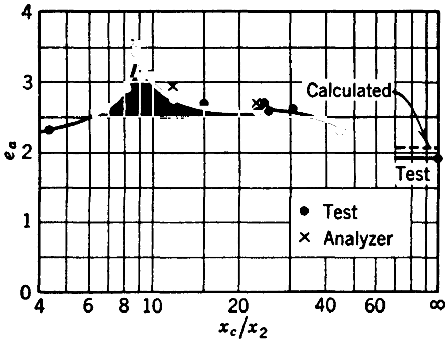


FIG. 2·11. Overvoltage due to line-to-line fault on synchronous machine—comparison of test and analyzer results. $x_q''/x_d'' = 1.52$.

2·4 COMPARISON WITH TEST RESULTS

In order to check the results obtained by calculation and by the differential analyzer several tests were made. In general, the maximum voltages recorded in test have been somewhat lower than those indicated by the analyzer. This is believed to be principally because the machines tested had rather low field time constants (about one second open circuit). The rapid decay of field flux prevented the high build-up of voltage at resonant points so that the maximum voltage always occurred in the first two or three cycles. Three methods of comparing test and calculated results were therefore used. One was to increase the voltage ordinates obtained in test oscillograms in inverse proportion to the field flux; another was similarly to decrease the analyzer curves or more simply to read the analyzer voltage peak at the point of maximum test voltage. Since the maximum voltage measured in test always occurred in a very short time, it was not necessary to correct for field flux in the last method.

Figure 2·11 shows a comparison of a curve of maximum test overvoltage with the overvoltage obtained by the differential analyzer in the same time. The overvoltage with no capacitance is also shown and compared with the calculated value. In this case the ratio $(e_c - 1)/(e_0 - 1)$ is $2.45/0.92 = 2.66$ instead of about six as would be obtained with infinite field time constant and a lower armature resistance.

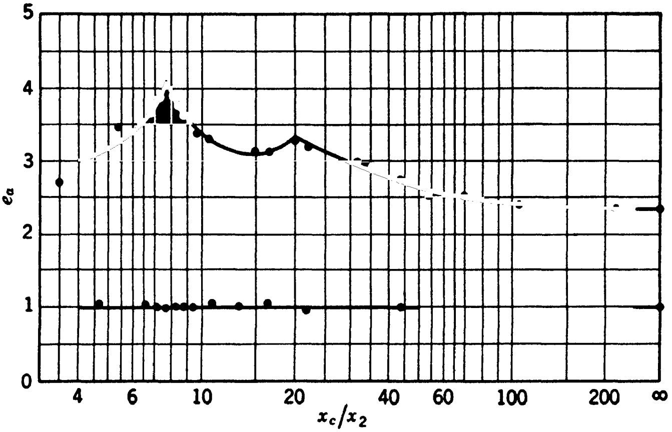
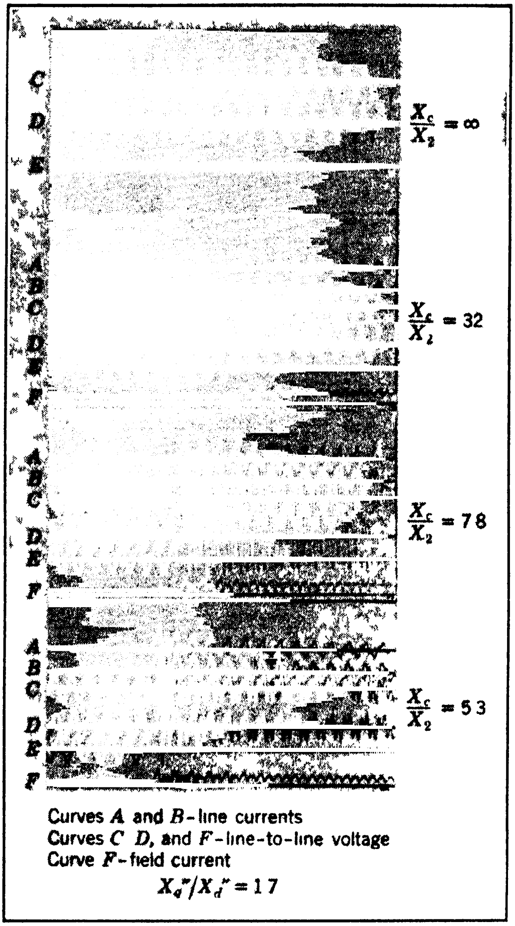


FIG. 2·12. Overvoltage due to line-to-line fault on synchronous machine—test results. Upper curve, $x_q''/x_d'' = 1.7$. Lower curve, $x_q''/x_d'' = 1.0$.

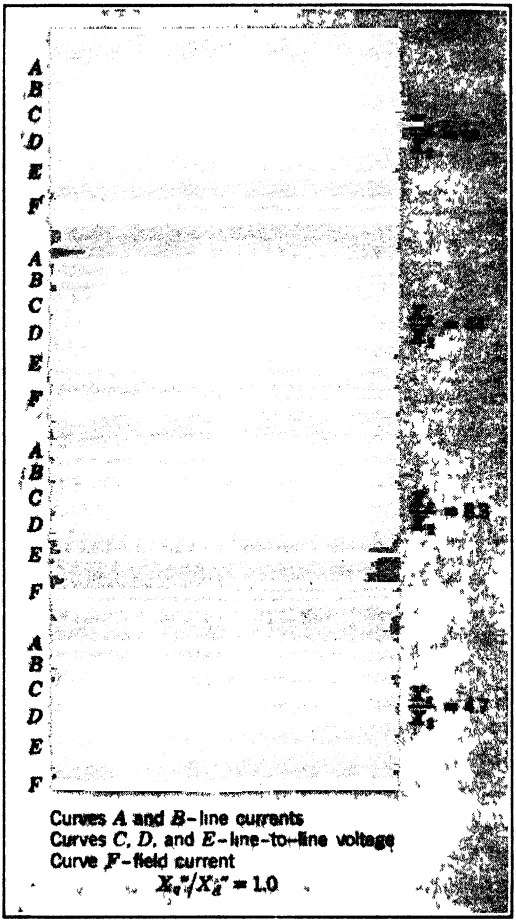
Figure 2·12 shows a comparison of test overvoltages obtained on the same machine with different amortisseur windings. It is seen that for $x_q''/x_d'' = 1.0$ there is no appreciable overvoltage, whereas for $x_q''/x_d'' = 1.7$ the voltage reaches 4.05 times normal at the third-harmonic resonance point. With no capacitance the maximum voltage is 2.32. Here the ratio $(e_c - 1)/(e_0 - 1) = 3.05/1.32 = 2.3$.

Figure 2·13(a) shows the oscillogram (third from top) from which the maximum overvoltage at the highest peak of Fig. 2·12 was obtained. The oscillogram shows line-to-line voltage, whereas Fig. 2·12 shows line-to-neutral voltage, and so the ratio of voltage after short circuit to voltage before short circuit read from the oscillogram is 0.866 times the ratio plotted in Fig. 2·12. A few other typical oscillograms are also given in Fig. 2·13.



(a)

Fig. 2.13. Oscillograms of line-to-line fault on salient-pole synchronous machine. Curves A and B, line currents; curves C, D, and E, line-to-line voltage; curve F, field current. (a) $x_q''/x_d'' = 1.7$; (b) $x_q''/x_d'' = 1.0$.



(b)

Fig. 2·13 (*Continued*).

In the tests it was found that the values of subtransient reactance to be used in computing overvoltage could be determined most easily and accurately by a static single-phase impedance test. Figure 2·14 shows the test ratios for the machine of Fig. 2·12. The high ratio is seen to be affected by saturation, and the value used in checking the test overvoltage curves was that corresponding to the calculated offset rms single-phase short-circuit current. The tests shown in Fig. 2·12 were taken at reduced voltage. For short circuits at full voltage the ratio x_q''/x_d'' is about 1.5 and the overvoltage is correspondingly reduced.

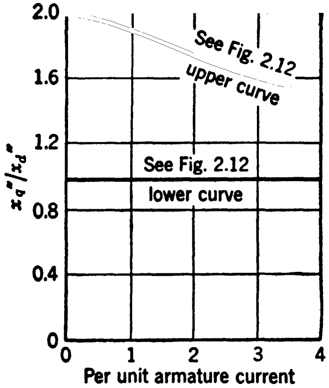


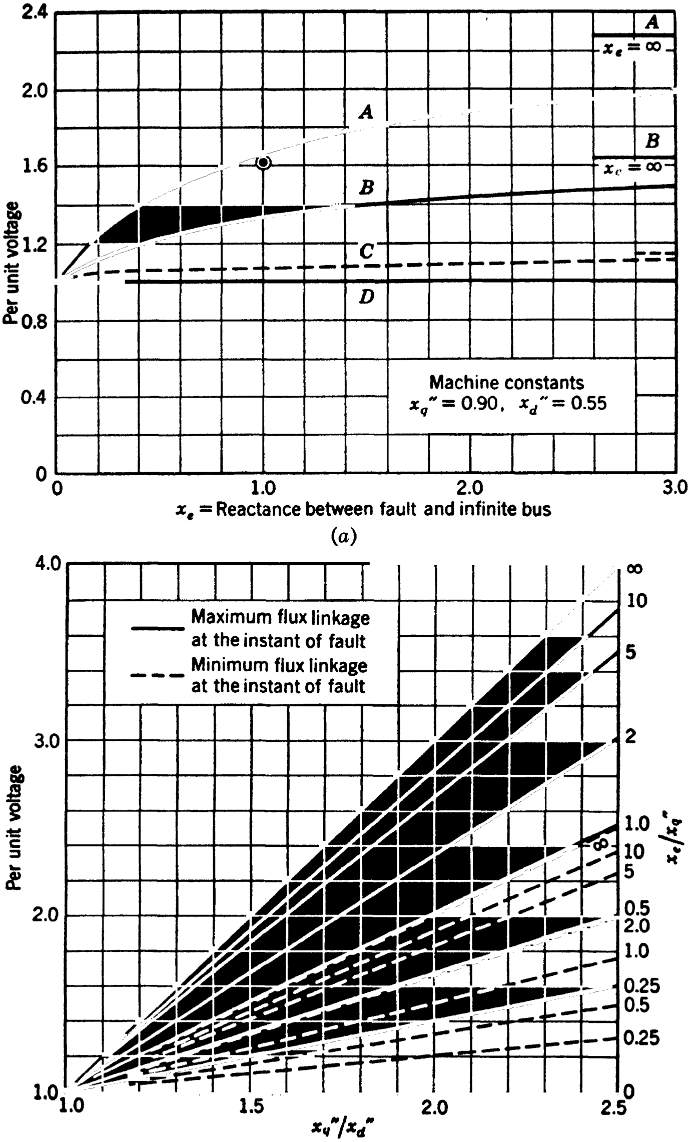
FIG. 2·14. Ratio x_q''/x_d'' determined by single-phase static tests.

2·5 DISCUSSION

The Ratio x_q''/x_d''. It has been brought out above that overvoltage is principally a function of the ratio x_q''/x_d''. However, except for faults directly at the machine terminals there will be a reactance in series with the machine reactance. Therefore, in general a low average subtransient reactance is also desirable since for the same ratio x_q''/x_d'' at the machine terminals the effective ratio at the fault will be lower with lower machine reactance. Thus, aside from their effect on the ratio as measured at the machine terminals, amortisseur windings will usually reduce the overvoltage produced.

The ratio x_q''/x_d'' is always highest at the machine terminals. It does not necessarily follow that faults directly at the terminals will always result in the highest voltages. In the circuit of Fig. 2·1, the reactance should be taken to the point of fault, whereas the capacitance is, to a first approximation, that of the whole open line or lines. For a particular case it is therefore conceivable that third-harmonic resonance may occur for faults out on the line and that the resultant voltage may be higher despite the decreased effective ratio x_q''/x_d''. In practice, however, one must usually take into consideration all possible connected amounts of line. The highest possible overvoltages will then occur for faults at the machine terminals.

Factors Limiting Overvoltage. The highest overvoltages indicated here will not in general occur in practice. Saturation will limit the voltages to a certain extent, and protective equipment will function.



(a)

(b)

FIG. 2·15. Line-to-line fault on salient-pole generator—maximum voltage at the point of fault, effect of reactance tie to system. (a) Curve A, no load, fault at maximum flux linkage; Curve B, no load, fault at minimum flux linkage; Curve C, symmetrical components, fundamental frequency only; Curve D, with $x_q'' = x_d''$. ⊙, 20° load angle, fault at maximum flux linkage.

The saturation effect, however, will not be nearly so great as might be supposed from an inspection of only the magnitude of the voltage peaks, since the fundamental component is usually only slightly higher than normal, the highest values recorded consisting principally of a third-harmonic component. Lightning arresters may be expected to be damaged by the high sustained voltages. Also, generally the machine will be connected to a system. In order to evaluate the effect of a connected system on overvoltage Fig. 2·15 has been included. Figure 2·15(a) shows how in a particular case where $x_q'' = 0.90$, $x_d'' = 0.55$, the overvoltage caused by line-to-line short circuit is reduced by the system tie reactance x_e. All the curves are for the condition of no load and equal voltages on machine and infinite bus. Curve A is for short circuit at maximum flux linkage in the shorted phases and approaches the value of equation [2·1], as the tie reactance approaches infinity. Curve B is for short circuit at minimum flux linkage. Curve C shows the voltage that would be obtained by the conventional symmetrical-component calculation. The symmetrical-component method is found to give substantially the fundamental-frequency component of voltage. The effect of generator load angle on the maximum overvoltage is indicated by the single point at $x_e = 1.0$.

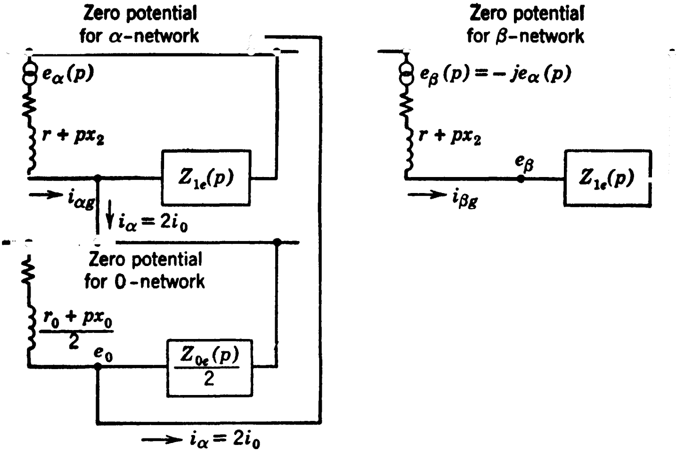


FIG. 2·16. Equivalent circuit for line-to-ground fault for determining harmonic currents and voltages.

Figure 2·15(b) shows more generally the effect of x_q''/x_d'' and the reduction in voltage caused by a reactance tie to an infinite bus. All resistances and capacitances were neglected in calculating the curves of Fig. 2·15.

Faults Involving Ground. A line-to-line fault was chosen here because the overvoltage depends primarily on the ratio x_q''/x_d'', whereas,

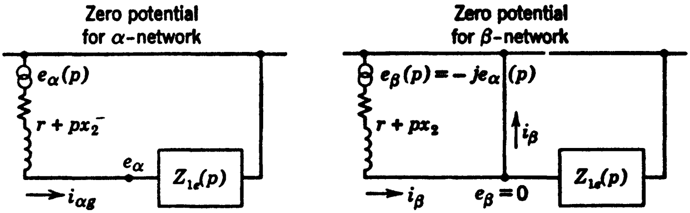


FIG. 2·17. Equivalent circuit for line-to-line fault for determining harmonic currents and voltages.

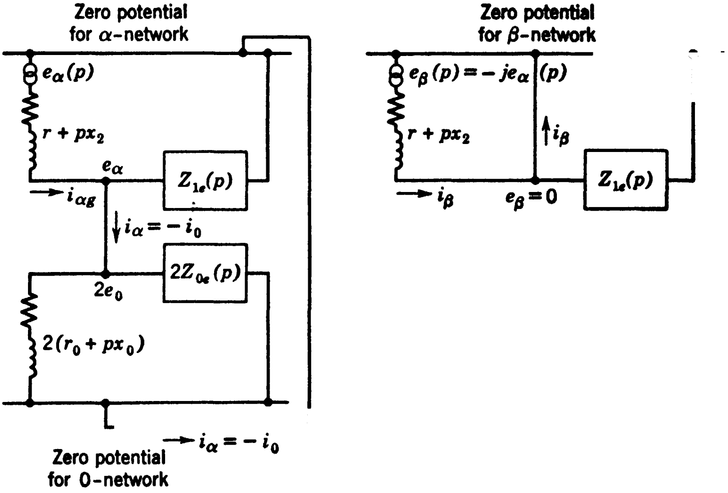


FIG. 2·18. Equivalent circuit for double line-to-ground fault for determining harmonic currents and voltages.

for ground faults, the overvoltage is affected also by the method of system grounding. The great majority of faults on power systems involve ground, the line-to-line fault being a less frequent occurrence.

Equivalent circuits for determining instantaneous currents and voltages following faults are given in Appendix I for line-to-ground, line-to-line, and double-line-to-ground faults. See Figs. 2·16, 2·17, and 2·18. From these circuits and either of the methods of calculation given in Appendix I, overvoltages following faults can be determined.

Figure 2·19 gives resonance regions to third-harmonic currents for the three types of faults when the fault occurs at the terminals of a salient-pole machine connected through impedance to any symmetrical external system in which the positive- and negative-sequence impedances are equal. Resistance is neglected.

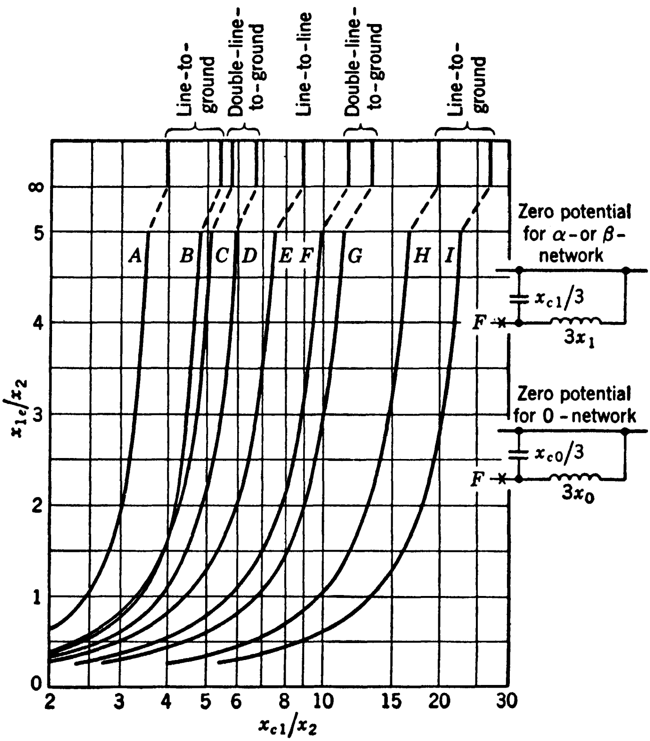


FIG. 2·19. Third-harmonic resonance regions for various types of faults. Curve *A*, line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = 0.6C_1$. Curve *B*, line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = C_1$. Curve *C*, double line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = 0.6C_1$. Curve *D*, double line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = C_1$. Curve *E*, line-to-line fault. Curve *F*, double line-to-ground fault, ungrounded system, $C_0 = 0.6C_1$. Curve *G*, double line-to-ground fault, ungrounded system, $C_0 = C_1$. Curve *H*, line-to-ground fault, ungrounded system, $C_0 = 0.6C_1$. Curve *I*, line-to-ground fault, ungrounded system, $C_0 = C_1$.

Equivalent circuits representing the external system in the positive- and zero-sequence network are shown in Fig. 2·19. These circuits consist of a capacitive reactance x_c and an inductive reactance x_e in parallel. In a complex external system, x_{c1}, x_{c0}, x_{1e}, and x_{0e} of the equivalent circuits in general will not be constant for all harmonics.

Figure 2·19 indicates that with a line-to-ground fault and the assumed ratios of x_{0e}/x_{1e} and x_0/x_2 the capacitance required to cause resonance on a grounded system is high. Even with open lines, a value of x_{c1}/x_2 as low as 5.4 would probably seldom be found in actual systems. For the ungrounded system, the amount of capacitance required for resonance is only about one-fourth to one-fifth as much and the ratio x_{c1}/x_2 lies in a not uncommon region for large systems with long transmission lines.

Other Functions of Amortisseur Windings. Limitation or reduction of overvoltage is not, of course, the only function of an amortisseur winding, and it is necessary to keep the other aspects of amortisseur applications in mind in order to determine properly the best winding for a particular machine. The damping action for rotor oscillations about a steady-state operating angle is determined principally by the amortisseur resistance. A low resistance gives in general the largest damping coefficient for slow oscillations. From the standpoint of stability the damping action of amortisseur windings is beneficial, but here a high-reactance winding which will limit the fault current is desirable. An amortisseur winding will also keep the field voltage on open circuit from reaching dangerous values. These functions do not involve the ratio x_q''/x_d''. However, the recovery voltage on clearing faults has been shown[4] to depend on this ratio.

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3 SYSTEM VOLTAGE RECOVERY CHARACTERISTICS— FUNDAMENTAL CONCEPTS

3·1 INTRODUCTION

In Chapters 1 and 2 we have considered what happens in a power system when a fault suddenly occurs. Although the frequency of occurrence of faults in power systems can be reduced to an acceptable minimum by proper design and operation of the system, such faults cannot economically be eliminated completely. Therefore their occurrence must be accepted, and the system must be designed in such a way that any faulted portion can be (1) located quickly by means of appropriate relays or other protective devices and (2) separated from the main portion of the system with the minimum of disturbance to that main portion of the system. In other words, the faults must be located and the short-circuit currents must be interrupted by the proper equipment. Devices for these purposes are known as protective devices.

Electrically, a power system is an oscillatory network, so that it is logical to expect that the interruption of fault-current flow will give rise to a transient in a manner similar to that which accompanied the sudden application of a fault in Chapter 1. In general, this transient will be composed of a fundamental-frequency term and one or more natural-frequency terms. There may or may not be harmonic terms of significance, depending on whether the effect of saliency is important or not.

It is the purpose of this chapter to inquire more carefully into the exact nature of such system voltage recovery characteristics under a variety of fault and system conditions. For some conditions, generalized curves will be included to show the effects of several significant factors upon the rate of rise and the magnitude of the transient recovery voltage.

3·2 AN INTRODUCTORY PROBLEM

In order to present the elements of the phenomena, consider the simple single-frequency circuit of Fig. 3·1. Initially in this circuit, assume that the switch is closed (i.e., a fault has occurred). Let the generated voltage be

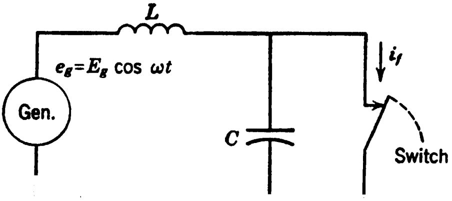


FIG. 3·1. Single-frequency circuit used to illustrate transient recovery voltage.

$$e_g = E_g \cos \omega t \tag{3·1}$$

and the fault current will then be

$$i_f = I_f \sin \omega t = \frac{E_g}{\omega L} \sin \omega t \tag{3·2}$$

where ω is 2π times the fundamental frequency in cycles per second and L is in henries.

If the switch is opened, the flow of fault current will be stopped and a voltage known as the transient recovery voltage will appear across the switch. Assume that this circuit interruption is timed so that the interruption is completed instantaneously at a time $t = 0$ when the value of $i_f = 0$ as shown in Fig. 3·2. The dashed curve labeled e_s is the voltage that would exist across the switch if it were open and steady-state conditions were attained. The voltage across the switch is, of course, zero during the period of fault-current flow. Immediately after the switch is opened, the voltage across the switch builds up to approximately twice the fundamental-frequency peak magnitude in accordance with the high-frequency oscillation of the circuit.

To simulate this condition mathematically, it is only necessary to inject in the faulted circuit a current such that the resultant current flowing through the switch is made zero. The voltage appearing then across the switch contacts is the recovery voltage. If at $t = 0$ we inject the current $-i_f$ in the direction of current flow indicated, or i_f in the opposite direction, the operational expression for the recovery voltage is

$$V_R(p) = i_f(p)Z(p) \qquad [3\cdot 3]$$

where $Z(p)$ is the operational impedance of the circuit viewed from the switch. For the circuit of Fig. 3·1, this operational impedance is

$$Z(p) = \frac{\dfrac{p}{C}}{p^2 + \dfrac{1}{LC}} \qquad [3\cdot 4]$$

where p is the operator d/dt, C is the capacitance in farads, and L is the inductance in henries.

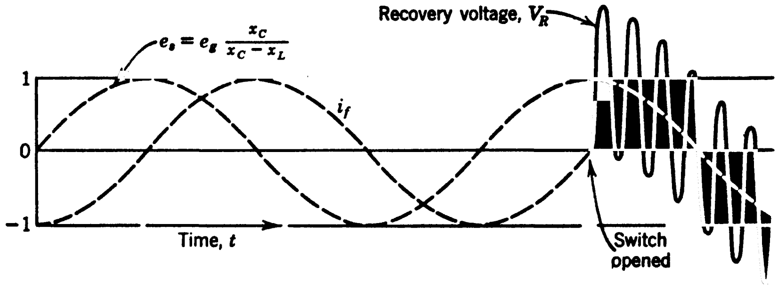


FIG. 3·2. Recovery voltage for circuit of Fig. 3·1.

Substituting equations [3·2] and [3·4] in equation [3·3] and solving operationally, we get

$$V_R = \frac{E_g}{1 - \dfrac{x_L}{x_C}}\left(\cos \omega t - \cos \frac{t}{\sqrt{LC}}\right) = E_s\left(\cos \omega t - \cos \frac{t}{\sqrt{LC}}\right) \qquad [3\cdot 5]$$

where E_s is the peak voltage of fundamental frequency appearing across the switch when it is open. The second term is the natural-frequency term, and, in magnitude, it is equal to the peak value of the fundamental-frequency term. The natural frequency in cycles per second is

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad [3\cdot 6]$$

where L is in henries and C in farads.

The recovery voltage V_R reaches a maximum value of essentially $2E_s$ at a time approximately ½ cycle of natural frequency after the switch is opened. This is true when the natural frequency is high with respect to the fundamental frequency and when losses are insignificant. Losses will reduce the maximum value of V_R, of course. If the natural frequency is high as compared to the fundamental, then equation [3·5] may be written

$$V_R \cong E_s\left(1 - \cos\frac{t}{\sqrt{LC}}\right) \qquad [3\cdot 7]$$

This expression is generally valid for the interval of time between the opening of the switch at $t = 0$ and the time at which the maximum value of V_R is reached. This is the transient voltage recovery period of much significance to the designer and user of circuit interrupting equipment.

3·3 COMPOUND TRANSIENTS

In the preceding section, it was assumed that the fault current preceding interruption contained only a fundamental-frequency term. In other words, the circuit was not in a transient state before the switch was opened. There are times when this is not the case, e.g., when an expulsion protector tube operates. Ordinarily, the total time from the instant of fault occurrence to the removal of the fault is less than one cycle of fundamental frequency. Overvoltage arresters in general operate within a similar period of time. As higher speed switching has been developed, the transient fault current has been of increasing importance to the designers and users of circuit breakers. Consequently, in general the transient accompanying the occurrence of the fault is of importance since it may not have disappeared when the second transient, due to the circuit interruption of the fault current, begins. Therefore it is of interest to inquire with regard to the effects of such compound transients.

The circuit of Fig. 3·1 will serve again for the purpose of illustration. Since a fault may occur at any instant, the general case must be considered so that the transient fault current is obtained for any instant of closing the switch. As before, let

$$e_g = E_g \cos \omega t \qquad [3\cdot 1]$$

Let the switch be closed at an instant defined by the angle θ, such that θ is positive if the switch is closed after a voltage crest. The angle θ is the angle between the instant of positive voltage crest and the instant of closing the switch. The equation for fault current, i_f, is then (neglecting the capacitance discharge current)

$$i_f(p) = \frac{e_g(p)}{pL}$$

where $e_g = E_g(\cos \omega t + \theta)$, and

$$i_f = \frac{E_g}{\omega L}[\sin(\omega t + \theta) - \sin\theta] \qquad [3\cdot 8]$$

From this expression, it is apparent that the fault current has a d-c component, the magnitude of which is proportional to the sin θ. If $\theta = 0$ (fault occurring at crest voltage), then the d-c component is zero. If $\theta = -90°$, then the d-c component is positive and equal to the crest value of the normal frequency current. If θ is $+90°$, the d-c component has a negative maximum value. Thus the normal frequency component may be completely offset with respect to the zero axis. This offset may occur with either polarity as shown in Fig. 3·3.

With resistance neglected as in Fig. 3·1, it is relatively easy to define the current zeros. If we wish to open the switch at the instant of the first zero occurring after closing the switch, the current given by equation [3·8] must be permitted to flow until a given time t_0 is reached corresponding to this instant of current zero. For any given value of θ as already defined, the first current zero occurs at the smallest value of t_0 (other than zero) that satisfies the equation

$$\sin(\omega t_0 + \theta) - \sin\theta = 0 \qquad [3\cdot 9]$$

or

$$\omega t_0 = \pi - 2\theta \qquad [3\cdot 10]$$

The current to be injected into the switch at this particular instant defined by $t = t_0$ is

$$i_f = \frac{E_g}{\omega L}[\sin(\omega t + \omega t_0 + \theta) - \sin\theta]$$

$$= \frac{E_g}{\omega L}[\sin(\omega t + \pi - \theta) - \sin\theta] \qquad [3\cdot 11]$$

where t is now measured from the instant of current interruption.

The recovery voltage transient is given by equation [3·3] where i_f is given by equation [3·11] and $Z(p)$ is given by equation [3·4].

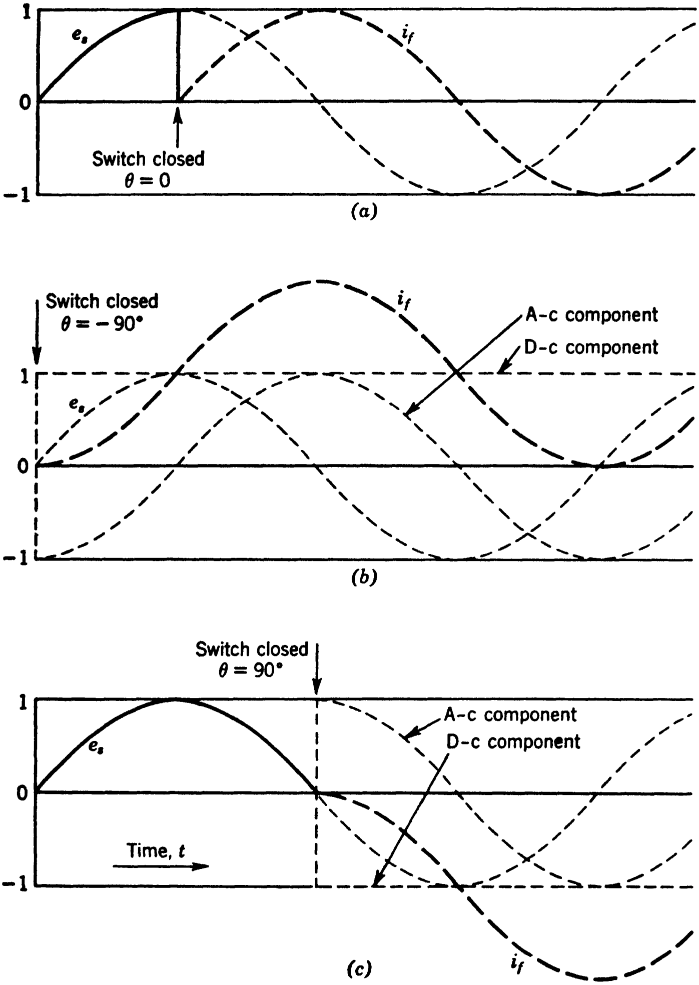


FIG. 3·3. Effect of instant of fault occurrence on the fault current i_f.

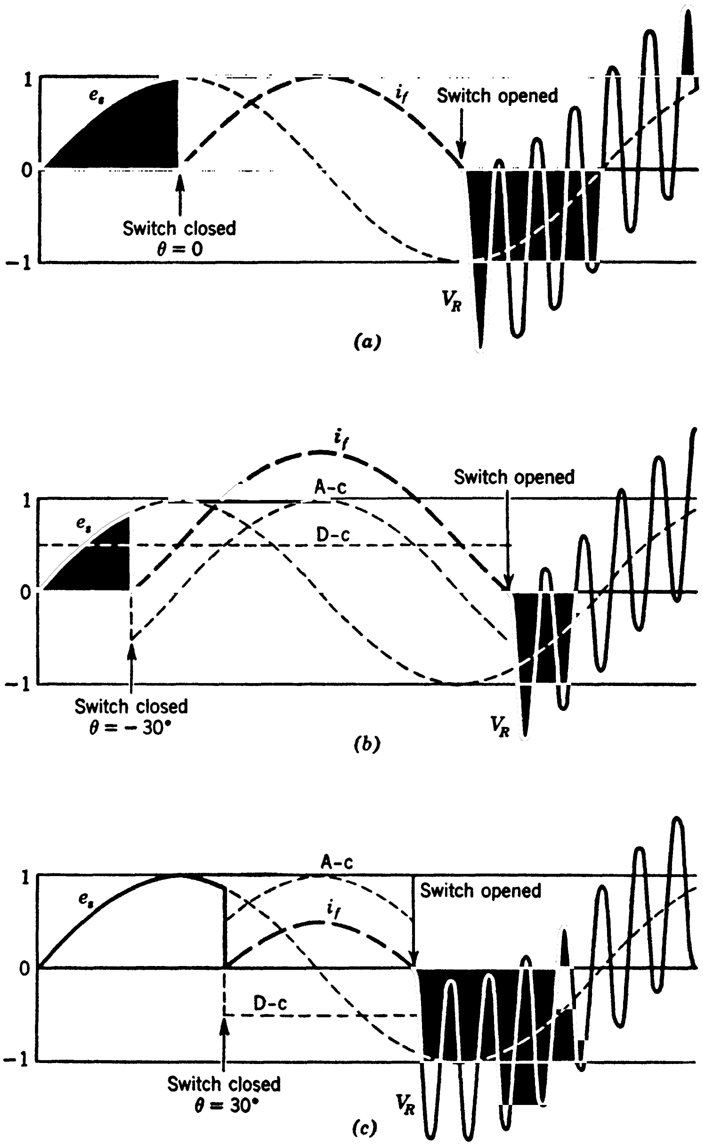


FIG. 3·4. Effect of fault current transient on voltage recovery transient.

With this substitution, the solution of equation [3·3] operationally yields

$$V_R = \frac{E_g}{\left(1 - \dfrac{x_L}{x_C}\right)}\Bigg[- \cos(\omega t - \theta)$$

$$+ \sqrt{\cos^2\theta + \frac{x_L}{x_C}\sin^2\theta}\,\cos\left(\frac{t}{\sqrt{LC}} - \alpha\right)\Bigg]$$

$$= E_s\left[- \cos(\omega t - \theta) + \sqrt{\cos^2\theta + \frac{x_L}{x_C}\sin^2\theta}\,\cos\left(\frac{t}{\sqrt{LC}} - \alpha\right)\right] \qquad [3\cdot 12]$$

where

$$\cos\alpha = \frac{\cos\theta}{\sqrt{\cos^2\theta + \dfrac{x_L}{x_C}\sin^2\theta}} \qquad [3\cdot 13]$$

$$\sin\alpha = \frac{\sqrt{\dfrac{x_L}{x_C}}\sin\theta}{\sqrt{\cos^2\theta + \dfrac{x_L}{x_C}\sin^2\theta}} \qquad [3\cdot 14]$$

For such compound transient phenomena, several curves of transient recovery voltage are plotted in Fig. 3·4 to show the general effect of θ. If $\theta = 0$ (the fault occurring at voltage crest), then the recovery voltage transient V_R is the same as that given by equation [3·5]. For any other value of θ, the maximum value of V_R is less.

If x_L/x_C is small, as will usually be true, then equation [3·12] may be written

$$V_R \cong E_s\left[- \cos(\omega t - \theta) + \cos\theta\cos\frac{t}{\sqrt{LC}}\right] \qquad [3\cdot 15]$$

From this expression, it is clear that the magnitude of the natural-frequency component of the recovery voltage is proportional to cos θ. In other words, the greater the amount of d-c current flowing in a fault preceding interruption, the less will be the magnitude of the natural-frequency oscillation in the recovery transient under the assumed conditions of analysis.

3·4 COMPOUND TRANSIENTS AND DISTRIBUTED CONSTANTS WITH ARC VOLTAGE

Certain types of interrupting devices which complete their function within one cycle of fundamental frequency after a fault occurs have, during the period of fault-current flow, a fairly constant arc voltage characteristic. An example of such a device is the expulsion protector tube. These devices are used for overhead line protection against outages arising primarily from lightning discharges.

The ability of such devices to clear the power-follow current or fault current that flows subsequent to the initial lightning sparkover depends on the severity of the voltage recovery transient. The fault can be initiated at any instant, and the fault current will vary as has been discussed in the preceding section. During the period of fault-current flow, the device itself offers a fairly constant arc voltage drop. This voltage is of such a polarity as to oppose the flow of current. At the first current zero, the device will interrupt the circuit under proper conditions. The recovery transient is modified by this arc voltage so that its effect should be included in the analysis. Furthermore, since an overhead line is generally involved as part of the circuit, the distributed-constant nature of the transmission line must also be considered.

To illustrate the effects of these factors, consider the system of Fig. 3·5(*a*) which consists of a generator, a transformer, and a transmission line. Figure 3·5(*b*) is a more revealing diagram with the above elements brought into the circuit. This equivalent circuit is easily arrived at if the system of Fig. 3·5(*a*) is a single-phase circuit. If it is not a single-phase circuit then the mutual surge impedances of the other phases must be included for correct results. For the present purposes, consider only a single-phase circuit. The protective device is located at the sending end of the transmission line.

Let θ be defined as before and let E_a be the constant arc voltage, represented by the battery in Fig. 3·5(*b*). Neglect the fault current transients due to the distributed-constant line. Let the surge impedance of the transmission line be Z. The differential equation for current in the fault is then

$$i_f(p) = \frac{e_g(p)}{pL} \tag{3·16}$$

where $e_g = E_g \cos(\omega t + \theta) - E_a$. Equation [3·16], when solved operationally, yields

$$i_f = \frac{E_g}{\omega L}[\sin(\omega t + \theta) - \sin\theta] - \frac{E_a t}{L} \qquad [3\cdot17]$$

This is the same as equation [3·8] with the addition of the E_at/L term. This arc voltage, E_a, introduces a component of fault current $-(E_at)/L$ which always subtracts from the sinusoidal component of fault current during the first loop. The quantity E_a would have to change sign for the second loop and continue to reverse in sign each time the current flow is reversed. However, for the interval of time from the closing of the switch until the first subsequent current zero, equation [3·17] is valid. In terms of the peak value of fundamental frequency it may be rewritten

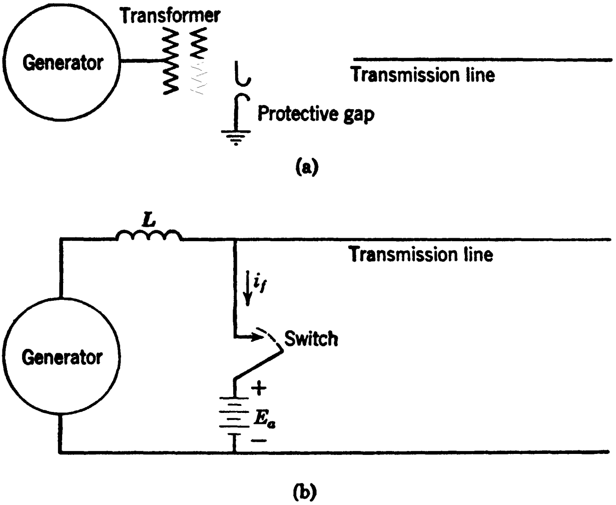


FIG. 3·5. Circuit for illustrating effect of compound transients and arc voltage in a distributed-constant system.

$$i_f = \sin(t + \theta) - \sin\theta - \frac{E_a}{E_g}t \qquad [3\cdot18]$$

if t is in radians of fundamental frequency. The components of fault current are shown in Fig. 3·6 for some typical constants, and the effects of E_a and θ on the duration of fault current flow (until the first current zero) are shown in Fig. 3·7. The curves of Fig. 3·6 are easily identified with the proper components of equation [3·18]. The curves of Fig. 3·7 indicate that the time of fault-current flow is a function of θ and E_a/E_g which is not easily expressed mathematically.

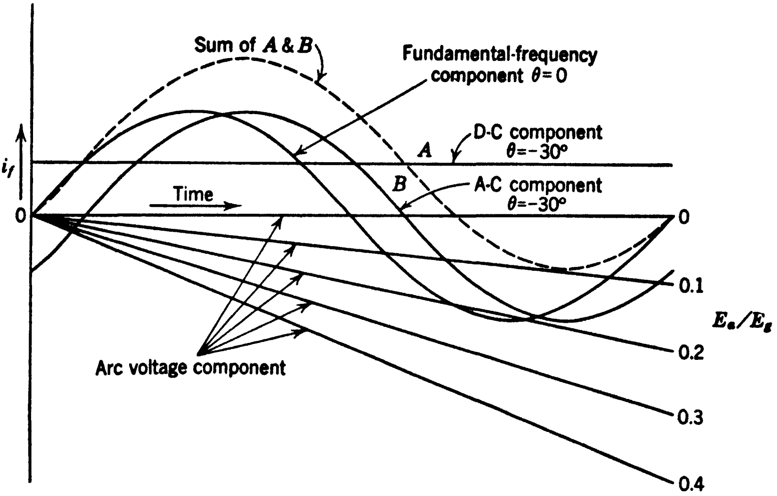

Sum of A & B
Fundamental-frequency component $\theta = 0$
D-C component $\theta = -30°$
A-C component $\theta = -30°$
A
B
i_f
0
Time
Arc voltage component
0
0.1
0.2
0.3
0.4
E_a/E_g

FIG. 3·6. Fault current components when constant arc voltage is present.

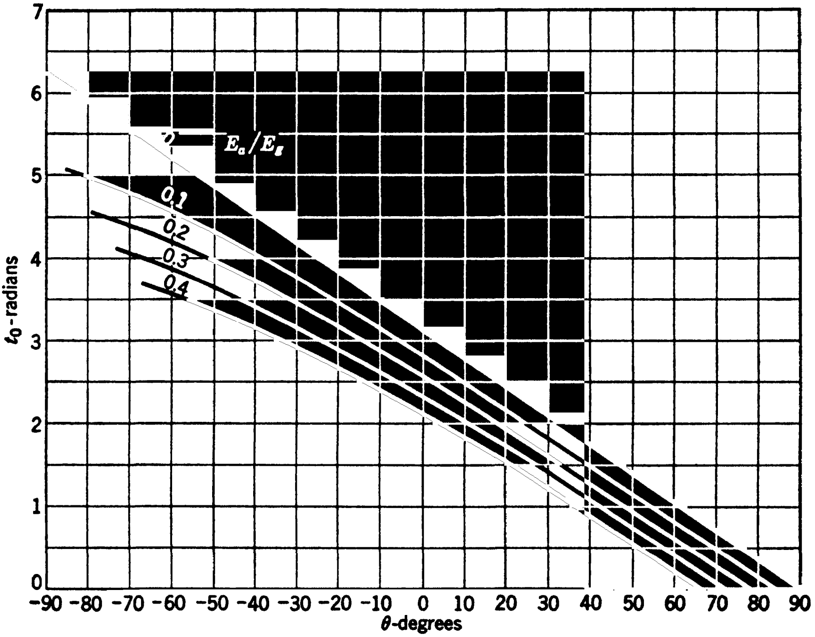


FIG. 3·7. Effect of arc voltage drop on time of fault current flow.

With fault interruption taking place at the first current zero subsequent to fault application, the recovery voltage transient *due to interruption* will be

$$V_R'(p) = i_f(p)Z(p) \qquad [3\cdot 19]$$

where

$$Z(p) = \frac{zp}{p + (Z/L)} \qquad [3\cdot 20]$$

and

$$i_f = \sin(t + t_0 + \theta) - \sin\theta - \frac{E_a}{E_g}(t_0 + t) \qquad [3\cdot 21]$$

This equation comes from equation [3·18] by substitution of t_0 as the value of time at which the fault current is zero. Substitution of equations [3·20] and [3·21] in equation [3·19] and solving operationally yield

$$V_R' = -\cos\alpha\cos(t + t_0 + \theta - \alpha) + \frac{E_a}{E_g} - \epsilon^{-(Z/x_L)t}\left[-\frac{Z}{x_L}\sin\theta + \frac{E_a}{E_g} - \frac{E_a}{E_g}t_0\frac{Z}{x_L} + \frac{Z}{x_L}\cos\alpha\sin(t_0 + \theta - \alpha)\right] \qquad [3\cdot 22]$$

This is the recovery voltage transient appearing across the switch in the circuit of Fig. 3·5. The battery was put into the circuit merely to simulate the arc characteristic of the protective device; i.e., both the switch and the battery together represent the protective device with the stipulation that the battery is in the circuit while current is flowing but is not in the circuit as soon as current flow stops. Therefore the battery voltage should disappear simultaneously with the opening of the switch. This means that V_R' as given by equation [3·22] is not the complete solution for the transient recovery voltage. In order to get the complete solution, to equation [3·22] must be added the arc voltage $-(E_a)/E_g$ which was there across the interrupting device before it opened the circuit. The complete solution is then

$$V_R = -\cos\alpha\cos(t + t_0 + \theta - \alpha) - \epsilon^{-(Z/x_L)t}\left[-\frac{Z}{x_L}\sin\theta + \frac{E_a}{E_g} - \frac{E_a}{E_g}t_0\frac{Z}{x_L} + \frac{Z}{x_L}\cos\alpha\sin(t_0 + \theta - \alpha)\right] \qquad [3\cdot 23]$$

where t is in radians and x_L is in ohms at fundamental frequency and

$$\sin \alpha = \frac{x_L}{\sqrt{Z^2 + x_L^2}}$$

$$\cos \alpha = \frac{Z}{\sqrt{Z^2 + x_L^2}}$$

If the interval of time under investigation is small, then it may be permissible to neglect the variation in the fundamental-frequency component of recovery voltage as a function of time and equation [3·23] becomes

$$V_R = M\epsilon^{-(Z/x_L)t} - N \qquad [3\cdot24]$$

where for any given physical circuit arrangement, M and N are functions of E_a and θ. In other words, for given values of E_a and θ, M and N are constants. A different set of values of E_a and θ will in general give different values to M and N.

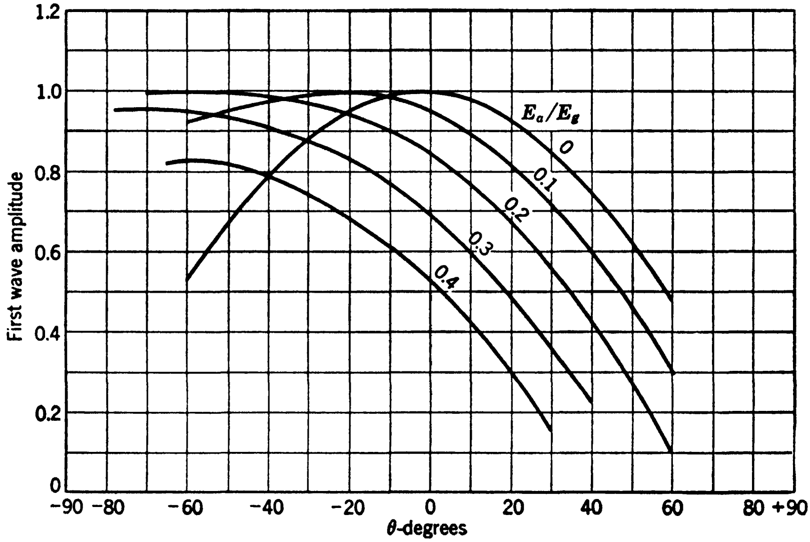


FIG. 3·8. Effect of arc voltage and instant of fault occurrence on first wave amplitude of recovery voltage.

Equation [3·24] gives the initial wave of voltage which travels down the transmission line. The analysis can be continued to include reflections from the open end or from junction points if desired. The calculations are very laborious and time consuming, however, and no attempt will be made here to carry them out. One simple case will be considered in Section 3·5. Some transient analyzer results will be shown in Chapter 4 and will be discussed at that time.

It will be of interest here to show how the arc voltage affects the first wave amplitude (values of N in equation [3·24]). Z is assumed

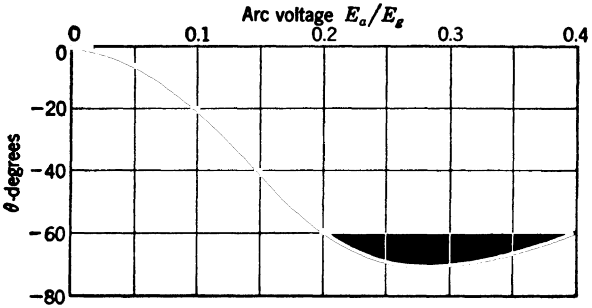


FIG. 3·9. Effect of arc voltage drop on instant of fault occurrence which gives maximum first wave amplitude.

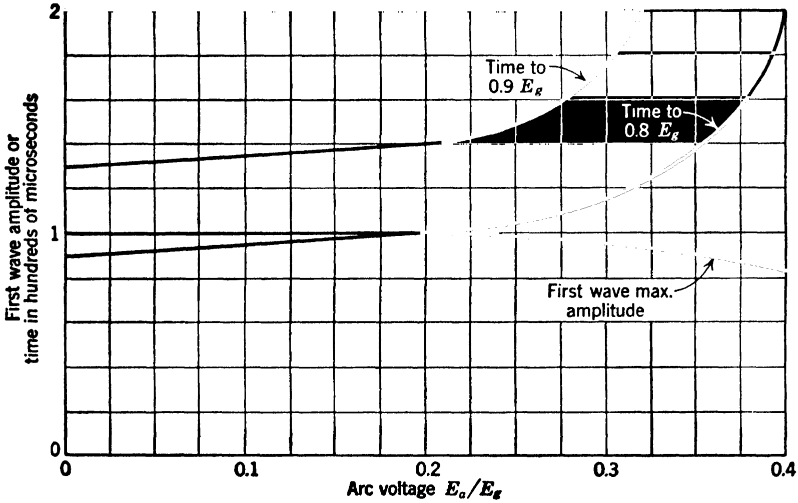


FIG. 3·10. Effects of arc voltage on recovery voltage transient.

to be a typical value of 500 ohms, and L is a value such that the short-circuit current corresponds to 6,250 rms symmetrical amperes in a 115-kv three-phase system. With arc voltage in the circuit, the amplitude of the first wave of voltage depends on both arc voltage and the fault angle θ as shown in Fig. 3·8. The relationship between arc voltage and θ to give maximum first wave amplitude is shown in Fig. 3·9. In general, with arc voltage present, the fault must be initiated before a voltage crest in order to give the maximum first wave amplitude. Figure 3·10 shows the effect of arc voltage on the first wave maximum amplitude as well as the effect on the time to 0.8 E_g and 0.9 E_g.

3·5 EFFECTS OF REFLECTIONS ON RECOVERY VOLTAGE

In this section a case will be considered similar to the foregoing Fig. 3·5 but with the line being assigned certain specific lengths so that reflections can be taken into account. The following assumptions are made for the analysis in this section.

1. Zero arc voltage drop.
2. No attenuation of voltage surges.
3. Line-to-ground fault at transformer terminals.
4. Station or source capacitance is negligible.
5. Mutual effects of phases is negligible.
6. The far end of the line is open.
7. Current transients have disappeared before interruption occurs.

The first wave of recovery voltage can be obtained from equation [3·23] by substituting $E_a/E_g = 0$, $\theta = 0$, and $t_0 = \pi$

$$V_R = \cos\alpha\cos(t-\alpha) - \epsilon^{-(Z/x_L)t}\cos^2\alpha \qquad [3\cdot25]$$

If Z is very large compared to x_L, and the interval of time that is of interest is very small compared with a period of fundamental frequency, then as a reasonable approximation

$$V_{R0} = 1 - \epsilon^{-(Z/x_L)t_0} = 1 - \epsilon^{-Bt_0} \qquad [3\cdot26]$$

where the zero subscripts associate this with the original wave of recovery voltage and $B = Z/x_L$. It is the total recovery voltage up to the time of return of the first reflection.

With attenuation neglected in the line, the wave reflected from the open end will be exactly the same as the original wave. This, upon returning to the source, impinges upon the source impedance pL which gives rise to a reflected wave and the sum of the two waves (incident and reflected) is

$$V_{R1}(p) = \frac{2p}{p+B}V_{R0}(p)$$

which yields

$$V_{R1} = 2Bt_1\epsilon^{-Bt_1} \qquad [3\cdot27]$$

The total voltage at the point of fault is then $V_{R0} + V_{R1}$ during the interval between the time of the return of the first reflection and the time of the return of the second reflection from the open end. The subscript 1 simply identifies this time interval. Time in equation [3·27] is, of course, zero until the time of return of the first reflection.

The reflected portion of equation [3·27] is

$$e_1'(p) = \left(\frac{p - B}{p + B}\right) V_{R0}(p)$$

so that

$$e_1' = -1 + \epsilon^{-Bt_1} + 2Bt_1\epsilon^{-Bt_1} \qquad [3\cdot28]$$

Again neglecting losses, this wave travels to the far end of the line and returns without any change in form. Impinging upon the source inductance, it produces a voltage

$$V_{R2}(p) = \left(\frac{2p}{p + B}\right) e_1'(p)$$

which, upon being solved, yields

$$V_{R2} = 2Bt_2\epsilon^{-Bt_2}(1 - Bt_2) \qquad [3\cdot29]$$

The second reflection from the fault point (or source) is

$$e_2'(p) = \frac{p - B}{p + B} e_1'(p)$$

which yields

$$e_2' = 1 - \epsilon^{-Bt_2} - 2B^2{t_2}^2\epsilon^{-Bt_2} \qquad [3\cdot30]$$

The additional voltage at the fault point caused by the return of this wave when reflected from the open end is

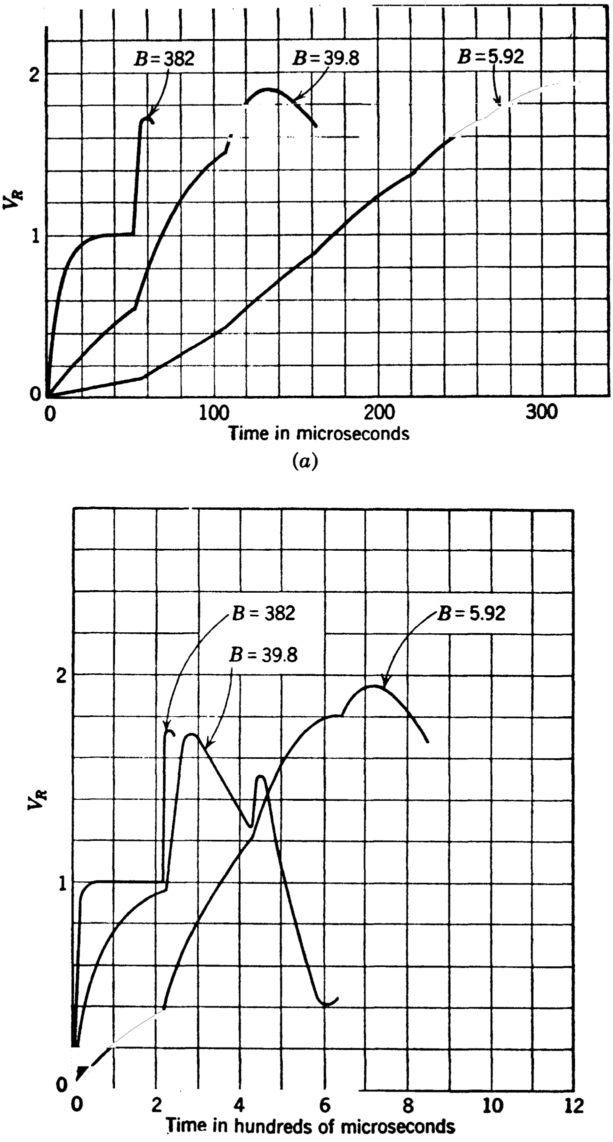
$$V_{R3}(p) = \left(\frac{2p}{p + B}\right) e_2'(p)$$

which, in turn, yields

$$V_{R3} = 2Bt_3\epsilon^{-Bt_3}(1 - 2Bt_3 + \tfrac{2}{3}B^2{t_3}^2) \qquad [3\cdot31]$$

This process can be continued as far as is necessary. If the length of line is short, then more reflections will be needed than if the line is long. In Fig. 3·11 are shown curves of total recovery voltage transients plotted from the foregoing analysis, assuming the traveling wave velocity equals the velocity of light.

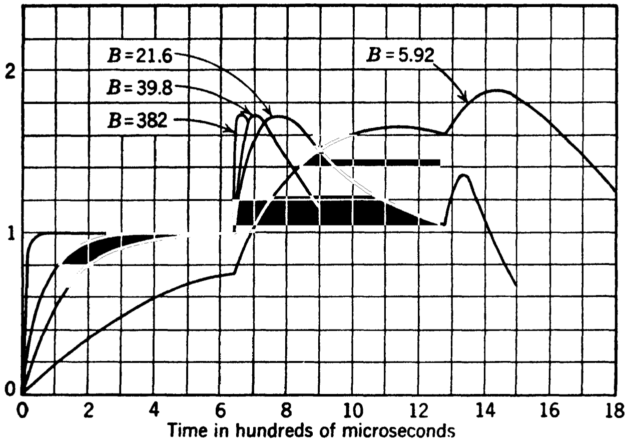
These curves apply for a single-phase system in accordance with the foregoing assumptions. They will apply approximately for a line-



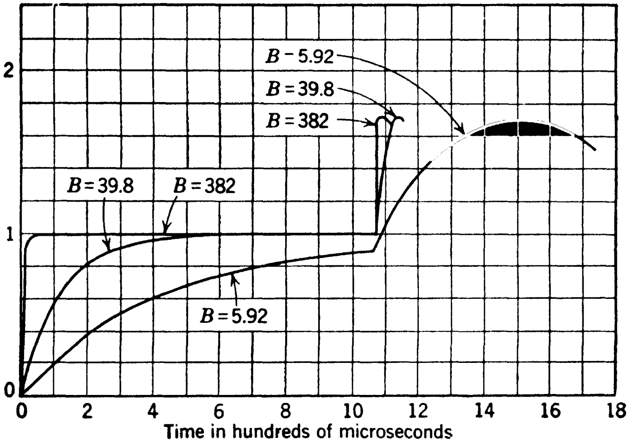
(a)

(b)

FIG. 3·11. (a) Recovery voltage vs. time. Line-to-ground fault, 5 miles of line. (b) Recovery voltage vs. time. Line-to-ground fault, 20 miles of line. (c) Recovery voltage vs. time. Line-to-ground fault, 60 miles of line. (d) Recovery voltage vs. time. Line-to-ground fault, 100 miles of line.



(c)



(d)

FIG. 3·11 (*Continued*).

to-ground fault on a three-phase system provided that, at the fault location, the positive, negative, and zero-sequence reactances are nearly the same in magnitude. In the three-phase system, this means that the source x_L is equal to the sum of the positive-, negative-, and zero-sequence reactances divided by 3.

The curves on this basis are fairly general in that the ratio of $B = Z/x_L$ is independent of system voltage. In other words, since the surge impedance of an overhead line is approximately 500 ohms, the quantity can be evaluated for any of the conditions shown in Fig. 3·11.

Therefore, in a system of given voltage, the range of values of B used in making the calculations corresponds to a range of rms symmetrical fault amperes. For instance, if $B = 39.8$ and $Z = 500$ ohms, then $x_L = 500/39.8 = 12.55$ ohms. In a 115-kv three-phase system, this corresponds to a fault current of $115{,}000/\sqrt{3}\,(12.55) = 5{,}270$ rms symmetrical amperes.

3·6 INTERRUPTION OFF NORMAL CURRENT ZERO

In certain instances, an interrupter may force a current zero to occur earlier than would be the case if the circuit elements alone were involved. That is, the arc voltage produced by the interruption may rise rapidly as the current is approaching a normal zero. In the extreme case, this is equivalent to opening the switch in Fig. 3·1 at any time regardless of the instantaneous value of current just preceding interruption. Practically, such interruptions may take place when a circuit breaker with large interrupting capacity is used to interrupt some very small current, such as the magnetizing current of a transformer, for example. High voltages may be produced under such circumstances. One illustration which has been the source of much analysis from time to time from this overvoltage point of view is the arc furnace transformer circuit in which frequent switching is done on the high-voltage side. Switching with the furnace electrodes withdrawn from the charge requires interruption of only magnetizing current, and in some instances the overvoltages produced have led to the installation of overvoltage protection or some other corrective means for overcoming the ensuing difficulties.

For the purpose of preliminary analysis, consider again the circuit of Fig. 3·1. The generator current will be defined

$$i_f = \frac{1}{x_L}\sin\,(t - \psi) \qquad [3\cdot32]$$

where ψ is the angle at which the switch is opened prior to a normal current zero. Then the recovery voltage transient is

$$\begin{aligned} V_R(p) &= i_f(p)Z(p) \\ &= \frac{x_C}{x_L}\left[\frac{p}{p^2 + (x_C/x_L)}\left(\cos\psi\,\frac{p}{p^2+1} - \sin\psi\,\frac{p^2}{p^2+1}\right)\right] \qquad [3\cdot33] \end{aligned}$$

which, solved operationally, becomes

$$V_R = \cos\,(t - \psi) - \sqrt{\cos^2\psi + \frac{x_C}{x_L}\sin^2\psi}\,\cos\left(\sqrt{\frac{x_C}{x_L}}\,t - \beta\right) \qquad [3\cdot34]$$

in terms of the normal peak voltage across the switch. The angle β is defined by

$$\cos\beta = \frac{\cos\psi}{\sqrt{\cos^2\psi + (x_C/x_L)\sin^2\psi}}$$

$$\sin\beta = \frac{\sqrt{(x_C/x_L)}\sin\psi}{\sqrt{\cos^2\psi + (x_C/x_L)\sin^2\psi}}$$

Equation [3·34] is the same as equation [3·5] if $\psi = 0$, which corresponds to normal zero current interruption. However, as ψ is increased (switch opened before normal zero), the recovery voltage transient increases in magnitude if x_C/x_L is large, which is usually true. If $x_C/x_L = \infty$, then even the smallest value of ψ will give rise to an infinite voltage. This would correspond to having no capacitance at all in the circuit of Fig. 3·1. If $x_C/x_L = 1.0$, then the amplitude of this natural-frequency component is independent of ψ. The amplitude of the natural-frequency oscillation is in the general case

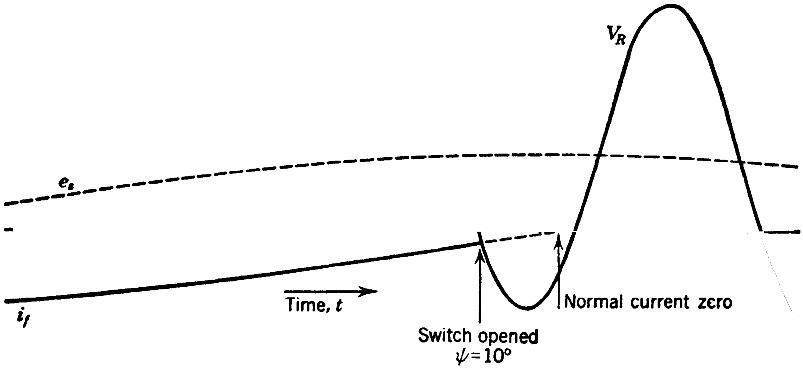


FIG. 3·12. Effect of circuit interruption before a normal current zero.

$$\sqrt{\cos^2\psi + \frac{x_C}{x_L}\sin^2\psi}$$

Therefore the peak value of recovery voltage across the switch is

$$V_{R\,\max} = 1 + \sqrt{\cos^2\psi + \frac{x_C}{x_L}\sin^2\psi} \qquad [3\cdot35]$$

if it is assumed that the two can have maximum value of the same polarity at some instant. If $\psi = 0$, this maximum value is 2 from equation [3·35] which, of course, checks that given by equation [3·5].

A detailed case of such "premature" circuit interruption is shown in Fig. 3·12. A plot of $V_{R\,\mathrm{max}}$ from equation [3·35] as a function of ψ with x_C/x_L as a parameter is shown in Fig. 3·13. It is apparent that, if x_C/x_L is high, even a very small angle ψ will give dangerously high overvoltages.

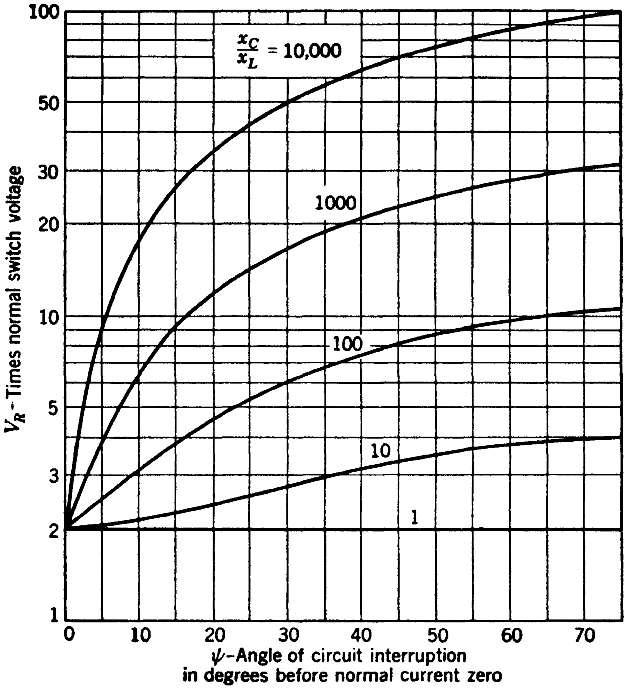


FIG. 3·13. Effect of premature circuit interruption upon maximum recovery voltage·

3·7 OTHER FACTORS

At this point it may be well to enumerate some of the factors that are of importance but have not been considered in the foregoing sections. Among these factors are the following:

1. Arc voltage as a function of current and time.
2. Damping.
3. Circuits with several natural frequencies.
4. Natural frequencies in the fault current preceding interruption.
5. Fault location.
6. Different line constants.
7. Type of fault.
8. Method of grounding the system.

9 Effect of load.

10 Effect of modifying the terminal impedances with capacitors.

11. Effects of restriking during the recovery period.

Certain of these factors can be included simply by continuing the analysis. However, to cover each case analytically becomes far too laborious and time consuming. As a consequence, the results shown in the next chapter are those obtained through the use of the transient analyzer with occasional reference to the theory and calculations included in this chapter.

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1 *Transients in Linear Systems*, Vol I, *Lumped-Constant Systems* by M F GARDNER and JOHN L BARNES, John Wiley & Sons, New York, 1942

2 *Traveling Waves on Transmission Systems*, by L V BEWLEY, John Wiley & Sons, 1933 (Second edition in press)

3 "Switching Surges with Transformer Load-Ratio Control Contactors," L F BLUME and L V BEWLEY, *A I E E Transactions*, **56**, 1937, pages 1464–1475

4 "Traveling Waves Initiated by Switching," by L V BEWLEY, *A I E E Transactions*, **58**, 1939, pages 18–26.

4 SYSTEM VOLTAGE RECOVERY CHARACTERISTICS— MINIATURE-SYSTEM APPROACH

4·1 INTRODUCTION

Because of the numerous factors mentioned in the preceding chapter which influence the recovery voltage transients in power systems, the miniature system or model technique of studying the effects of these factors was developed. It consists simply of reproducing in miniature the actual circuit or system to be studied. In this miniature system, faults of various kinds can be applied, switching can be done, constants can be varied, etc. Results can be obtained quickly and permanent records can be obtained by photographic means.

In this chapter, some of the results of investigations with miniature systems will be given; and the effects of several factors not readily calculable will be summarized.

4·2 TRANSMISSION LINE REPRESENTATION

In Chapter 1, the three-phase miniature-system transmission line was described briefly. Some additional discussion is pertinent in order to visualize more clearly the degree of approximation inherent in the method of substituting a semidistributed-constant network for the actual transmission line of uniformly distributed constants.

Figure 4·1 shows several voltage recovery curves, each having been obtained with a different method of representation in miniature of an actual system. The actual system was that shown in Fig. 3·5(*a*) with 90 miles of transmission line connected to the source generator, the line being open at the far end. The line-to-ground fault (with no arc voltage) was left on at the sending-end terminal sufficiently long in each case so that interruption took place from a steady-state condition (that is, there were no initiating transients; only symmetrical fundamental-frequency current was flowing prior to circuit interruption at normal current zero). The curve *A* in Fig. 4·1 is the recovery-voltage transient obtained when nine π sections such as the one shown in Fig. 1·19 were connected in series to represent the 90 miles of line. It approximates the behavior of a uniformly distributed-constant line (curve *D*). This approximation is very good, even to the timing of the return of the reflected wave from the far end of the line. Since no losses were assumed in calculating curve *D*, it is to be expected that the crest voltage after reflection would be higher than that actually obtained in any of the miniature-system setups.

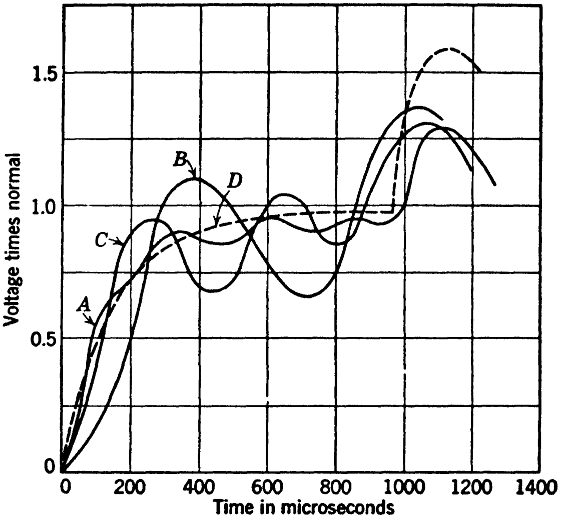


FIG. 4·1. Effect of method of transmission line representation on transient voltage recovery. System as in Fig. 3·5(*a*) with 90 miles of line. See text for identification of curves.

Curve *C* obtained for a weighted-constant[7] double-π representation shows a tendency to give a higher first crest voltage than that obtained either with the nine π sections or by distributed-constant calculations. There is a pronounced voltage drop after this first crest is reached before the voltage continues on upward to the maximum crest value. It will be observed that the axis of oscillation of this first period is the true exponentially rising recovery characteristic.

Curve *B* shows the recovery characteristic for an equivalent double-π representation (i.e., two 45-mile equivalent π sections in series). This gives a still higher first crest voltage, but the axis of oscillation appears to be approximately the exponentially rising true recovery characteristic.

Several significant points of interest are brought out in this figure. All three methods of miniature representation considered yield essentially the same time to maximum crest voltage, and only slight discrepancies appear to exist for the magnitude of the maximum crest voltage. The greatest discrepancies occur during the initial period before the reflection returns from the far end of the line. With the nine-π-section representation there are several oscillations, small in magnitude, which deviate only slightly from the true recovery characteristic. Experience, as illustrated by these curves as well as by traveling wave theory, indicates that for any given length of line, such π representation is satisfactory for many purposes provided that enough sections are used. If two 10-mile equivalent π sections are used to represent a 20-mile line, for example, one cannot expect very accurate duplication of true initial recovery transients. However, maximum crest and time to maximum crest would not be in great error.

4·3 A SPECIFIC ILLUSTRATION—PROTECTOR-TUBE RECOVERY VOLTAGES

For the particular system shown in Fig. 4·2, the operation of a protector tube was simulated by applying a line-to-ground fault and clearing it at the first subsequent current zero. It will be observed that this system has two sources (they were electrically identical) of short-circuit current and is a reasonable physical duplicate of a large actual system. By means of synchronous switching equipment, it was possible to apply the fault at any instant on the voltage wave and

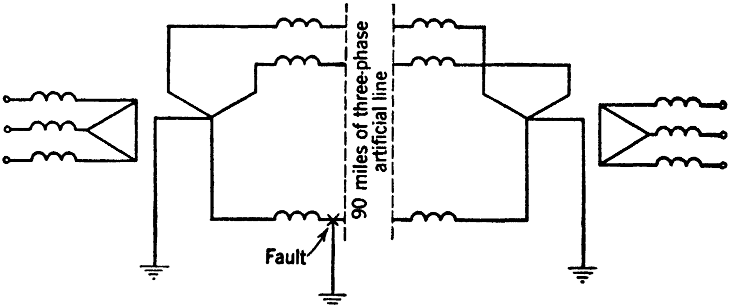


FIG. 4·2. Miniature system used in obtaining oscillograms shown in Fig. 4·3.

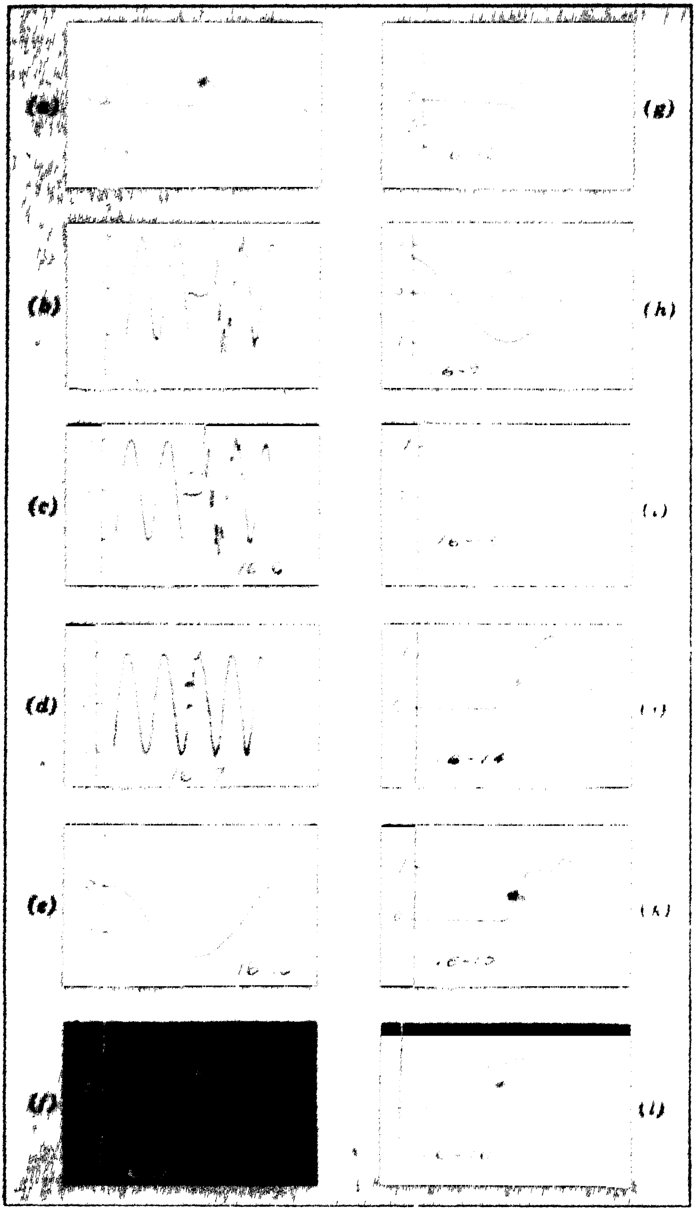


FIG. 4·3. Oscillograms of power system recovery-voltage phenomena for the system shown in Fig 4·2 See text for identification of oscillograms interrupt it at the first current zero. The resulting transients of fault current and recovery voltage are shown in Fig. 4·3. These oscillograms were taken of the cathode-ray oscillograph screen with the sweep properly synchronized to give a stationary trace.

The oscillograms show the complete transients for several instants of fault application. The single-line-to-ground fault current corresponds to 1,830 rms symmetrical amperes in a 138-kv system. Figure 4·3(*a*) shows the maximum recovery voltage obtainable. This maximum value was obtained by adjusting the fault-angle control of the switching device until the maximum recovery voltage peak was reached. For calibration purposes, a half-cycle of normal fundamental-frequency line-to-neutral voltage is shown. Thus, magnitude and time to maximum crest of the recovery voltage can be determined directly from such a record.

Figures 4·3(*b*), (*c*), and (*d*) show a fault occurring before normal voltage crest, at voltage crest, and after voltage crest, respectively. The time scale was changed for these so that several cycles of fundamental frequency are shown. Figures 4·3(*e*), (*f*), and (*g*) illustrate the effect of such variation in fault angle on the magnitude and duration of fault-current flow, the three conditions being for a fault occurring before normal voltage crest, at voltage crest, and after voltage crest, respectively. Figure 4·3(*h*) shows a half-cycle of sustained fault current for comparison.

Figures 4·3(*i*), (*j*), (*k*), and (*l*) are recovery voltages for the same system but for other angles of fault application. These illustrate the effect of this factor on the recovery voltage of this particular system for a given fault location. The time scale in these four oscillograms is half as much per-unit length as it is in Fig. 4·3(*a*).

4·4 GENERALIZED RESULTS

In discussing transient recovery voltages which occur when fault currents are interrupted by protector tubes, it should be emphasized that generally there are two basic differences between these voltages and those obtained in connection with circuit breaker operation. In the first place, because of the appreciable transmission line capacitance, expulsion protector-tube recovery voltages are generally of a slower nature than those very fast ones sometimes obtained in a circuit breaker operation which severs one part of a system from another. As a second point of difference, circuit breaker interruptions more often occur from a steady-state condition of fault current (i.e., no fault-current natural frequencies), whereas a protector tube interrupts generally from a transient condition. The transient oscillations in the fault current are clearly evident in Figs. 4·3(*e*), (*f*), and (*g*). These are actually traveling waves of current. Ordinarily a circuit breaker is called upon to operate after fault current has been flowing for several cycles, a duration which is sufficient for the natural-frequency oscillations to disappear from the fault current and for the d-c offset to be significantly reduced. Protector tubes may operate in a single half-cycle, and therefore in many instances the natural-frequency oscillations in the fault current may not be damped out when interruption takes place. The amount of this transient disturbance present (and also the amount of d-c offset) for a given case is a function of the point on the voltage wave at which the fault occurs.

Figure 4·4(*a*) shows several single-line-to-ground fault recovery voltage transients, each one for a particular value of the angle of fault application as indicated. These were obtained for the system shown in Fig. 3·5(*a*) with 60 miles of line. The fault angle is designated as θ_A which is so defined that a value of $\theta_A = 0$ corresponds to the time at which the steady-state voltage on phase *a* is a maximum, the single-line-to-ground fault always being applied on phase *a*. Values of θ_A from $-90°$ to 0 indicate application of the fault before voltage crest is reached; values of θ_A from 0 to 90° indicate application of the fault after the voltage crest. Figure 4·4(*c*) shows the crest values of the curves in Fig. 4·4(*a*) plotted as a function of θ_A. The maximum crest recovery voltage was obtained for $\theta_A = -15°$. Here, line losses were high enough (line resistance was 40 per cent of line reactance) so that the natural-frequency oscillations in the fault current were damped out before interruption took place.

Figure 4·4(*b*) shows several recovery-voltage transients as obtained on the same miniature system representing 100 miles of low loss line (line resistance was 10 per cent of line reactance) and source reactance as indicated. In Fig. 4·4(*d*) the crest values reached in Fig. 4·4(*b*) are plotted as a function θ_A. Here, the maximum recovery voltage of 1.7 was reached for either of two values of θ_A, namely, $-2°$ and 30°, while in between these two peaks lies a low value of 1.2 at 18°. This characteristic of alternate peaks and valleys in the crest recovery voltage versus θ_A curve is particularly marked for low loss lines and for low values of fault current.

From observations of the transient fault current, it was possible to associate these peaks and valleys with the oscillations in the fault current which were of a frequency essentially inversely proportional to the length of line represented in the miniature system. Corresponding field tests indicate that this frequency corresponds to the frequency with which a voltage (or current) surge travels four times over the line length involved. (The natural period of a line open at one end is the time required for a surge to travel over the entire length four times.) The number of such traversals which can take place during

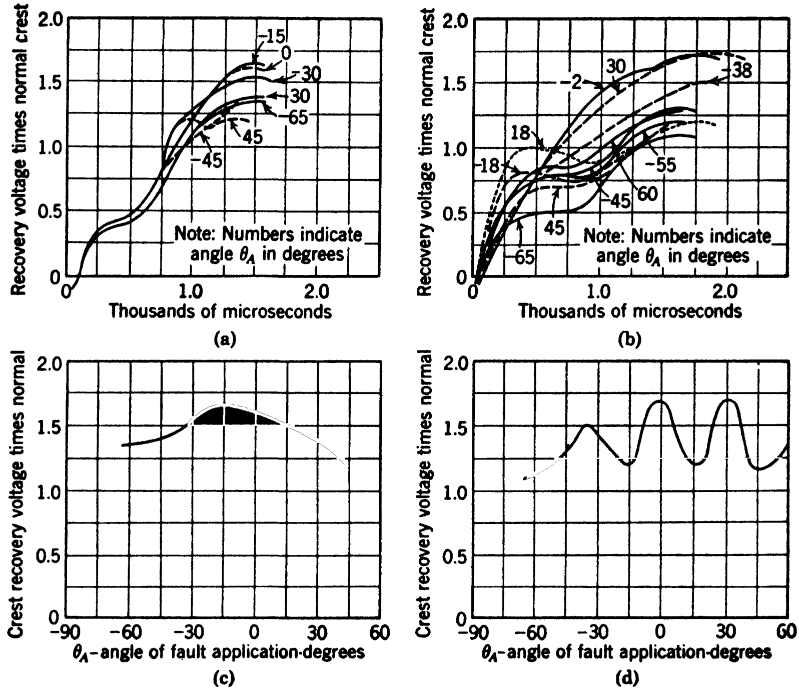


FIG. 4·4. Effect of fault angle on recovery voltage. $X_0/X_1 = 0.4$ at source supplying 650 amperes rms symmetrical line-to-ground fault current in 115-kv system.

the time that the fault is on is a function of the angle at which the fault is applied. Further analysis indicated that each peak of recovery voltage in Fig. 4·4(*d*) could be associated with fault-current interruption at an instant of minimum rate of change of fault current, and each valley with interruption at a maximum rate of change of fault current. Also, since initial rate of rise of recovery voltage is proportional to the rate of change of fault current just prior to interruption, it was possible to associate a maximum initial rate of rise with fault interruption at a maximum rate of change of current and a minimum initial rate of rise with fault interruption at a minimum rate of change of current. If a system has losses low enough so that these fault-current natural-frequency oscillations exist when the first normal current zero is reached, these alternate peaks and valleys in the crest recovery voltage versus θ_A curve will occur.

The significance of the curves of Fig. 4·4 is that in general it is necessary to determine a recovery-voltage envelope obtained by applying the fault over a range of values of θ_A. It is conceivable that a single-line-to-ground fault could occur for any value of θ_A between $-90°$ and $+90°$. However, it has been found that the recovery-voltage envelope is generally determined by values of θ_A between $-20°$ and $+20°$, approximately, in other words, for faults occurring within a range of $\pm 20°$ of voltage crest.

The entire recovery voltage envelope is of importance, and for successful operation it is essential that the recovery dielectric strength within the tube should be greater at all times than the voltage recovery characteristic.

It was found that the initial period of recovery voltage is quite definitely a function of fault location along the line, whereas the maximum crest voltage reached and the time to maximum crest voltage were not greatly affected by fault location. The reason for this is that the initial transient period following interruption is governed by the rate of change of fault current just prior to interruption. Fault current changes with fault location along the line, being greater if near the source and becoming less the more remote the fault location is with respect to the source of voltage. Therefore, the initial voltage recovery rate is greater if the fault is near the voltage source. Since the time to maximum crest recovery voltage is a measure of the natural frequency of oscillation of the entire system, it is not subject to much change with change in fault location. Therefore, time to maximum crest appears to be more descriptive as a system characteristic than the first crest or time to first crest.

The initial rate of rise of recovery voltage can be evaluated approximately in many instances by a relatively simple calculation, based on injecting the symmetrical fault current back into the network at the point of fault. If the short-circuit current at any point along a line is known, then the initial rate of rise is

$$IZ\omega \times 10^{-6} \text{ volt per microsecond} \qquad [4\cdot1]$$

where I = crest value of the symmetrical fault current in amperes.
$\omega = 2\pi f$ (f is system frequency).
Z = surge impedance in ohms viewed from the fault point.

The initial rate of rise calculated from this expression is the maximum that can be obtained (neglecting natural-frequency components in the fault current) since a symmetrical current wave is assumed. For any instant of fault occurrence which would give a d-c offset component in the fault current, the initial rate of rise would be less. For any system having low losses such that the natural-frequency oscillations would not be damped out when interruption occurred, the maximum initial rate would be higher. Under such conditions, the maximum initial rate would be difficult to calculate.

Single-line-to-ground faults are more likely to occur than any other type, and also, since in the solidly grounded system the double-line-to-ground faults that are most likely to occur are generally no more severe from the standpoint of tube operation, most of the results presented are for single-line-to-ground faults. The system selected for study included source reactances and the unloaded line open at the far end as shown in Fig. 4·5. Most of the work was done at the sending end of the transmission line.

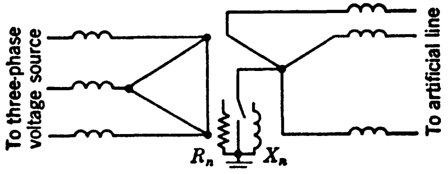


FIG. 4·5. Equivalent circuit for source impedances.

Figure 4·6 shows the effect of line length for several values of fault current in amperes referred to a 115-kv system. The curves are valid for a system of any voltage provided that the fault currents are changed in proportion to the change in system voltage. In other words, it is possible to define a single system such as these in terms of ohms of reactance in the source (x_1 and x_0) and miles of line. The results have more practical significance when they are plotted with actual amperes as a parameter. The results in this figure indicate that lower currents and shorter line lengths at a given system voltage tend to give higher crest values of recovery voltage in solidly grounded systems. Time to crest increases with line length. This can be explained by the fact that voltage reflections from the far end of the line return less frequently than for shorter lengths, and therefore a longer time is required for this building-up process. Higher currents reduce the time to crest since the short-circuit current in the miniature systems studied is a measure of source reactance, which in turn increases the natural frequency of the system and consequently results in a shorter time to crest voltage when interruption takes place.

Figure 4·7 is a more comprehensive summary of results obtained for various lengths of line, for various system voltages, and for three

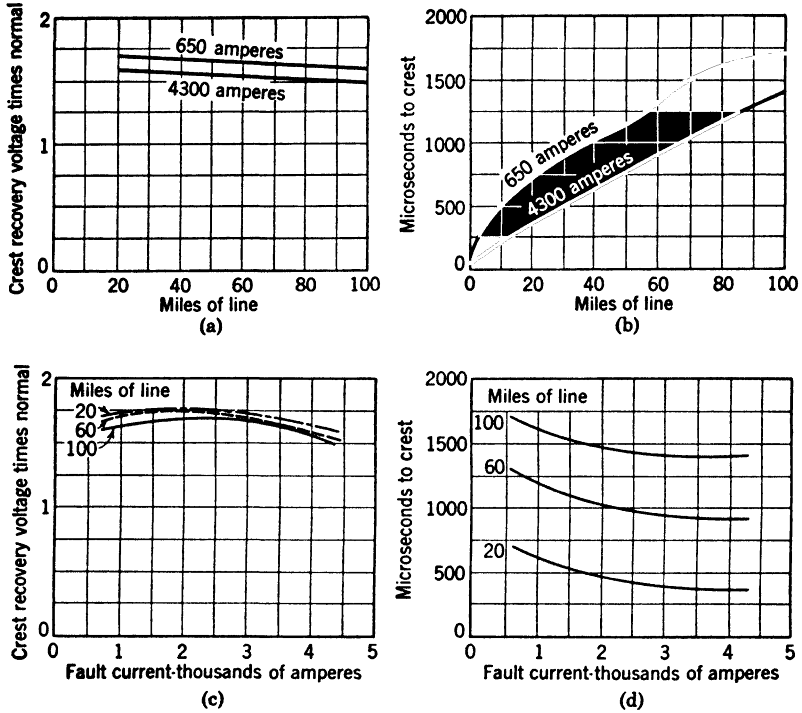
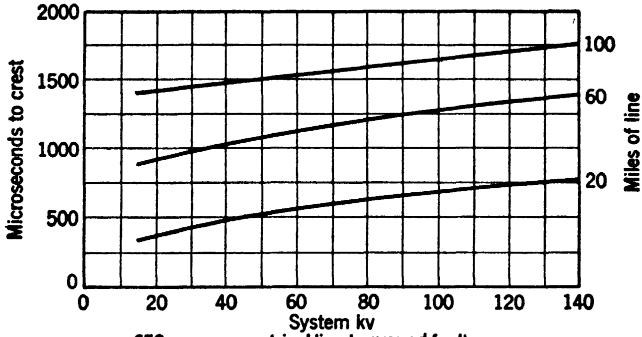
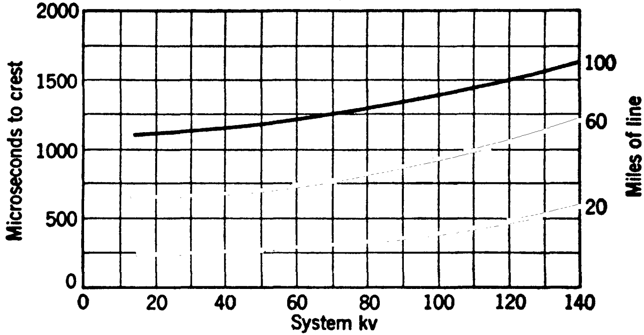


FIG. 4·6. Effect of line length and fault current on recovery voltage. $X_0/X_1 = 0.4$ at source. Currents are rms symmetrical values for line-to-ground fault in 115-kv system.

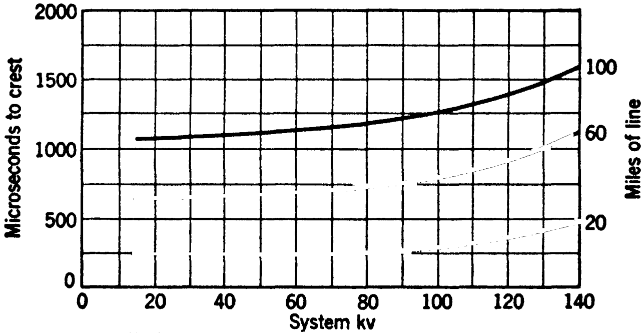
values of fault current. These curves are of importance in that they show clearly the effect of varying any one of the three most significant factors. The curves are valid for the relatively simple solidly grounded systems studied, but the qualitative effects of these factors can be shown to be valid in general for the more complex systems as well. Crest recovery voltages are not summarized since the results indicated that, in solidly grounded systems, line-to-ground fault recovery voltages do not vary greatly unless influenced by losses. This summary in Fig. 4·7 excludes the effects of these factors. Conse-



650 rms symmetrical line-to-ground fault amperes
(a)



2200 rms symmetrical line-to-ground fault amperes
(b)



4300 rms symmetrical line-to-ground fault amperes
(c)

FIG. 4·7. Time to maximum crest recovery voltage for solidly grounded systems. $X_0/X_1 = 0.4$ at source. Single-line-to-ground fault.

quently the crest recovery voltages for any of these cases is probably between 1.5 and 1.75, with 1.65 to 1.7 being typical for higher voltage systems and 1.55 to 1.6 being typical for lower voltage systems.

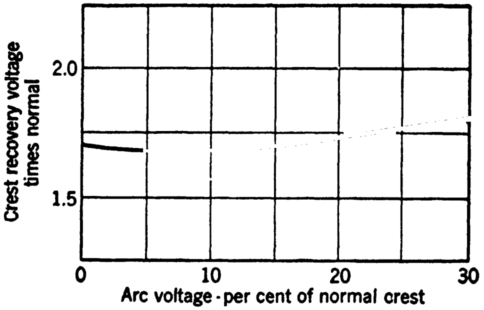


FIG. 4·8. Effect of arc voltage. $X_0/X_1 = 0.4$ at source. 100 miles of line. 650 amperes rms symmetrical line-to-ground fault current in 115-kv system at source.

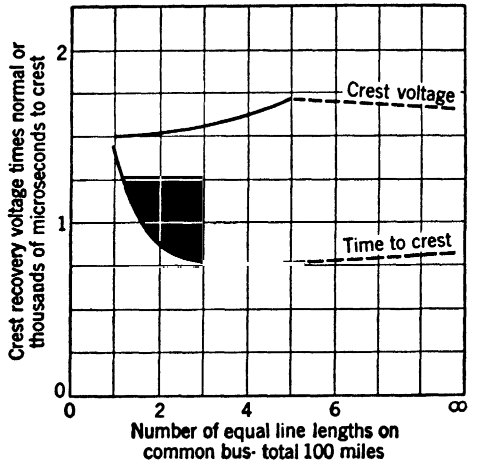


FIG. 4·9. Effect of line grouping. $X_0/X_1 = 0.4$ at source. 4,300 amperes rms symmetrical line-to-ground fault current in 115-kv system at source.

Figure 4·8 shows the effect of varying the magnitude of constant arc voltage in the fault. Over a reasonable range, this factor has little effect on the crest value of recovery voltage. However, it was shown in Chapter 3 that varying this factor does change the fault angle which gives the maximum overshoöt and the rate of rise during the initial part of the transient recovery period.

Figure 4·9 shows the effect of line grouping on a common bus. This curve shows that for a given aggregate number of miles of line, the time to maximum crest recovery voltage is materially reduced as the line is broken up into more short sections radiating from a bus. A minimum value is reached, however, which corresponds to lumping the capacitance of all lines at the point of fault. This fact suggests the possibility of calculating recovery voltages approximately for certain cases using lumped constants when no long lines are involved.

Figure 4·10 shows the effect of neutral grounding through resistance.

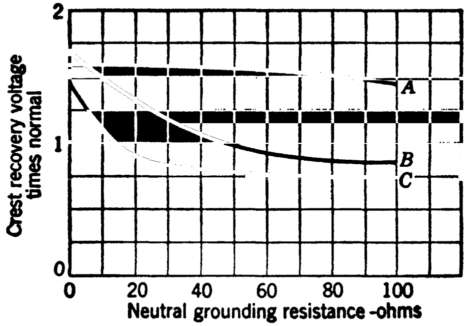


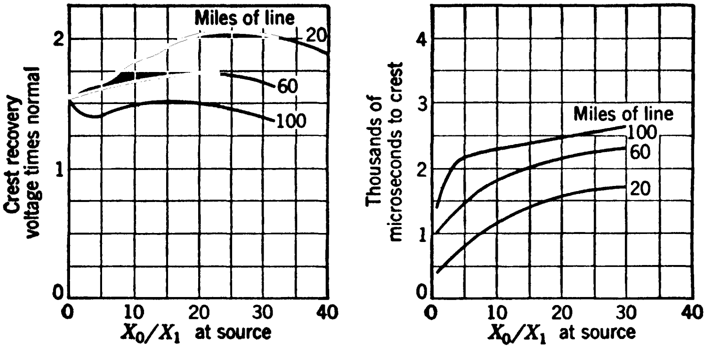
FIG. 4·10. Effect of neutral grounding resistance. 100 miles of line. $X_0/X_1 = 0.4$ at source. (*A*) 650, (*B*) 220, (*C*) 4,300 rms symmetrical line-to-ground fault amperes in 115-kv system for zero neutral resistance.

The qualitative effect is very similar to that observed when tower footing resistance is inserted (see Fig. 4·14). This is to be expected since the circuit connections differ only slightly for the two conditions.

The effect of reactance grounding is shown in Fig. 4·11. Both maximum overshoot and time to crest are significantly altered by this factor. As more reactance is added, time to crest becomes greater because of the lower zero-sequence natural frequency. This soon becomes the predominant factor in determining both time to crest and maximum overshoot, particularly for the short lines where the effect of the distributed constants of the line become relatively small.

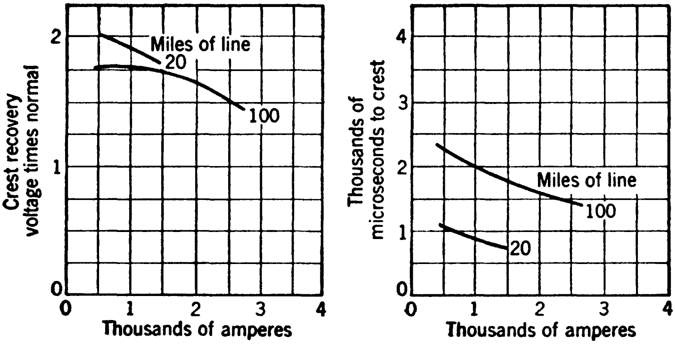
Figure 4·12 shows the effect of line length and short-circuit amperes when a line is fed from both ends. Qualitatively the effect is quite similar to that shown in Fig. 4·7 for lines fed from one end only.

Figure 4·13 shows the effect of fault location along a line energized from both ends. There is little change in crest recovery voltage, but there is some change in time to maximum crest as the fault point is moved along the line. This effect was much less pronounced for lines fed from only one end. However, a fault at the mid-point of a 100-mile line fed from both ends is the same in effect as a fault at the far end of two 50-mile lines in parallel and bussed at both ends with



(a) (b)

Fig. 4·11. Effect of neutral grounding reactance. Single-line-to-ground fault of 4,300 amperes rms symmetrical at source in 115-kv system for $X_0/X_1 = 0.4$.



(a) (b)

Fig. 4·12. Effect of fault current. Equal generating capacity at both ends of line. Solidly grounded system. Currents are rms symmetrical amperes for line-to-ground fault at one end in 115-kv system.

twice the fault current. Therefore, it appears that the effect shown in Fig. 4·9 is playing an important part.

The effect of tower footing resistance for a single-line-to-ground fault on a line fed from both ends is shown in Fig. 4·14. The crest voltage is seen to be reduced as the tower footing resistance is increased, and the extent of this effect is greater for the larger available short-circuit currents.

In conclusion, the following general statements can be made:

1. Different angles of fault application materially affect system voltage recovery characteristics. When a fault is removed in less than a cycle after its occurrence, the transient recovery characteristic is best described as an envelope of the maximum transient voltage at any time for any instant of fault application.

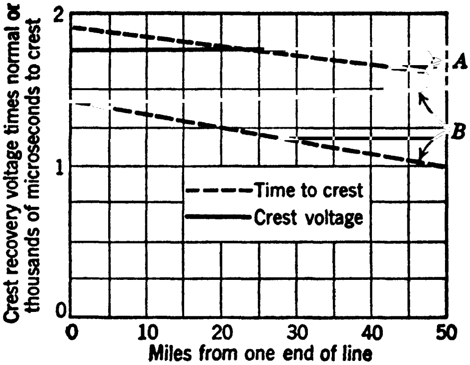


FIG. 4·13. Effect of fault location; 100 miles of line with equal generating capacity at both ends; solidly grounded system. (*A*) 1,240 rms symmetrical line-to-ground fault amperes at either end in 115-kv system. (*B*) 2,740 rms symmetrical line-to-ground fault amperes at either end in 115-kv system.

2. Crest recovery voltages obtained in the solidly grounded neutral systems investigated varied from 1.5 to 1.75 when tower footing resistance was neglected. The effect of losses in the system (including tower footing resistance) is to decrease the overshoot for single-line-to-ground faults. It appears that 1.75 is near an upper limit which may be reached in higher voltage systems, whereas 1.5 would be more likely to be reached as a maximum in lower voltage systems. This is in general agreement with field tests and theoretical considerations. System loads and interconnections generally would tend to reduce the maximum overvoltage reached in an actual system.

3. Neutral grounding resistance produces a decrease in maximum overshoot in a manner very similar to that produced by tower footing resistance.

4. Neutral ground reactance increases the time to crest recovery voltage, since the zero-sequence natural frequency is reduced as more reactance is added in the neutral.

5. The magnitude of the constant arc voltage corresponding to the arc voltage across a protector tube (over a practical range) appears to have little effect upon the voltage recovery characteristics.

6. Length of line, system voltage, and fault current appear to be the most significant factors in determining the voltage recovery characteristics in solidly grounded neutral systems.

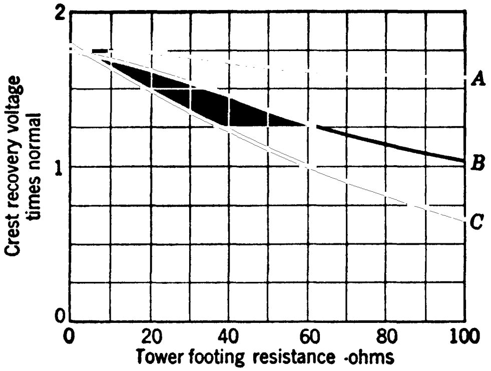


FIG. 4·14. Effect of tower footing resistance. Equal generating capacity at both ends of line. Solidly grounded system. (*A*) 100 miles of line; 414 rms symmetrical line-to-ground fault amperes at either end in 115-kv system. (*B*) 100 miles of line; 1,240 rms symmetrical line-to-ground fault amperes at either end in 115-kv system. (*C*) 20 miles of line; 1,460 rms symmetrical line-to-ground fault amperes at either end in 115-kv system.

4·5 SOME RECOVERY VOLTAGE CHARACTERISTICS FOR LOW-VOLTAGE 60-CYCLE CIRCUITS

In order to show some of the low-voltage or distribution-system voltage recovery characteristics, this section is included. The results are of interest in connection with the operation of overvoltage protective devices, as well as circuit interrupting devices such as fuses, in these systems.

The system studied is shown in Fig. 4·15. The single-phase lines as well as the three-phase portion and the ground or neutral return wires were No. 6 copper. A spacing of 36 inches between the phase wire and the neutral and between phases on the three-phase line was assumed.

The impedance of the single-phase line was calculated to be $4.46 + j1.6$ ohms per mile. The capacitance to ground was 0.011 μf per mile. The positive-sequence impedance of the three-phase line was $2.23 + j0.76$ ohms per mile with a positive-sequence capacitance of 0.016 μf per mile. The zero-sequence impedance of the three-phase line was $2.95 + j2.75$ ohms per mile with a zero-sequence capacitance of 0.01 μf per mile. Substation transformer three-phase ratings of 50, 100, and 500 kva were considered. The system voltage was 6,900 volts from line to neutral (12 kv line to line at the substation transformer). The high-voltage side of the substation transformer was considered an infinite bus, and the system was not loaded.

The study was made with no intentional arc voltage during the fault-current conducting period. Its purpose was to determine the severity of recovery voltages for systems such as that shown in Fig. 4·15.

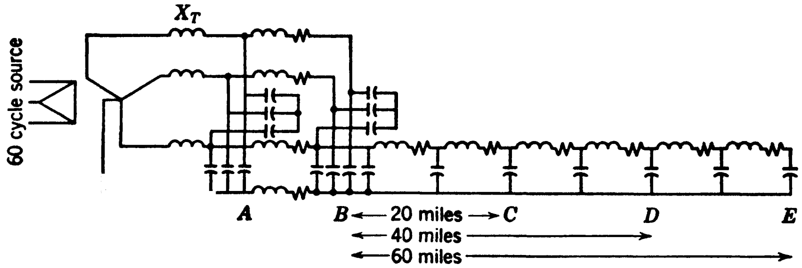


Fig. 4·15. Representative circuit for 6,900 volt REA distribution circuits.

The severity of these voltages was determined in terms of the following criteria.

1. The maximum recovery voltage that would appear across the interrupting device following a flashover and a subsequent clearing at the first current zero. (M_M on curves.)

2. The minimum time to reach the crest of this recovery voltage transient. (T_M on curves.)

3. The minimum time for this recovery voltage to reach 90 per cent of normal crest voltage. ($T_{0.9}$ on curves.)

Cases studied (refer to Fig. 4·15):

1(*a*) With no three-phase line (*A* and *B* coincident), recovery voltages were observed for clearing a fault adjacent to the transformer with 20 miles of single-phase line, 40 miles of single-phase line, and 60 miles of single-phase line.

(*b*) These tests were repeated for a fault 20 miles from the transformer, 40 miles from the transformer, and 60 miles from the transformer.

In these tests if the high-voltage side of the transformer is considered an infinite bus and if there is no three-phase transmission line on the low side of the transformers, there will be no mutual coupling between phases and the effect of unbalance caused by different

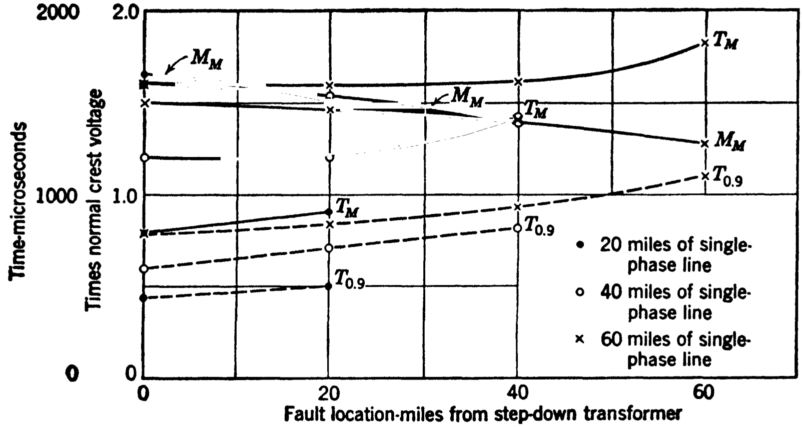


FIG. 4·16. Recovery voltages on 6,900 volt REA distribution circuits. No three-phase line. 50 kva transformer.

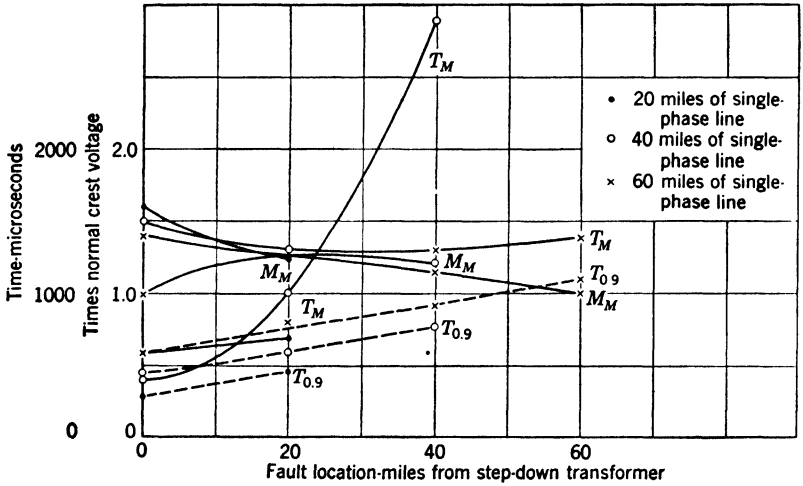


FIG. 4·17. Recovery voltages on 6,900 volt REA distribution circuits. No three-phase line. 100-kva transformer.

lengths of single-phase line connected to each of the three phases need not be considered.

2. With ten miles of three-phase line connected between the transformer and the single-phase line, the above tests were repeated.

In all these cases, recovery voltages were observed for the step-down transformers rated 50 kva, 100 kva, and 500 kva. For each case, the angle of application of the fault was varied so as to produce the most severe recovery voltage condition. Arc voltage during the period of fault-current flow was always made essentially zero.

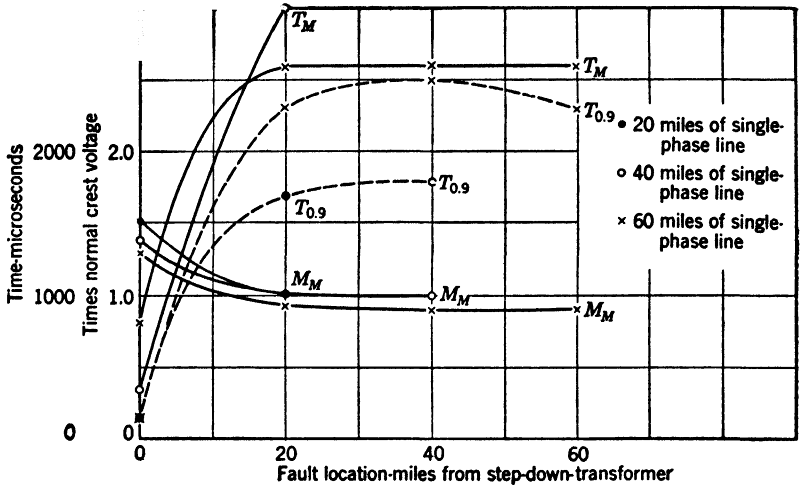


FIG. 4·18. Recovery voltages on 6,900 volt REA distribution circuits. No three-phase line; 500-kva transformer.

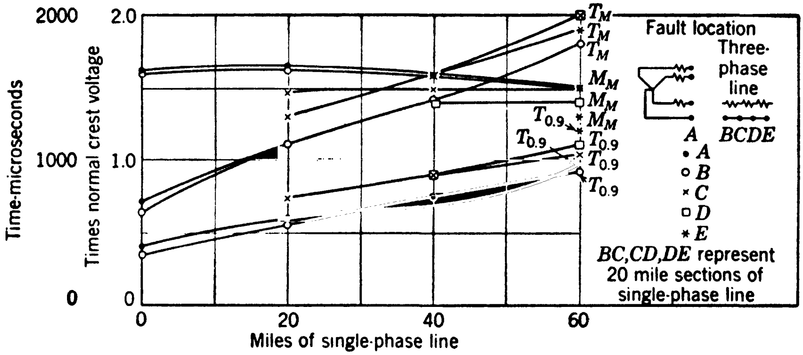


FIG. 4·19. Recovery voltages on 6,900 volt REA distribution circuits. Ten miles of three-phase line; 50-kva transformer.

The curves in Figs. 4·16, 4·17, and 4·18, show the time to 0.9 normal crest, time to maximum crest, and the magnitude of the maximum crest for various lengths of single-phase line when there is no three-phase line connected on the low-voltage side of the transformers. Figure 4·16 gives the data obtained with a transformer rated 50 kva, Fig. 4·17 for a transformer rated 100 kva, and Fig. 4·18 for a transformer rated 500 kva.

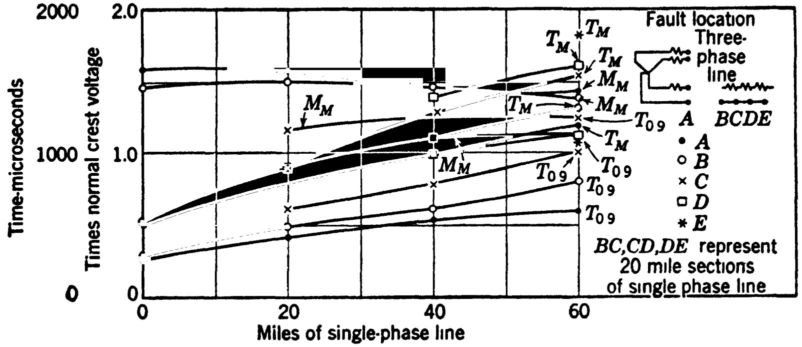


FIG. 4·20. Recovery voltages on 6,900 volt REA distribution circuits. Ten miles of three-phase line; 100-kva transformer.

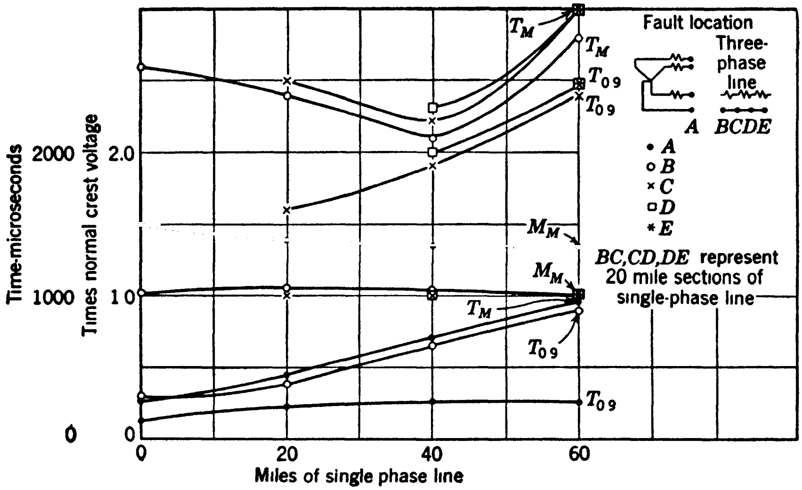


FIG. 4·21. Recovery voltages on 6,900 volt REA distribution circuits. Ten miles of three-phase line; 500-kva transformer.

These curves show that in general the time to 0.9 normal crest voltage increases as the number of miles of single-phase line is increased and this time also increases as the fault is assumed farther away from the step-down transformer.

In general the magnitude of the maximum crest of recovery voltage decreases as the number of miles of single-phase line increases and the magnitude of this crest also decreases as the distance between the fault and the substation or step-down transformer is increased.

The time to maximum crest increases as the distance between the fault and the step-down transformer is increased.

Figures 4·19, 4·20, and 4·21 show the effect of fault location and the effect of different lengths of single-phase line when ten miles of three-phase line connect the transformer and the point at which single-phase distribution begins.

In general these curves show that the severity of the recovery voltage is lessened as the fault becomes more remote from the step-down transformer and the severity of the recovery voltage is lessened as more single-phase line is added.

A tabulation of fault currents in rms amperes for the various fault locations and conditions is shown in Table 4·1.

TABLE 4·1

TABULATION OF FAULT CURRENTS (RMS AMPERES)

Transformer Kva	Fault Location *A*	*B*	*C*	*D*	*E*
		No Three-Phase Line			
50		43.6	34.0	27.5	22.7
100		87.2	48.7	33.0	25.7
500		436.0	70.0	43.0	33.0
		10 Miles of Three-Phase Line			
50	43.6	33.0	30.8	25.4	21.9
100	87.2	61.6	35.2	27.9	22.8
500	436.0	110.0	42.7	31.4	24.8

Various lengths of single-phase line were added to the two unfaulted phases to observe the effect of unbalance in the system capacitance on the recovery voltage following the clearing of a fault. It was found that the addition of single-phase line in the two unfaulted phases had very little effect on the recovery voltage on the faulted phase. Assuming an infinite bus on the high-voltage side of the substation transformer there would have to be some mutual coupling between phases before the length of line on one phase would affect appreciably any other phase. As this mutual coupling is present only in the three-phase line and as the three-phase line is short, there is very little coupling between phases.

A comparison of the results shown in this section with results of field tests on similar systems shows that in general the results shown herein are noticeably more severe. This is to be expected for several reasons, viz.:

1. The severity of recovery voltage depends on the angle of fault incidence if interruption occurs at the first current zero, as has been pointed out. In field testing, interruptions may not occur for a number of cycles where interruption is by fuses, for instance. Consequently, the transient recovery voltage obtained in field testing does not necessarily correspond to that which was obtained for the fault angles which gave the most severe envelope of recovery voltage, the circuit being interrupted always at the first current zero.

2. The miniature-system line was unloaded. The presence of load distributed along the line will have a pronounced effect upon the severity of recovery voltage.

3. No arc voltage was introduced in the miniature system. Test results for actual system tests indicate that some interrupting devices introduce appreciable arc voltage, especially during the half-cycle preceding interruption. This causes a phase shift of current such that interruption takes place earlier in the cycle at a lower instantaneous value of fundamental-frequency voltage, so that, generally speaking, recovery voltages are reduced in severity.

4. No damping in the arc path immediately after interruption was considered in the miniature system. That some finite resistance exists during the scavenging of the products of arcing is to be expected. The damping effect may not be noticeable for the slower recovery rates, but it is probably quite effective for the highest recovery rates.

It should be emphasized in connection with this entire chapter, that these results are strictly *circuit* characteristics. In any actual system, the arc characteristics of the *interrupting device* may modify these characteristics appreciably.

4·6 EFFECT OF NEUTRAL GROUNDING IMPEDANCE ON SYSTEM TRANSIENT VOLTAGE RECOVERY CHARACTERISTICS

The foregoing sections in this chapter have dealt with the recovery voltages following interruption of a single-line-to-ground fault. The recovery voltage transients for other types of faults are also of considerable importance. In this section the recovery voltage for the first phase to clear of a double-line-to-ground fault will be discussed in detail, with some additional comments on three-phase faults.

As has been shown in Chapter 3 and in the first part of this chapter, the transient recovery voltage of the system with respect to ground is the sum of the fundamental-frequency component and the natural-frequency component. At the instant of interruption, the system voltage starts to return to normal and, if the arc is successfully extinguished, eventually, after the transient has disappeared, normal voltage will be restored.

At the instant of final zero current (instant of current interruption), the recovery voltage can be divided into two components. One is the fundamental-frequency component whose magnitude is equal to the *instantaneous* fundamental-frequency restored voltage, and the other is the natural-frequency component which is equal in magnitude but directly opposed to the instantaneous fundamental-frequency component.

It is significant that the magnitude of the natural-frequency component of voltage at the final current zero is equal to the instantaneous restored voltage and, depending on the power factor of the circuit, may vary from the minimum of zero to the maximum which is equal to the crest of the restored voltage wave.

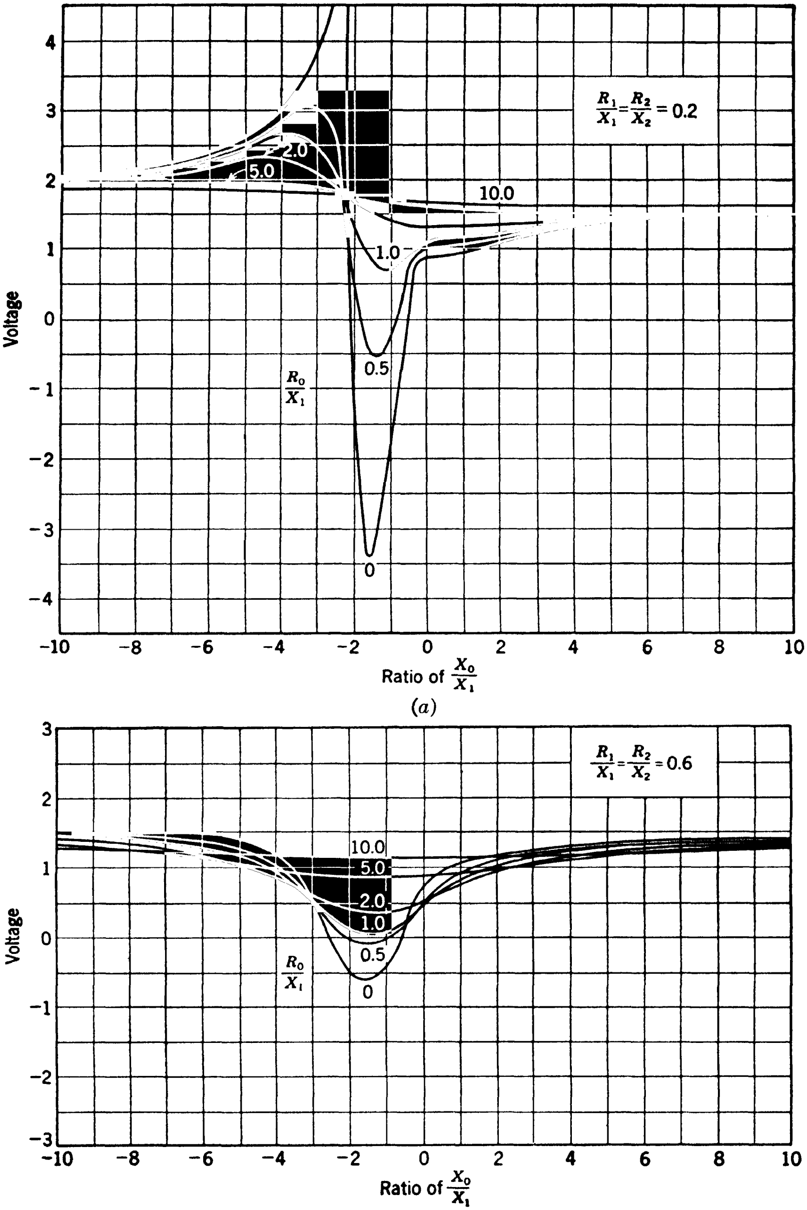
Following interruption, the natural-frequency voltage will oscillate about the fundamental-frequency restored voltage until it is damped out, leaving only the fundamental-frequency restored voltage.

If, as in the preceding sections, the fault is a single-line-to-ground fault, the fundamental-frequency restored voltage will be the normal voltage of that phase to ground. If the original fault is a double-line-to-ground fault, then the fundamental-frequency restored voltage will be the voltage to ground of the cleared phase existing with a line-to-ground fault on the other still faulted phase.

The instantaneous restored voltage is the crest of the fundamental-frequency restored voltage multiplied by the sine of the angle between this voltage and the fault current.

The curves shown in Figs. 4·22 and 4·23 are of practical value in the application of interrupting devices. The instantaneous restored voltages shown give an indication of the severity of the duty imposed on the interrupting devices. Thus, other factors being equal, a lower instantaneous restored voltage means a less severe duty on the interrupting device. The practical value of these curves in Fig. 4·22 lies in the fact that the transient recovery voltage will not exceed twice the crest of the fundamental-frequency instantaneous restored voltage given by the curves.

When the first phase of a double-line-to-ground fault clears, the fault becomes a single-line-to-ground fault. Thus the figures in



(*a*)

(*b*)

FIG. 4·22. Instantaneous restored voltage on phase *b*, when phase *b* is the first to clear after a double line-to-ground fault on phases *b* and *c* to ground. Voltage is times normal line-to-neutral crest voltage.

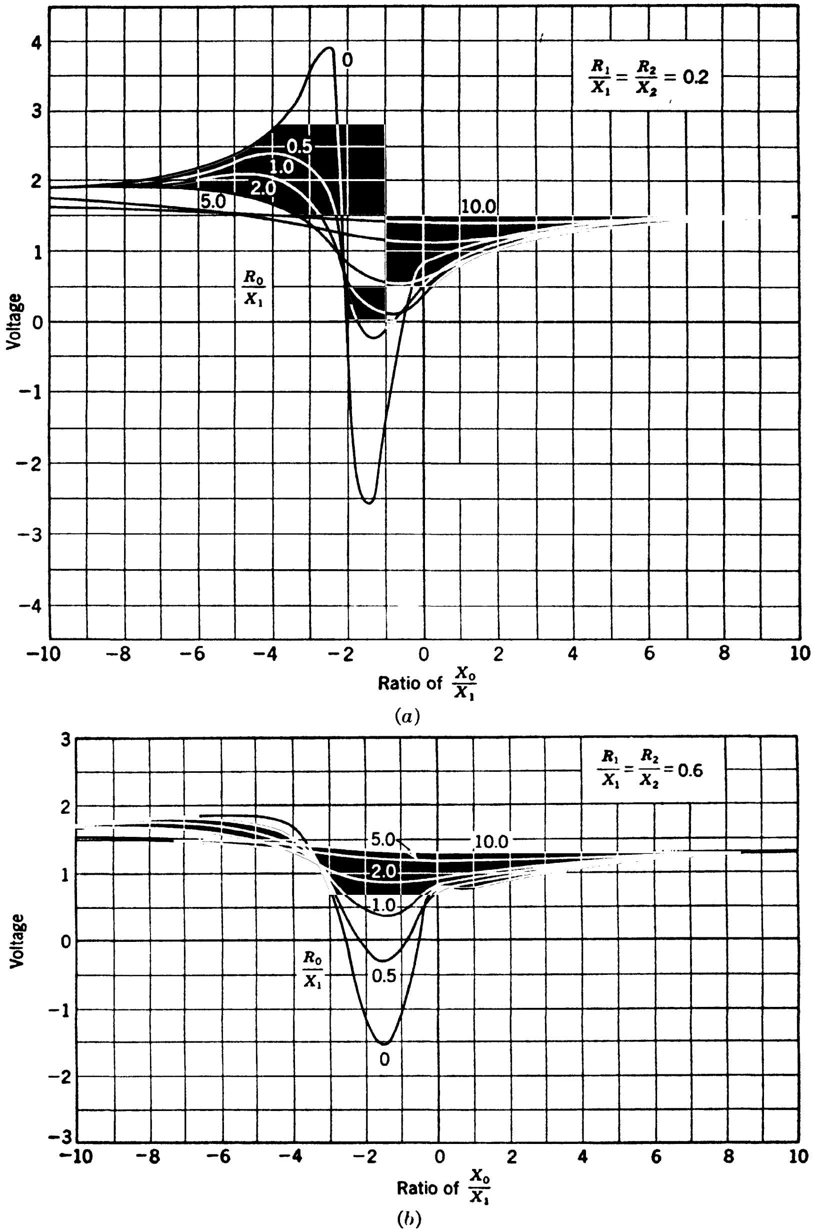


FIG. 4·23. Instantaneous restored voltage on phase *c*, when phase *c* is the first to clear after a double line-to-ground fault on phases *b* and *c* to ground. Voltage is times normal line-to-neutral crest voltage.

Chapter 1 for a single-line-to-ground fault represent also the crests of the restored voltage after the first phase of a double-line-to-ground fault has cleared.

The instantaneous restored voltage when the first phase of a double-line-to-ground fault clears is shown by the curves of Figs. 4·22 and 4·23. These curves have been calculated by means of symmetrical components based on the following considerations:

1. Positive- and negative-sequence impedances are equal.
2. For clearness, the three-phase voltages have been designated *a*, *b*, and *c*, with the double-line-to-ground fault on phases *b* and *c* to ground. Symmetrical systems were assumed with *abc* rotation.
3. The fault current interrupts at a normal fundamental-frequency current zero.
4. Effects of saturation, arc voltage, corona, and harmonic voltages are negligible.
5. The effects of resistance and reactance in the positive- and zero-sequence networks are included.
6. The curves give the instantaneous restored fundamental-frequency voltage to ground at the fault location.

These curves serve to illustrate the one general conclusion which will be further endorsed later that recovery voltages in solidly grounded systems as defined in Chapter 1 are less severe than they are in any other type of system when all types of faults are considered. This is a most significant point to those concerned with the design and application of protective devices in power systems.

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5 EFFECTS OF ARC RESTRIKING ON RECOVERY VOLTAGE

5·1 INTRODUCTION

It has been recognized for many years that voltage surges of considerable magnitude could be produced under certain system conditions by switching operations with or without arcing faults. At times the finger of suspicion has been pointed at the arcing fault alone as a source of high overvoltage. As yet, no experimental evidence has been disclosed to confirm such suspicion. The highest voltage surges experienced on power systems have gradually become associated more closely with the effects of arc restriking within circuit breakers or other interrupting devices.

Voltage surges have been investigated in some detail in the past, and various theories have been developed to explain their occurrence and to predict their magnitude. In general, however, for most of the theories expounded, the prediction of surge magnitudes was on a rather insecure basis.

There were two reasons, principally, for this insecurity: (1) the inability to define adequately the arc characteristic, and (2) the inadequacy of long-hand methods of circuit analysis even when the arc characteristics were *assumed* to be of the simplest form imaginable. The first of these reasons still exists as a very real stumbling block. For instance, not enough is known about a simple arc in air to make it possible to predict *with precision* whether a certain circuit arrangement with an arc between two points in that circuit will (*a*) clear itself, (*b*) clear itself momentarily and restrike, (*c*) clear itself and restrike without producing overvoltages, or (*d*) clear itself and restrike with attendant overvoltages. The problem is fundamentally one of expressing reliably the *transient* volt-ampere characteristic of an arc. Even for a simple arc in air under standard atmospheric conditions, this cannot be done with quantitative precision although much excellent work has been done and is being done in an attempt to understand better the quantitative dynamics of the electric arc.

The second reason for the insecurity of earlier predictions of surge magnitudes has been largely overcome since about 1935 through the introduction and use of circuit analyzers as already mentioned in preceding chapters. These analyzers have made it possible to study *circuit* characteristics with relative ease and precision. Using such devices, one could *assume* certain arc characteristics and at least make a consistent study based upon such assumed characteristics. Thus it became possible to obtain quantitative results which would permit comparisons to be made between different circuit arrangements.

Many such circuit studies have been made by a number of investigators. Although the results obtained depend on the assumed arc characteristics, it has been possible to draw several significant conclusions concerning the relative merits of many circuit arrangements and power system operating procedures which appear to be clearly substantiated by actual system operating experiences.

In the preceding chapter, some results of studies were summarized and discussed in which circuit interruption was assumed to be complete and instantaneous at a current zero. In this chapter results of several studies will be summarized for which it was assumed that the circuit interruption was not complete, but rather, restriking of the arc occurred in various ways so that an intermittent arcing condition was obtained.

5·2 THE TRANSIENT VOLTAGE RECOVERY PERIOD

It is relatively easy to study the effects of arcing in electric circuits if it is assumed that the arc is either an open circuit or a short circuit. Such an assumption was put into practice in the foregoing chapter, for during the period of current flow the arc voltage was taken to be zero (a short circuit) while, at a current zero, the arc disappeared and no current flowed thereafter. The circuit "recovered" in accordance with the dynamic behavior dictated by the constants of the circuit. This voltage recovery transient was a function of time and was the voltage impressed across the arc path. The arc path cannot regain its ultimate dielectric strength instantaneously, however, for it takes time for the arc products to disappear and in effect "heal" the path of rupture. The restoration of dielectric strength is therefore, in general, a function of time also. If at any time during this recovery interval the voltage across the arc path exceeds the dielectric strength of that path, the arc will restrike and a new dynamic state ensues not covered by Chapter 4. This period is therefore characterized by a race between the recovery of dielectric strength and the voltage recovery transient of the system. If the former wins, Chapter 4 will apply; if the latter wins, the consequences remain to be evaluated. It is the purpose of this chapter to evaluate the consequences of such restriking

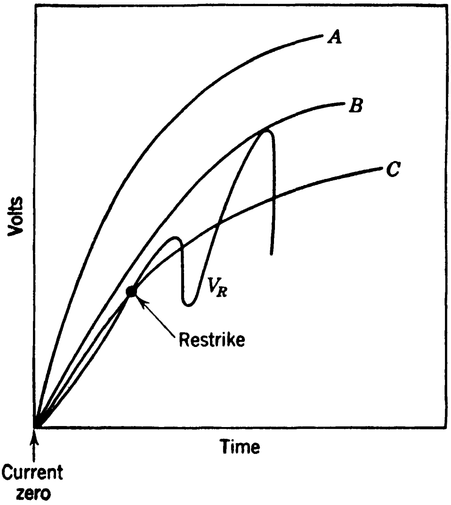

FIG. 5·1. Elements of arc restriking during the transient recovery voltage period. *A*, *B*, and *C* are assumed transient dielectric strength recovery characteristics of the arc path. V_R is an assumed transient recovery voltage. If dielectric strength is recovered in accordance with curve *A* or *B*, there is no restriking, although *B* is nearly critical. If it is recovered in accordance with *C*, a restrike occurs at the point indicated and interruption is not completed.

of the arc for a variety of assumed characteristics for the dielectric strength recovery. The general elements involved are shown in Fig. 5·1. The arc path is to be either a short circuit or an open circuit as before, but restriking of the arc is assumed to take place in accordance with certain fixed rules which will be stated for each case.

In this chapter then, the arc characteristic (or interrupting device) enters only so far as its assumed dielectric recovery characteristic determines the point at which restriking takes place, the resulting recovery voltage being otherwise completely determined by the circuit characteristics. Certain types of interrupting devices may modify considerably this resultant recovery characteristic, and therefore the form and magnitude of the resulting overvoltages.

5·3 CASES STUDIED

Five types of arc restriking have been studied and are described in this section. The miniature system is shown in Fig. 5·2. The significance of system constants is clear from this figure. For each type of arcing fault studied, these system constants were varied over a considerable range in order that curves of results could be obtained for general use.

Type 1. Continuously sustained arc restriking of one phase to ground at the constant voltage level that results in the highest voltages on the unfaulted phases, the restriking voltage being the same in magnitude for either polarity. Such restriking may approximate the arcing conditions over an insulator string in the open air, wherein the restriking voltage may wander more or less slowly over the range that can be sustained, as the arc path wanders. In general, the highest restriking voltage that can be sustained gives the highest voltages on the unfaulted phases. This type of arc restriking is illustrated in Fig. 5·3. In this case successive restriking of alternating polarity took place at half-cycle intervals, whereas in some of the other cases studied the arc restruck two or three times with the same polarity in each half-cycle. In all cases restriking took place whenever the recovery voltage reached the predetermined restrike voltage level, regardless of polarity.

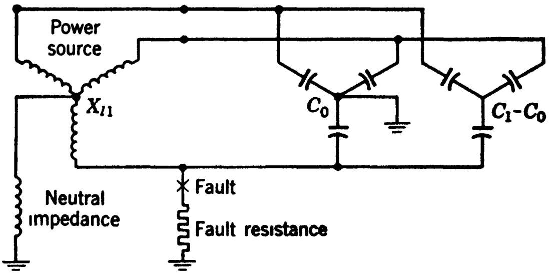

FIG. 5·2. System studied for Figs. 5·3 to 5·13 and 5·16 to 5·19.

Type 2. Continuously sustained arc restriking at the voltage levels that result in the highest voltages on the unfaulted phases, without regard to the equality of the restriking voltage of either polarity. This form of restriking was studied as a convenient representation of possible irregularities in the restriking characteristics. That is, as this particular form of irregularity either increases or does not affect the maximum attainable voltages, there is a basis for concluding that transient variations in the restriking characteristics either are or are not important, respectively.

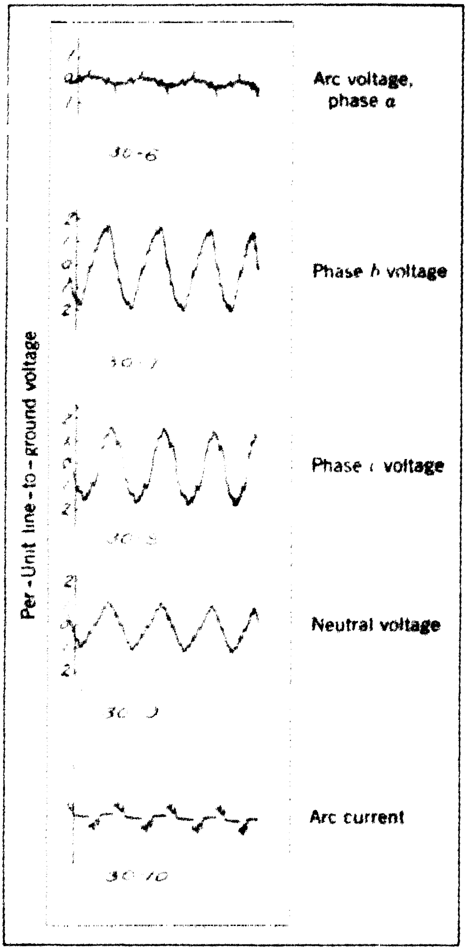

FIG. 5·3. Sustained arcing fault (Type 1) on reactance-grounded system.

Type 3. Continuously sustained arc restriking of one phase to ground at regular time intervals, in this case at half-cycle intervals. Data on this type of restriking are presented as an indication of what may occur with extremely random, rapid, and large variations in the restriking or flashover voltage. Such restriking seems to be unlikely to occur in an actual system but could occur under unusual circumstances. The data are included here to illustrate the fact that the recovery characteristic assumed is important, particularly for isolated-neutral systems. Figure 5·4 illustrates the building-up process leading to the ultimate sustained maximum voltages of this type.

Type 4. Controlled restriking after interruption of a solid-line-to-ground fault. This is not a continuously sustained restriking condition but consists of imposing one or two individually controlled restrikes subsequent to circuit interruption from a steady-state fault condition.

Type 5. This is very similar to Type 4, the only difference being that the system is initially without a fault The fault occurs, and circuit interruption takes place at the first subsequent current zero instead of from a steady-state fault condition. Restriking of Types 4 and 5 may more properly be associated with the behavior of certain circuit interrupting devices than with arcs in air.

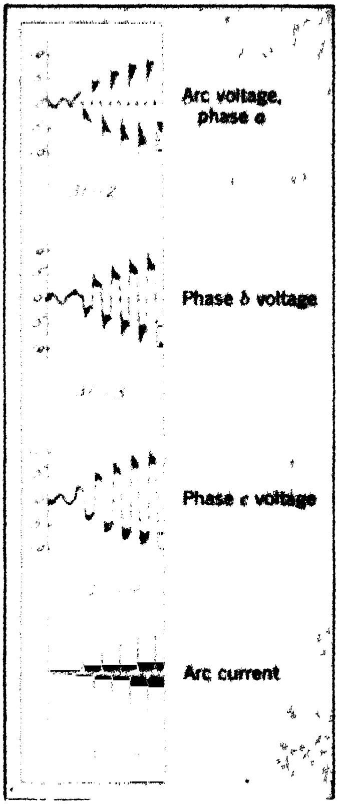

FIG. 5·4. Build-up of voltage caused by arcing fault (Type 3) in isolated neutral system.

5·4 COMPARISON OF EFFECTS OF VARIOUS ASSUMED RESTRIKING CHARACTERISTICS

Figures 5·5 to 5·9 inclusive give the maximum voltages appearing on either unfaulted phase for restriking of Type 1. Curves *C* and *D* of Fig. 5·5, for systems with isolated neutral, may be compared with curve *E* to show that these sustained voltages are of the same order of magnitude as the transient voltages appearing on sudden application of a solid fault. Figures 5·6 to 5·9 indicate that for this type of restriking (*a*) the voltages appearing on systems grounded through reactance are usually greater than those on systems grounded through resistance, (*b*) the voltages are greater with $C_1 = C_0$ than with $C_1 = 2C_0$, and (*c*) the voltages on resistance-grounded sys-

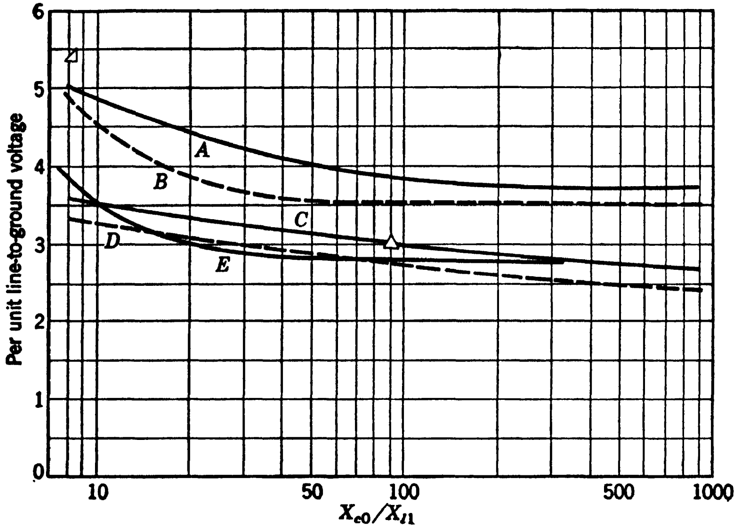

FIG. 5·5. Maximum sustained unfaulted phase voltage for arcing fault in isolated-neutral system of Fig. 5·2. Curve A, Type 2, $C_1 = C_0$; curve B, Type 2, $C_1 = 2C_0$; curve C, Type 1, $C_1 = C_0$; curve D, Type 1, $C_1 = 2C_0$; curve E, $C_1 = C_0$. Maximum transient voltage accompanying application of solid fault for comparison. Points △ were obtained with 100 miles of three-phase artificial line, 10-mile sections.

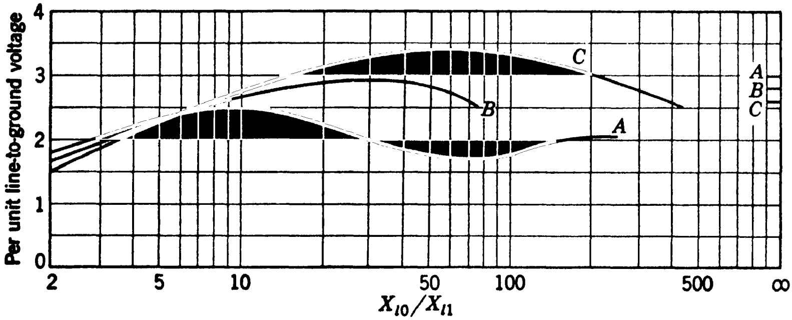

FIG. 5·6. Maximum sustained unfaulted phase voltage for arcing fault of Type 1 in reactance-grounded system of Fig. 5·2, $C_1 = C_0$. Curve A, $X_{c0}/X_{l1} = 90$; curve B, $X_{c0}/X_{l1} = 280$; curve C, $X_{c0}/X_{l1} = 1{,}120$. (Ground-fault neutralizer at $X_{l0}/X_{l1} = X_{c0}/X_{l1}$.)

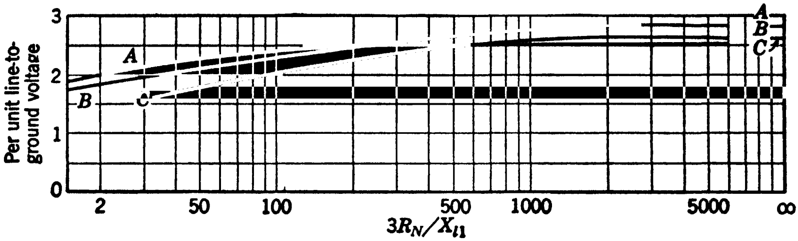

Fig. 5·7. Maximum sustained unfaulted phase voltage for arcing fault of Type 1 in resistance-grounded system of Fig. 5·2, $C_1 = C_0$. Curve A, $X_{c0}/X_{l1} = 90$; curve B, $X_{c0}/X_{l1} = 280$; curve C, $X_{c0}/X_{l1} = 1{,}120$.

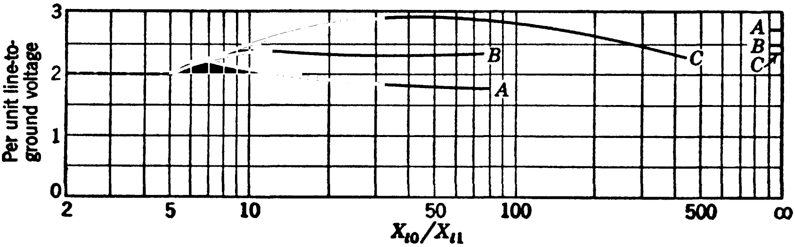

Fig. 5·8. Maximum sustained unfaulted phase voltage for arcing fault of Type 1 in reactance-grounded system of Fig. 5·2, $C_1 = 2C_0$. Curve A, $X_{c0}/X_{l1} = 90$; curve B, $X_{c0}/X_{l1} = 280$; curve C, $X_{c0}/X_{l1} = 1{,}120$.

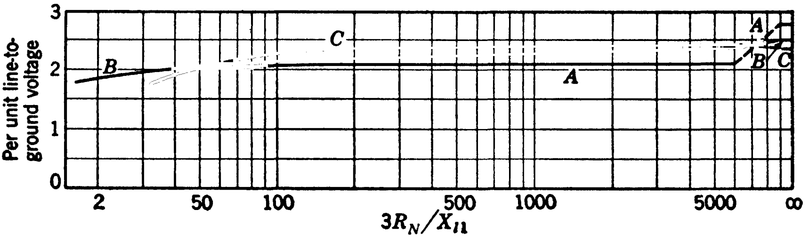

Fig. 5·9. Maximum sustained unfaulted phase voltage for arcing fault of Type 1 in resistance-grounded system of Fig. 5·2, $C_1 = 2C_0$. Curve A, $X_{c0}/X_{l1} = 90$; curve B, $X_{c0}/X_{l1} = 280$; curve C, $X_{c0}/X_{l1} = 1{,}120$.

tems are relatively independent of the system natural frequencies in the range studied.

The relative effect of arcing of Type 2 can be observed by comparing curves *A* and *B* with curves *C* and *D* in Fig. 5·5. Such a comparison indicates that possible irregularities in the restriking characteristic may result in a considerable increase in voltage.

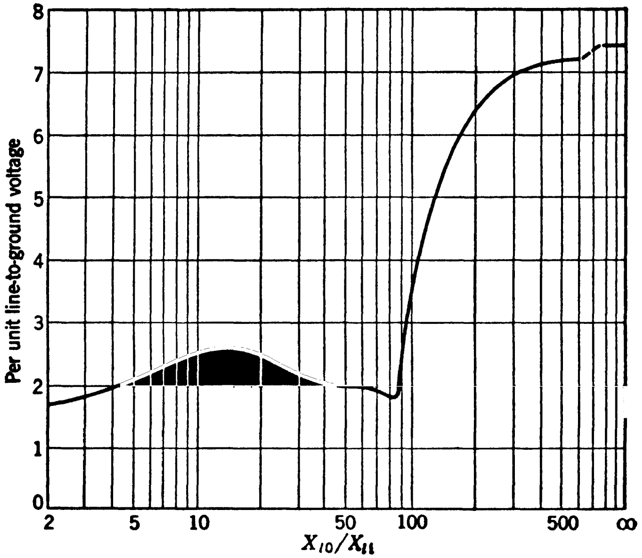

FIG. 5·10. Maximum sustained unfaulted phase voltage for arcing fault of Type 3 in reactance-grounded system with 100-mile three-phase artificial line. $C_1 = 1.6C_0$, $X_{c0}/X_{l1} = 89$. (Ground-fault neutralizer at $X_{l0}/X_{l1} = X_{c0}/X_{l1}$.)

The data of Figs. 5·5 to 5·9 were obtained with an arcing circuit having an inherent dielectric strength recovery characteristic given approximately by the relation:

$$\text{Breakdown voltage} = V(1 - e^{-t/T})$$

where V = maximum breakdown voltage.
t = time in microseconds.
T = 2,400 microseconds.

Thus if successive restriking occurred several times in a half-cycle the restriking voltages of the various restrikes were not all exactly alike, depending on the length of the interval from a current zero to the subsequent restrike. Several check points were taken with the time constant T made very small so that the restriking voltage was truly constant, and little difference could be observed in the magnitude of the unfaulted phase voltage.

Sustained arc restriking of Type 3 gives rise to overvoltages as shown in Figs. 5·10 and 5·11. Such restriking has been considered by a number of other investigators. No actual system field tests have indicated that such a building-up process can occur to any appreciable extent. Conceivably it could possibly occur under some very unusual conditions, but it is most unlikely to occur for an arc in air. The data are included here principally to show that the form of the restriking characteristic, as well as the circuit characteristics, is very important, particularly for systems with isolated neutral. Figure 5·4 illustrates the building-up process and clearly shows that, for an arc between fixed electrodes or terminals, such a build-up would be unlikely to take place because of the requirement that the arc path

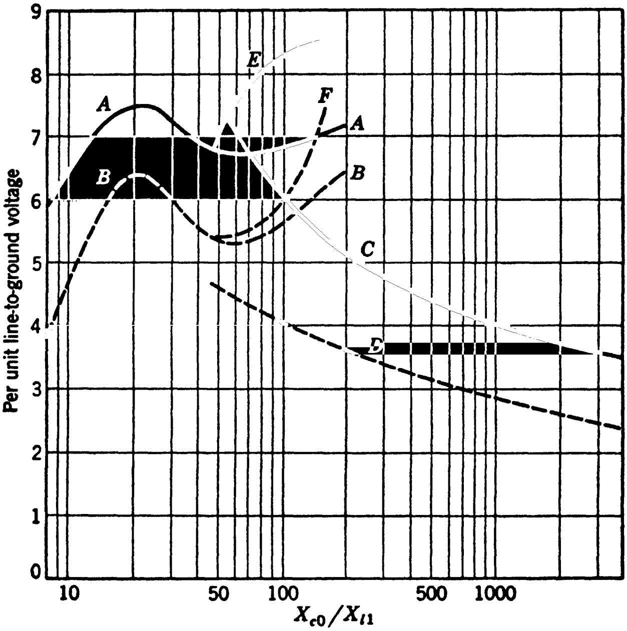

Fig. 5·11. Maximum sustained voltage for arcing fault of Type 3 in isolated-neutral system of Fig. 5·2, $C_1 = 1.6C_0$. Curve *A*, unfaulted phase voltage, 100-mile, three-phase artificial line; curve *B*, neutral voltage, 100-mile line; curve *C*, unfaulted phase voltage, lumped capacitance; curve *D*, neutral voltage, lumped capacitance; curve *E*, unfaulted phase voltage, decreasing line length with X_{l1} constant; curve *F*, neutral voltage, decreasing line length with X_{l1} constant.

must be capable of recovering dielectric strength *faster* after each successive interruption of the arc current. If the arcing terminals are moving apart, as for example would be true in a circuit breaker during

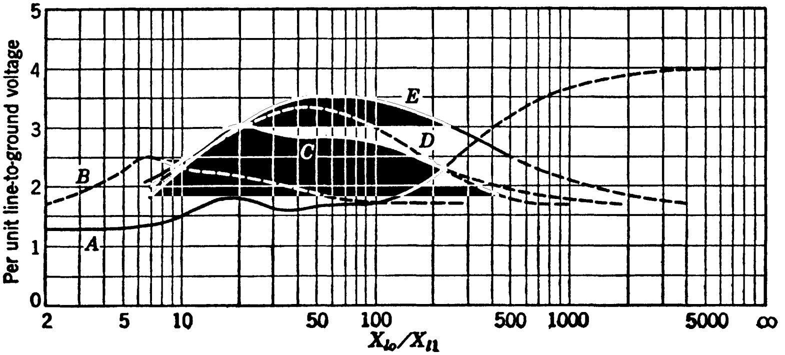

FIG. 5·12. Maximum unfaulted phase voltage for restriking of Type 4 in reactance-grounded system of Fig. 5·2, $C_1 = 1.6C_0$. Two restrikes. Curve A, $X_{c0}/X_{l1} = 90$, 100-mile artificial line; curve B, $X_{c0}/X_{l1} = 300$, 30-mile artificial line; curve C, $X_{c0}/X_{l1} = 900$, 10-mile artificial line; curve D, $X_{c0}/X_{l1} = 1{,}800$, lumped capacitance; curve E, $X_{c0}/X_{l1} = 3{,}900$, lumped capacitance. (Ground-fault neutralizer for $X_{l0}/X_{l1} = X_{c0}/X_{l1}$.)

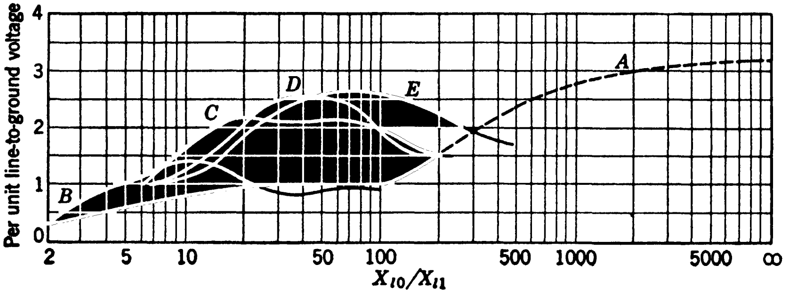

FIG. 5·13. Maximum neutral-to-ground voltage for restriking of Type 4 in reactance-grounded system of Fig. 5·2, $C_1 = 1.6C_0$. Two restrikes. Curve A, $X_{c0}/X_{l1} = 90$, 100-mile artificial line; curve B, $X_{c0}/X_{l1} = 300$, 30-mile artificial line; curve C; $X_{c0}/X_{l1} = 900$, 10-mile artificial line; curve D, $X_{c0}/X_{l1} = 1{,}800$, lumped capacitance, curve E, $X_{c0}/X_{l1} = 3{,}900$, lumped capacitance. (Ground-fault neutralizer at $X_{l0}/X_{l1} = X_{c0}/X_{l1}$.)

interruption, then such a building-up process may more logically be anticipated. This matter will be discussed more fully in connection with switching surges in Chapter 6.

Results of tests on miniature systems for arcing of Type 4 are shown in Figs. 5·12 and 5·13. This type of restriking approximates those conditions which may arise on interruption of a faulted conductor by a circuit interrupting device as described in reference 2. The sequence of switching operations applying for results shown in

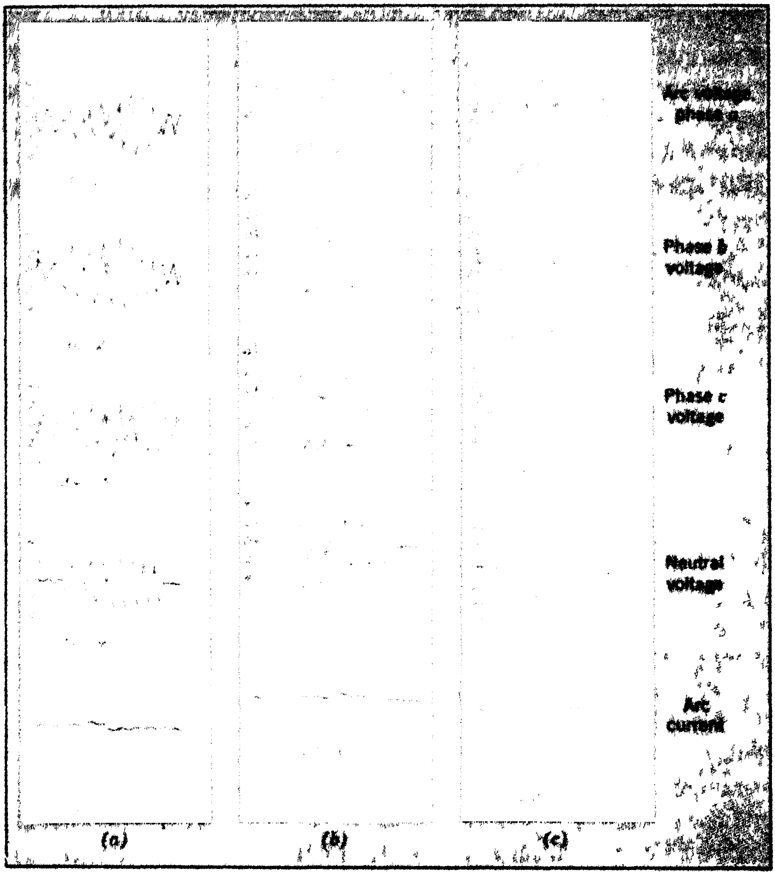

FIG 5·14. (*a*), (*b*), and (*c*) Various restriking sequences (Type 5) following interruption of transient line-to-ground fault current Reactance-grounded system of Fig 5·15.

Figs. 5·12 and 5·13 is as follows. First the fundamental-frequency fault current flowing in the arc is interrupted at a current zero. The normal recovery voltage then appears across the switch contacts, and the arc restrikes at some point on this recovery voltage wave. The consequent restrike current is interrupted at its first subsequent current zero, a recovery voltage again appears, restriking takes place a second time, the restrike current is again interrupted at its first subsequent current zero, and the final recovery voltage appears. The points at which the restrikes occur were adjusted in the miniature-system studies so as to give the maximum voltage on any phase to ground.

Figure 5·14 shows some results of restriking of Type 5. The oscillograms in this figure indicate that for circuits of lower natural frequency (i.e., large inductance and/or capacitance) the most probable manner in which restriking takes place tends to become rather difficult to specify and at the same time becomes of increasing importance in determining the magnitude of voltages finally reached. One might take as a guide to the selection of a restriking sequence the principle that the dielectric recovery characteristic of the interrupting device never decreases but always increases, or at least remains constant as a function of time. Then restriking could never occur as in Fig. 5·14(*b*) just after a crest of any oscillatory component of the recovery voltage (i.e., when the voltage is decreasing in magnitude) but could occur either before or at a crest as in Fig. 5·14(*a*). On the other hand, restriking would not necessarily have to occur at or before such a crest, if a later higher crest of either polarity provided a suitable opportunity. It must not be inferred that a decrease of effective voltage breakdown strength with time after a circuit interruption is impossible. In fact, there have been some indications that such action may occur in circuit breakers, especially if their construction and operation allow the dielectric strength to decrease momentarily at any point (e.g., by the formation of a bubble of ionized gas) as the contacts part.

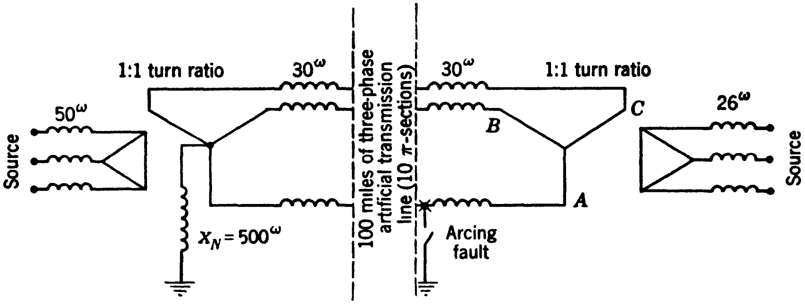

FIG. 5·15. System diagram for results shown in Fig. 5·14.

Comparison of Figs. 5·14(*a*), (*b*), and (*c*) shows the importance of the restriking sequence and illustrates the facts that (*a*) the highest voltages require, in general, a rather long time (about 3 or 4 cycles) to build up, and (*b*) a monotonic dielectric recovery characteristic (Fig. 5·14(*a*)) imposes an appreciable limitation on the magnitude of the voltage surges attainable. (Compare also Figs. 5·5 to 5·9 with Figs. 5·10 and 5·11 to illustrate this point.)

For most of the systems and types of arcing studied in this chapter the maximum surge voltages occur with isolated-neutral systems, minimum points occur with systems grounded either solidly or through ground-fault neutralizers, and a second maximum point occurs at a value of neutral reactance equal to about: (*a*) one-ninth of the ground-fault neutralizer value for arcing of Types 1 and 2; (*b*) one-sixth of the ground-fault neutralizer value for arcing of Type 3, and (*c*) one-fiftieth of the ground-fault neutralizer value for arcing of Types 4 and 5.

This second maximum point will occur at greater fractions of the ground-fault neutralizer value for systems having very low ratios of X_{c0}/X_{l1} (that is, low natural frequencies).

5·5 EFFECTS OF VARIOUS FACTORS IN SYSTEM CONFIGURATION

Distribution of System Capacitance. The curves of Figs. 5·5 to 5·9 are for a system as shown in Fig. 5·2. Figure 5·5, for a system with isolated neutral, shows also voltages obtained when the lumped capacitance of Fig. 5·2 is replaced by a 100-mile transmission line. It is seen that for a rather large source reactance ($X_{c0}/X_{l1} = 8.3$) the surge voltages with the line are greater than with lumped capacitance, whereas for a smaller source reactance ($X_{c0}/X_{l1} = 90$) the reverse is true. (The ratio of zero-sequence capacitive reactance to positive-sequence inductive reactance was determined without considering the line inductance.) These results suggest that X_{c0}/X_{l1} and C_1/C_{c0} are not the only factors affecting the phenomena and that therefore an attempt to generalize very much from results obtained from a study of restriking in a simple lumped-constant system may sometimes lead to erroneous results. Although the trend of the curve of voltage as a function of X_{c0}/X_{l1} given in Fig. 5·5 appears to be generally indicative, the maximum voltages possible in an actual system may vary considerably on either side of these curves, depending on system connections.

Ratio of Positive-Sequence Capacitance C_1 to Zero-Sequence Capacitance C_0. Figure 5·5 and comparison of Figs. 5·6 and 5·8 and 5·7 and 5·9 show that higher voltages are in general obtained for smaller ratios of C_1/C_0.

It must be pointed out that the effective ratio C_1/C_0 may be less than 1, not only for generators, but also for transmission lines and systems in general. That is, there is presented to the fault an effective ratio C_1/C_0 which depends on its location along the line and also on the magnitude of the neutral impedance. For example, for a fault at one end of a single unloaded line supplied at the other end from a power source whose reactance is small compared to that of the line, one might expect very roughly on the basis of an equivalent π line an effective ratio C_1/C_0 of about half of 1.6 or 0.8, if the source is ungrounded. This ratio is obtained as follows. The ratio C_1/C_0 for a transmission line as ordinarily defined neglecting line inductance is about 1.6. However, in the positive-sequence circuit viewed from the fault, half of the total positive-sequence capacitance of the equivalent π line is shunted at the source end by the low source reactance and so may not be very effective so far as the disturbances produced by the fault are concerned. On the other hand, in the zero-sequence circuit there is no inductance shunting the part of the capacitance at the source end, if the neutral is isolated. Thus the total zero-sequence capacitance may be effective. If the source neutral is grounded through an impedance the effective capacitance ratio is correspondingly increased, since the neutral impedance is in shunt with the line capacitance at the source end in the zero-sequence circuit. These considerations bring to light two points:

1. Although it is true that the system performance is completely determined by the positive-, negative-, and zero-sequence networks viewed from the point of fault, one must be cautious in the interpretation and use of the conclusion that apparently follows, that it makes no difference whether, e.g., a high ratio of X_{l0}/X_{l1} results from inherent properties of the generating network or from an impedance inserted in the neutral, since insertion of such an impedance may change the effective ratio C_1/C_0 for a given system.

2. The effective ratio C_1/C_0 and the capacitance distribution are in general intimately related, so that it may not be possible to specify the probable transient performance by merely stating an overall system capacitance ratio and noting whether the system capacitance should be classed as lumped or distributed. Instead, the actual system configuration may have to be described, and in specific instances studied by miniature circuit representation, to obtain a good estimate of its performance.

The voltages of Figs. 5·5 to 5·11, 5·12, and 5·13 may thus be exceeded for systems with the effective value of C_1 less than C_0. The greatest differences may be expected with isolated-neutral systems. Voltages as high as 16 times normal have been obtained for certain miniature-system configurations with restriking of Type 3 and with $X_{c0}/X_{l1} \cong 200$. (Compare with 5·11, where $C_1/C_0 = 1.6$ and the corresponding voltage magnitude is 5.2 times normal.) In the limit one approaches the condition of negligible effective C_1 for a fault supplied from a rather extensive system through a small concentrated single reactance. This reactance may be a transformer or reactor, or even a short length of transmission line or cable. The qualifying words "small" and "short" should here be interpreted as indicating (*a*) a sufficiently high natural frequency with the fault applied and (*b*) a comparison to the reactance beyond (from the fault) the principal system capacitance. In line-to-line faults this latter reactance is the system negative-sequence reactance and is seldom large; in ground faults, it is the reactance to ground, and in the isolated-neutral case it is infinitely large. Possibly cable could be omitted in the statement above because of its usually high ratio of resistance to reactance.

This discussion of capacitance ratio C_1/C_0 would be incomplete without mention of a small system, as for example an industrial plant, supplied through a transformer from a large power system. Here the effective ratio of C_1/C_0 may be considerably greater than unity for faults near the supply bus if the supply transformer reactance is small compared to the power system reactance, and thus the allowable neutral reactance will be high. Faults sufficiently far out along feeders so that the supply system reactance becomes small compared to the transformer and feeder reactance revert to the case of C_1/C_0 near unity and therefore require consideration of a proper neutral reactance as indicated in reference 2.

Fault Resistance. Figure 5·16 shows the effect of fault resistance in reducing overvoltage caused by arcing of Type 2. Figure 5·17 similarly shows the effect for restriking of Type 4. Both figures are for systems with isolated neutral.

The data of Fig. 5·16 are for the system of Fig. 5·2. A similar study in which 100 miles of artificial three-phase transmission line was substituted for the lumped capacitance of Fig. 5·2 has shown that fault resistance may at first slightly increase the voltages if: (*a*) X_{c0}/X_{l1} is sufficiently large, e.g., 90; and (*b*) the line has sufficient resistance, e.g., if the ratio of positive-sequence resistance to positive-sequence reactance in the line is on the order of 0.4 and the corresponding ratio of zero-sequence quantities is about unity.

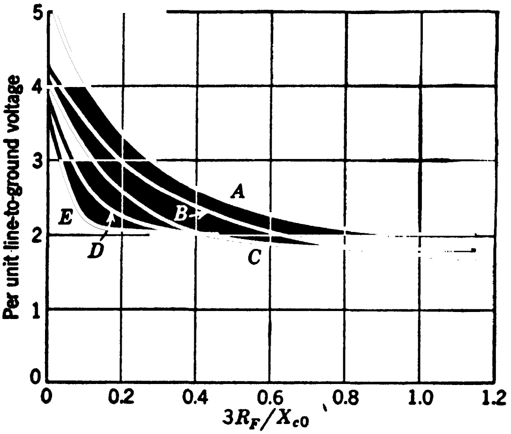

FIG. 5·16. Effect of fault resistance R_F on maximum sound phase voltage for arcing fault of Type 2 in isolated-neutral system of Fig. 5·2, $C_1 = C_0$. Curve A, $X_{c0}/X_{l1} = 8.2$; curve B, $X_{c0}/X_{l1} = 29$; curve C, $X_{c0}/X_{l1} = 89$; curve D, $X_{c0}/X_{l1} = 290$; curve E, $X_{c0}/X_{l1} = 890$.

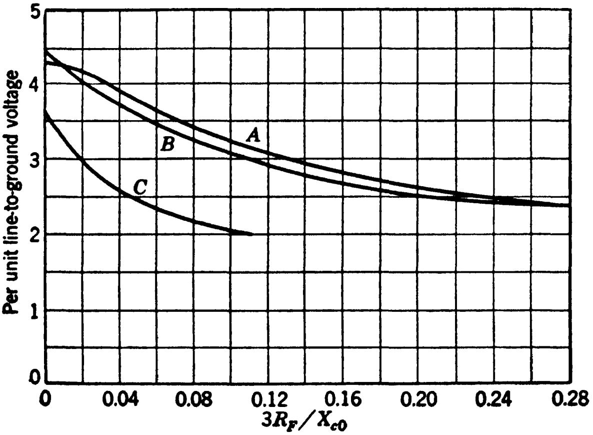

FIG. 5·17. Effect of fault resistance R_F on maximum sound phase voltage for restriking of Type 4 in isolated-neutral system of Fig. 5·2, $C_1 = C_0$. Two restrikes. Curve A, $X_{c0}/X_{l1} = 89$, lumped capacitance; curve B, $X_{c0}/X_{l1} = 89$, 100 miles of artificial line; curve C, $X_{c0}/X_{l1} = 890$, lumped capacitance.

Non-Linear Resistor in Shunt with Neutral Reactor. It has been pointed out [2] that resistance in shunt with the neutral reactance is beneficial as far as surges resulting from restriking of Type 4 are concerned, and that a resistance sufficiently small to produce critical damping of the ground circuit is more than sufficient to prevent successive restriking. It is evident that the shunt resistor can be a non-linear element such as Thyrite, and Fig. 5·18 shows the effect of the degree n of non-linearity on the amount of resistance required. The ratio of required resistance R at normal leg voltage to critical damping resistance R_c for a system with high neutral reactance X_{ln} and high natural frequencies is shown to increase as the non-linearity increases.

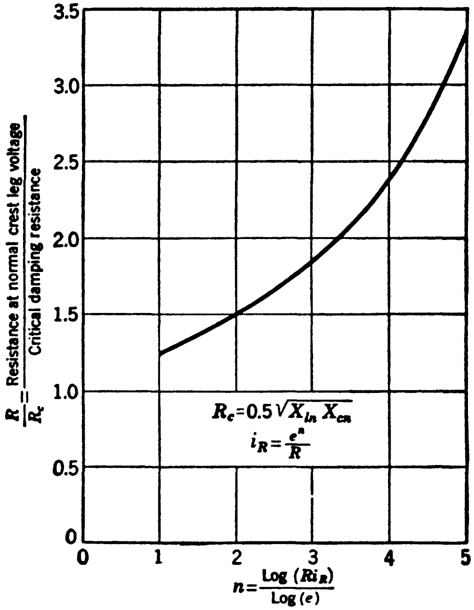

FIG. 5·18. Effect of degree of non-linearity on resistance required in shunt with neutral reactor to prevent successive restriking of Type 4, $C_1 = C_0$. R = shunt resistance at normal leg voltage crest; R_c = critical damping resistance for ground circuit; X_{ln} = inductive reactance of ground circuit = $X_{l0}/3$ for a three-phase system; X_{cn} = capacitive reactance of ground circuit = $X_{c0}/3$ for a three-phase system; i_R, e = resistor current, voltage.

A Gapped Resistor Such as a Lightning Arrester. Figure 5·19 shows a typical example of the voltages obtained with a reactance-grounded system and with restriking of Type 4, as affected by the gap sparkover setting and Thyrite resistance in series with the gap. The ground circuit critical damping value is indicated on the abscissa as $R_c \cong 900$ ohms.

An arrester is seen to be effective in holding down the surge voltage magnitude. Thus, even when the surge energy may be sufficient to cause arrester damage, damage to other equipment may still be prevented by the presence of the arrester.

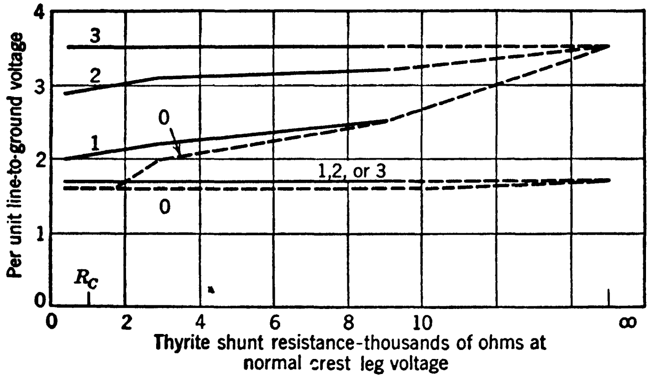

FIG. 5·19. Effect of gapped Thyrite in shunt with neutral reactor on maximum sound phase voltage for restriking of Type 4 in reactance-grounded system of Fig. 5·2. $X_{l0} = 557$ ohms; $X_{l1} = 13.9$ ohms; $X_{c0} = 53{,}000$ ohms; $X_{c1} = 33{,}100$ ohms; $X_{l0}/X_{l1} = 40$ ohms; $X_{c0}/X_{l1} = 3{,}820$ ohms; R_c = ground circuit critical damping resistance. Four upper curves—2 restrikes. Two lower curves—no restrikes. Numbers on curves are breakdown voltage of series gap in multiples of normal crest leg voltage.

5·6 EFFECTS OF NEUTRAL GROUNDING IMPEDANCE

It is evident that there are many factors that affect the choice of a proper method of system neutral grounding and that all of them must be considered carefully if the best overall method is to be obtained. The discussion here is concerned only with the effects of neutral impedance on overvoltages produced by arcing faults of the types described in the foregoing sections. It should be emphasized that there are other types of arcing and switching surge phenomena which can occur, each kind being somewhat differently affected by the method of system neutral grounding. The types of restriking under discussion are to be thought of as being associated with arcs in general, i.e., arcs in air, or in an interrupting device such as a fuse or circuit breaker.

The results summarized thus far and results of numerous other miniature-system studies indicate that it is possible to prevent the building up of excessive voltages due to arcing of the types discussed in this chapter by (1) keeping the zero-sequence reactance under the critical value shown by Fig. 5·20 (see reference 2), (2) using a ground-fault neutralizer for the neutral reactor, or (3) shunting the reactor by a suitable resistor.

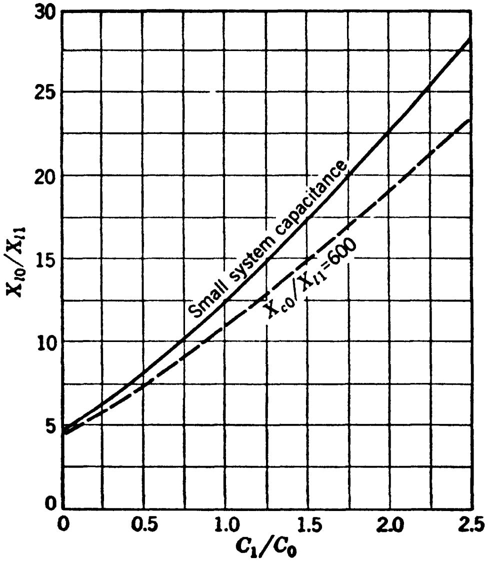

FIG. 5·20. Critical ratio of zero-sequence reactance X_{l0} to positive-sequence reactance X_{l1}, as a function of ratio of positive-sequence capacitance C_1 to zero-sequence capacitance C_0. Successive restriking may result in increased overvoltage if X_{l0}/X_{l1} exceeds the critical value.

If the neutral reactance is obtained by means of a small grounding transformer rather than by a neutral reactor, it may still be possible to use an effective shunt resistor when a near-by large transformer is in operation, by putting a resistor in the neutral-to-ground circuit of the large transformer. If the reactance of the large transformer is sufficiently small compared to that of the small grounding transformer, the resistor will be effectively in parallel with the neutral reactance in the zero-sequence circuit. Any case for which such a scheme is proposed should be studied to find out, in the first place, if it is feasible when the transformer size and system connections are considered. In the second place, the required value of resistance should be carefully determined. The fact that there is some transformer reactance in series with this resistor, the two elements being in parallel with the main neutral grounding reactor, renders the resistor less effective in its ability to damp out oscillations. The inherent reactance in an ordinary cast grid neutral resistor may be sufficient to impair its damping ability in this regard. This fact requires careful evaluation in connection with any neutral grounding scheme that employs a reactor and resistor in parallel. Frequently, the low value of neutral resistance required to obtain the desired damping effect may make it preferable to omit the reactor entirely and use the resistor alone.

5·7 CONCLUSIONS

The principal conclusions to be drawn from the material in this chapter based on transient analyzer studies are:

1. The results obtained cannot always be generalized to make them applicable to all systems. Certain general principles and effects have been discussed in order to show the trends in voltage surge severity which may be expected, but in particular instances it may be advisable to study the system in question (possibly by means of miniature-circuit representation), rather than to estimate the probable effects from general data. This is particularly true in evaluating quantitatively the effects of arresters or other resistors in reducing the magnitude of voltage surges.

2. For most of the systems and types of arcing studied, resistance grounding results in lower transient overvoltages than reactance grounding except at, and in the region immediately below, the ground-fault neutralizer impedance.

3. For either reactance- or resistance-grounded systems the voltages caused by arcing faults are not greatly different from those arising from simple application or removal of a solid fault.

4. For isolated-neutral systems voltages very much higher than those arising from simple application or removal of a solid fault may occur with certain types of restriking (Type 3).

5. Frequently the effective capacitance ratio C_1/C_0 and reactance ratio X_{t0}/X_{t1} viewed from the fault, together with the approximate circuit natural frequencies, can be used as guides to the evaluation of the probable severity of switching surges.

6. For reactance-grounded systems the voltages caused by switching surges and arcing faults have their maximum at a neutral reactance from about one-fiftieth to one-third of the ground-fault neutral-

izer value, depending on the type of arc restriking and on the circuit natural frequencies.

7. From the standpoint of arcing faults, methods of system grounding that should be satisfactory are: (*a*) solid grounding; (*b*) grounding through moderate values of resistance; (*c*) grounding through a ground-fault neutralizer; and (*d*) grounding through moderate values of reactance, if a study of the system in question, based on the considerations treated above, indicates that unduly high surges will not result. If a very low neutral reactance is used (for example, if the resulting ratio X_{t0}/X_{t1} is less than about 5) a detailed study should not be necessary.

8. For the interruption of capacitance fault currents (ground faults in isolated-neutral systems), circuit breakers should be used that have a minimum tendency to restrike.

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2. "Effect of Restriking on Recovery Voltage," by C. CONCORDIA and W. F. SKEATS, *A.I.E.E. Transactions*, **58**, 1939, pages 371–375.

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5. Discussion of reference 3 by P. A. JEANNE, *A.I.E.E. Transactions*, **58**, 1939, page 395.

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6 SWITCHING SURGES—CAPACITIVE CIRCUITS

6·1 INTRODUCTION

From the preceding chapter it is clear that restriking of an arc need not necessarily be followed or accompanied by destructive overvoltages. This is a comforting observation in view of the difficulties encountered in attempting to build interrupting devices that do not restrike. It may seem somewhat surprising that most circuit interrupting devices do restrike. Those that do not are the exceptions if there are any. Even the fastest high-speed circuit breakers are expected not always to complete the interruption of currents of all magnitudes at the first current zero subsequent to the parting of the contacts. To anyone who has observed the opening of an ordinary disconnecting switch when it was carrying current, the phenomenon of restriking will be remembered as having been observed audibly, visibly, and possibly also by the sense of smell.

If restriking of an arc, in a circuit breaker for instance, takes place before the normal recovery voltage has reached an appreciable magnitude, there should be no overvoltages produced of any serious magnitude. If, on the other hand, restriking is *delayed* until the recovery voltage has reached a higher value, approaching the maximum transient value, then the consequences *may* for certain system connections, or for certain switching operations, be serious in that overvoltages of still greater magnitude *may* be produced. The restrike itself is identified with a sudden reduction in voltage across the circuit breaker contacts. Thus a voltage surge, or voltage change, occurs which may or may not result in overvoltages on the system.

One of the common types of switching surges is associated with "line dropping," i.e., switching a line out of service. It is the purpose of this chapter to outline the fundamental nature of the mechanism whereby overvoltages may be produced during interruption of capacitive circuits. The significance of the problem from the standpoint of the circuit breaker designer as well as the system designer becomes clearer when it is pointed out that, every time a faulted line or cable is switched out of service, one or two phases will open a capacitive circuit, depending on the type of fault. Only for a three-phase-to-ground fault will there be no capacitive circuit interruption. Furthermore, the similarity between the circuit encountered in the line-dropping problem and the circuit encountered in clearing ground faults on isolated-neutral systems will be discussed.

6·2 THE BASIC CAPACITIVE CIRCUIT

The elementary basic circuit is shown in Fig. 6·1. It consists of a source of generated sinusoidal voltage e, a capacitor C, a small inductance L, and a switch S, all in series. The natural frequency $f = 1/2\pi\sqrt{LC}$ is assumed to be high as compared with the generator frequency. The switch S is assumed to be originally in a closed position so that a steady-state sinusoidal current flows prior to the initial opening of the switch. A half-cycle of this steady-state capacitor current is shown as i_s in Fig. 6·1(c). At the current zero following this half-cycle, the switch S, is suddenly opened at $t = t_0$, as indicated in Fig. 6·1(c). Interruption is assumed to be ideally instantaneous, so that current flow is completely stopped for the moment. Since current flow has stopped, the charge on the capacitor C remains unchanged and the capacitor voltage V_c remains constant as shown in Fig. 6·1(d). In the meantime the source voltage e is reversing, and the voltage across the switch V_s is therefore changing with time as shown by Fig. 6·1(e). A half-cycle after the initial opening of the switch, the voltage across the switch is twice the normal crest value of e, or $2e_{\text{max}}$. If the switch restrikes, or if the switch is closed at this instant, $t = t_1$, a current i_1 shown in (c) will flow, which is essentially of natural frequency determined by $f = 1/2\pi\sqrt{LC}$. The magnitude of it is essentially $2e_{\text{max}}/\sqrt{L/C}$.

If the switch is left closed, this natural-frequency current would continue to flow and, of course, the steady-state component would flow also. However, if the switch S is opened at $t = t_2$, some significant consequences are observed. During the interval $t_1 < t < t_2$, the capacitor voltage which was e_{max} reversed to a value of $-3e_{\text{max}}$. The change in voltage across the capacitor was therefore $4e_{\text{max}}$, or twice the voltage that existed across the switch at the instant of restriking,

$t = t_1$. With the switch opened again ideally at $t = t_2$, this voltage of $-3e_{\max}$ is left on the capacitor and V_c remains constant for the moment following $t = t_2$.

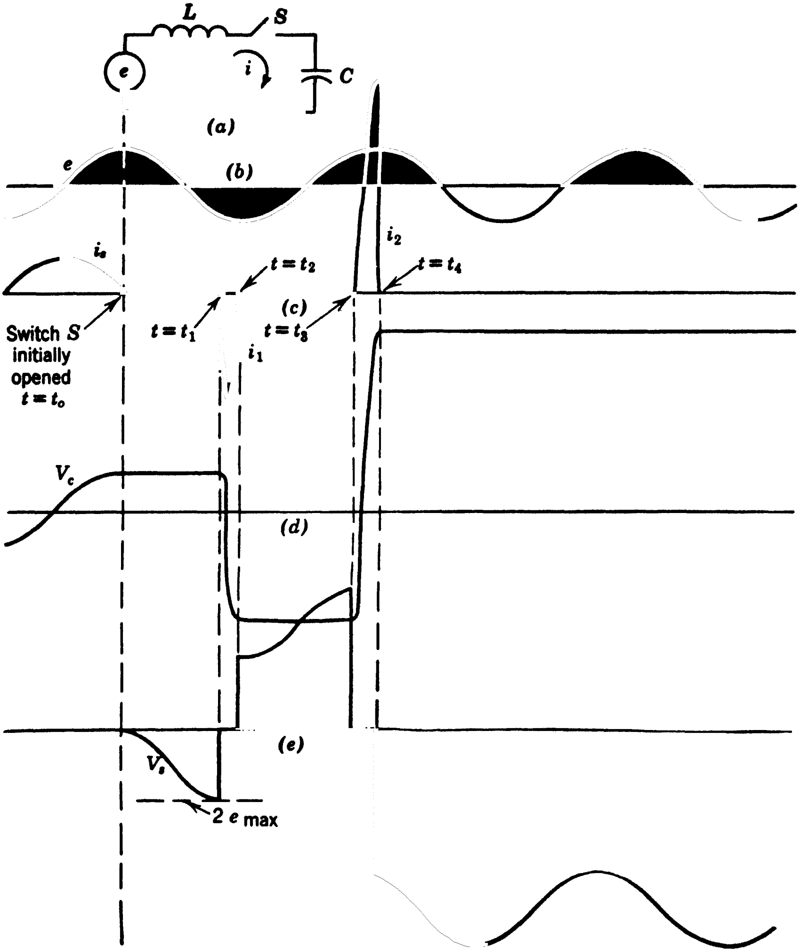


FIG. 6·1. Elements of capacitive circuit interruption with restriking. Two restrikes. (*a*) Basic circuit; (*b*) generated voltage e; (*c*) current i; (*d*) capacitor voltage V_c; (*e*) switch voltage V_s.

In the meantime, however, the source voltage e reverses again so that at $t = t_3$, which is essentially a half-cycle after the second opening of the switch, the voltage V_s is $4e_{\max}$. If a restrike occurs at this instant, or if the switch is again closed at $t = t_3$, a current i_2 will flow. Again it is the same natural frequency as i_1, but the magnitude is twice as great, being essentially of maximum value $4e_{\max}/\sqrt{L/C}$. If the switch were left closed, this natural-frequency current and the steady-state current, of course, would continue to flow. However, again assume that the switch is opened at the first subsequent current zero, $t = t_4$ following the restrike at $t = t_3$. During the interval $t_3 < t < t_4$, the capacitor voltage which was $-3e_{\max}$ reversed to a value of $+5e_{\max}$. The change in voltage across the capacitor was therefore $8e_{\max}$, or twice the voltage that existed across the switch at the instant of restriking, $t = t_3$. With the switch opened again ideally at $t = t_4$, this voltage, $+5e_{\max}$, is left on the capacitor and V_c remains constant at this value for the moment following $t = t_4$.

This sequence of events, of *systematically* opening and closing switch S, could be carried on indefinitely to any number of restrikes, and voltages could be built up without limit theoretically if there were no losses in the circuit. In any such simple circuit, as long as L is small and the natural frequency is large as compared to the frequency of the generated voltage, such systematic restriking results in a capacitor maximum voltage sequence of 1, 3, 5, 7, $\cdots$ etc.; the corresponding switch maximum voltage sequence is 0, 2, 4, 6, $\cdots$ etc.

In this building-up process, the facts that (1) restrikes were *delayed* and (2) that subsequent interruptions of the current took place at the *first* current zero following a restrike must be emphasized. This is the worst sequence from the standpoint of producing overvoltages, i.e., it produces the most overvoltage per restrike of any switching sequence for a circuit of this basic type. Interrupting devices in general are not aware of this fact, although some in actual field tests have shown an uncanny ability to hit closely upon such a sequence as will be shown later. It is only fair to point out that, if restriking occurs earlier, that is if t_1 is more nearly equal to t_0, then the voltage across the switch V_s is correspondingly reduced, and serious overvoltages are not produced even though interruption occurs at the first subsequent current zero. Thus, restrikes can take place without producing overvoltages. It is purely a question of *delays* of restrikes and subsequent timing of interruptions in the capacitive circuit as outlined above that will determine whether or not overvoltage will be built up.

6·3 EFFECT OF SWITCHING ONLY A PART OF TOTAL CAPACITANCE

Consider the circuit in Fig. 6·2. It is exactly the same as Fig. 6·1 (*a*) except that a capacitance C_a is left unswitched on the bus. Assume as in Fig. 6·1 that the switch S is opened instantaneously at a current zero at $t = t_0$. After a half-cycle has passed, the conditions are exactly as they appear in Fig. 6·1 before $t = t_1$. If the switch is closed now at $t = t_1$, there will immediately be a redistribution of charge between C and C_a which will be instantaneous; i.e., there will be a very high current for a very short time at the first instant. Thus the capacitor voltage V_c, which was left at $+e_{\max}$ during the interval from $t = 0$ to $t = t_1$, is suddenly at $t = t_1$ reduced from $+e_{\max}$ to a lower value. The *change* in voltage, ΔV_c, on closing the switch S at $t = t_1$ can best be evaluated by applying superposition and inquiring as to what voltage appears across C when we suddenly impress a voltage V_s across the switch, L being considered an open circuit. Since this voltage will divide between C_a and C inversely in proportion to the capacitance values, it is evident that the change in voltage is

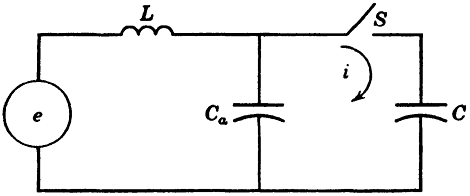


FIG. 6·2. Capacitive circuit interruption leaving part of capacitance on bus. It is assumed that natural-frequency oscillations will be damped out of the L–C_a loop during the half-cycle between restrikes.

$$\Delta V_c = V_s \frac{C_a}{C_a + C} \qquad [6\cdot1]$$

Thus, immediately following the occurrence of a restrike at $t = t_1$, the instantaneous capacitor voltage V_c is

$$V_c = e_{\max} - 2e_{\max}\frac{C_a}{C_a + C}$$

$$= e_{\max}\left(1 - 2\frac{C_a}{C_a + C}\right) \qquad [6\cdot2]$$

The oscillation in V_c following this restrike starts from this initial value, continues on to negative values, and overshoots the $-e_{\max}$ value by an amount that is the difference between the value given by equation [6·2] and $-e_{\max}$, or

$$V_c = e_{\max}\left(1 + 2\frac{C}{C_a + C}\right) \qquad [6\cdot3]$$

This value will occur at the instant of first current zero subsequent to the restrike as at $t = t_2$ in Fig. 6·1. If the switch S is opened suddenly at this time, then this value of voltage will stay on the capacitor C until a second restrike occurs.

The maximum values of switch and capacitor voltage following initial interruption for any number of restrikes can be expressed in the form of a series for this circuit. Let V_{s0}, V_{s1}, V_{s2}, etc., equal the maximum voltage across the switch expressed as multiples of $e_{\max}$ for no restrikes, 1 restrike, 2 restrikes, etc. Similarly, let also V_{c0}, V_{c1}, V_{c2}, V_{c3}, etc., equal the maximum voltage across the capacitor C following no restrikes, 1 restrike, 2 restrikes, etc. Then the following table can be constructed:

$$\begin{tikzcd}
V_{s0} = 2 \arrow[dr] & V_{c0} = 1.0 \arrow[l] \\
V_{s1} = 2 + 2\left(\frac{C}{C_a + C}\right) \arrow[dr] & V_{c1} = 1 + 2\left(\frac{C}{C_a + C}\right) \arrow[l] \\
V_{s2} = 2 + \left[2 + 2\left(\frac{C}{C_a+C}\right)\right]\left(\frac{C}{C_a+C}\right) \arrow[dr] & V_{c2} = 1 + \left[2 + 2\left(\frac{C}{C_a+C}\right)\right]\left(\frac{C}{C_a+C}\right) \arrow[l] \\
\vdots & \vdots \arrow[l]
\end{tikzcd}$$

It is clear that, in general,

$$V_{sn} = 2(1 + A + A^2 + A^3 + \cdots A^n) \qquad [6\cdot4]$$

and

$$V_{cn} = 1 + 2(A + A^2 + A^3 + \cdots A^n) \qquad [6\cdot5]$$

where $A = C/(C_a + C)$. These series hold for the idealized circuit under discussion for any number of restrikes.

For any number of terms in the series, $S_n = 1 + A + A^2 + A^3 + \cdots A^n$, the value of the sum of the n terms is, as is shown in any introductory textbook on series, $S_n = (A^{n+1} - 1)/(A - 1)$. Therefore, for any given number of restrikes n, from equations [6·4] and [6·5] the maximum voltages are

$$V_{sn} = \frac{2(A^{n+1} - 1)}{A - 1} \qquad [6\cdot4']$$

$$V_{cn} = \frac{2A^{n+1} - A - 1}{A - 1} \qquad [6\cdot5']$$

If A is less than 1, then for an infinite number of restrikes

$$V_{s\infty} = \frac{2}{1 - A} \qquad [6\cdot 6]$$

$$V_{c\infty} = \frac{1 + A}{1 - A} \qquad [6\cdot 7]$$

If $A = 1.0$, the series does not converge and the voltages approach an infinite magnitude as n approaches infinity. Thus, for an infinite number of restrikes with $C_a = 0$ and $A = 1$ (which is true in Fig. 6·1, where all the capacitance is switched off), there is no limit to the voltages that can be built up due to restriking in accordance with this analysis. Where C_a is appreciable, A is less than 1 and both series will converge. The limiting values of voltage are given by equations [6·6] and [6·7].

Generally, interest centers in the relative effects of 1, 2, or 3 restrikes although, for ground faults in isolated neutral circuits, a larger number may be of some interest, especially if operation of the system is felt imperative for long periods with ground faults on the system, or if a capacitive circuit is opened by means of a slow switch (i.e., one taking quite a number of half-cycles to complete the interruption). The following table shows the calculated effect of partial capacitance switching for 1, 2, and 3 restrikes, using equations [6·4′] and [6·5′].

	1 Restrike		2 Restrikes		3 Restrikes	
A	V_{s1}	V_{c1}	V_{s2}	V_{c2}	V_{s3}	V_{c3}
1.0	4.0	3.0	6.0	5.0	8.0	7.0
0.75	3.5	2.5	4.625	3.625	5.47	4.47
0.5	3.0	2.0	3.5	2.5	3.75	2.75
0.25	2.5	1.5	2.625	1.625	2.656	1.656

The maximum voltages are given in multiples of $e_{\max}$. It is clear that the maximum overvoltage possible per restrike is decreased considerably as a smaller fraction of the total capacitance is switched off the bus. This is a fact of considerable practical importance in power system operation for generally there are several lines radiating from a bus. When any one line is being switched, the maximum possible overvoltage for a given number of restrikes will therefore be reduced by the presence of the other lines. Only for the last line, or the last cable, or the last capacitor bank will A approach the value 1.0 and the worst conditions for producing overvoltages prevail.

6·4 GROUND FAULTS IN ISOLATED-NEUTRAL SYSTEMS

It is easily shown that the circuit of Fig. 6·2 applies for a ground fault in an isolated-neutral system. Consider the three-phase circuit of Fig. 6·3(a). The symmetrical component equivalent circuit is shown in Fig. 6·3(b). If positive- and negative-sequence constants are equal, then Fig. 6·3(c) represents a simpler version, directly valid for the voltage across C_0, which is very useful here. It is immediately apparent that this circuit is identical with that of Fig. 6·2 and that

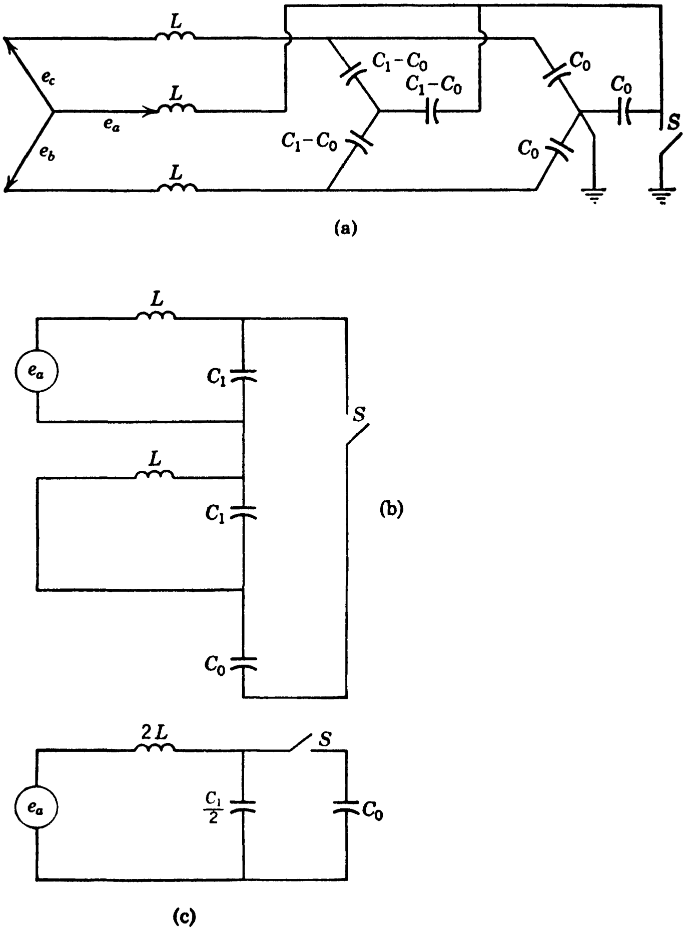


FIG. 6·3. Equivalent circuit for single-line-to-ground fault in isolated-neutral system.

the analysis of the foregoing section applies directly if, for C_a, the value of $C_1/2$ is substituted, and, for C, the value of C_0 is substituted. In a typical isolated-neutral system, the value of $C_1/2$ should be about $0.8C_0$. Therefore,

$$A = \frac{C_0}{0.8C_0 + C_0} = \frac{1}{1.8} = 0.555$$

and for 2 restrikes one would expect from equation [6·4] the maximum voltage across the switch (which is voltage to ground) to be 3.73. Thus it appears from these theoretical considerations that arcing faults in isolated-neutral systems *may* be productive of overvoltages.

It is interesting and significant at this point to evaluate the effect of shunt capacitors for power factor correction in isolated-neutral systems. The factor A is determined by $A = C_0/[(C_1/2) + C_0]$, in which shunt capacitors contribute to C_1 but not to C_0 as ordinarily applied. Therefore A is generally made very much smaller by the addition of such capacitors since they would usually be much larger than the inherent zero-sequence capacitance of a typical isolated-neutral system. In fact, it would appear that power factor correction capacitors might be excellent protection against overvoltages that might possibly be encountered during clearing of ground faults in isolated-neutral systems. As the value of A approaches zero, the maximum voltage obtainable for any number of restrikes is 2.0, although on the unfaulted phases a somewhat higher voltage might be expected due to the somewhat higher than normal voltage transient on these phases when the fault restrikes.

It will be recognized that this type of arcing was classified as Type 3 in the preceding chapter. In isolated-neutral systems some relatively high voltages due to this type of arcing up to 16 times normal have been observed under controlled miniature-system conditions for an infinite number of restrikes. One might wonder how such high voltages could be obtained in view of the foregoing discussion, but it must be emphasized that the analysis in this section is based on a very simple lumped-constant circuit, whereas the miniature-system approach made it possible to consider many very complicated circuits. This analysis is intended merely as a guide rather than a rigorous basis for analysis for systems in general. It should check miniature-system results approximately for simple lumped-constant circuits, however. Such checks can be made by referring to the results in Fig. 5·17, curves A and B for no fault resistance, and curve A of Fig. 5·12 for $X_{l0}/X_{l1} = \infty$. The agreement is very reasonable, in view of the simplifying assumptions made in the analysis in this section.

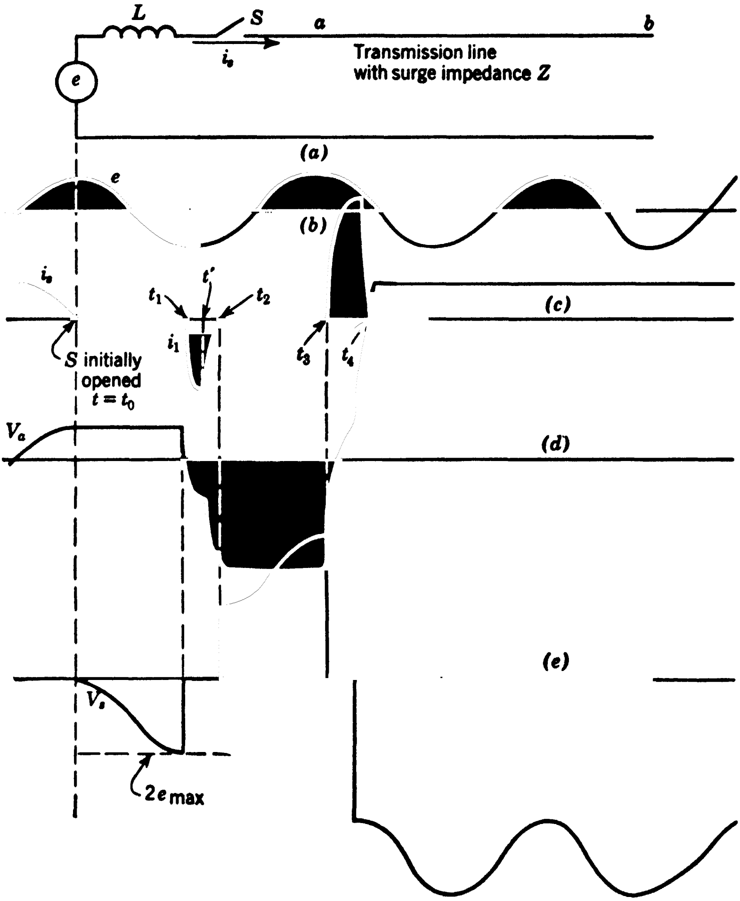


FIG. 6·4. Capacitive circuit interruption with distributed line constants.

6·5 DISTRIBUTED-CONSTANT EFFECTS

Consider next the single-phase circuit of Fig. 6·4, consisting of a source of sinusoidal voltage e, an inductance L, and a transmission line with surge impedance Z. With the switch S in the closed position, the unloaded transmission line is capacitive at fundamental frequency for practical line lengths, so that, prior to the initial opening of the switch, the current i_s flowing from the generator is capacitive. Let L be small as before. When the switch S is opened initially at a current zero as in Fig. 6·4(c), the voltage on the transmission line is left at the $e_{\max}$ value, as shown in Fig. 6·4(d).. V_a is the voltage to ground at the sending end and up to this point may be considered uniform throughout the length of the transmission line without significant error. (The error depends on line length.) After this initial opening of the switch, the voltage across the switch increases in magnitude as shown in Fig. 6·4(e) until a maximum of $2e_{\max}$ is reached just as for the cases considered previously in this chapter.

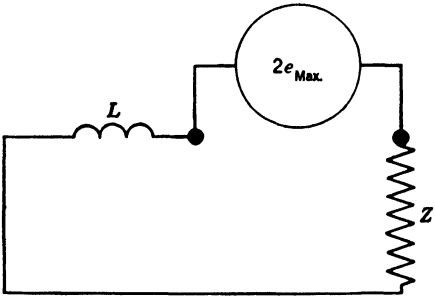


FIG. 6·5. Equivalent circuit viewed from switch for a restrike with distributed line constants. System of Fig. 6·4(a).

If the switch is closed (or a restrike occurs) at the instant of maximum voltage, V_s suddenly drops to zero. The effects can be simulated mathematically by impressing suddenly a voltage $2e_{\max}$ at the switch S. The circuit viewed from the switch for such a suddenly impressed voltage is as shown in Fig. 6·5. The current that flows as a result of this suddenly impressed voltage is obtained by solving the differential equation.

$$Lpi + Zi = -2e_{\max} \tag{6·8}$$

From this equation,

$$i = -\frac{2e_{\max}}{Z}(1 - \epsilon^{-(Z/L)t}) \tag{6·9}$$

The change in V_a due to this restrike is equation [6·9] multiplied by Z, or

$$\Delta V_a = -2e_{\max}(1 - \epsilon^{-(Z/L)t}) \tag{6·10}$$

The value of V_a during this transient period just following the restrike is

$$\begin{aligned} V_a &= e_{\max} - 2e_{\max}(1 - \epsilon^{-(Z/L)t}) \\ &= e_{\max}(-1 + 2\epsilon^{-(Z/L)t}) \end{aligned} \qquad [6\cdot 11]$$

From this expression, it is apparent that V_a does not change instantly, but exponentially in accordance with a time constant L/Z. It approaches the value -1 as t increases as shown in Fig. 6·4(d).

The change in voltage, ΔV_a, given by equation [6·10] travels away from the generator end as a traveling wave with the velocity of light. At the open end b, the wave is totally reflected positively, and, neglecting any attenuation in the line, the same wave returns to the switch S after having traveled out to b. The voltage at a due to this wave impinging upon the terminal a is

$$\Delta V_R(p) = \left(\frac{2p}{p + B}\right) \Delta V_a(p)$$

from which

$$\Delta V_R = -4e_{\max} Bt\epsilon^{-Bt} \qquad [6\cdot 12]$$

in which $B = Z/L$ and time is now measured from the instant of return of the initial wave to the generator end of the line. Equation [6·12] is exactly the same as that given by equation [3·27]. This voltage must be added to that already existing at a and given by equation [6·11]. Thus, if it is assumed that equation [6·11] has essentially reached its final value of $-e_{\max}$ (this depends on the length of line and the time constant $1/B$), then equation [6·12] added to it will cause an additional increase in magnitude with time.

In order to show how the voltage on the line at point a reaches a value of essentially $-3e_{\max}$, it will be necessary to inquire with regard to the transient current zero following the return of ΔV_a as given by equation [6·10]. If the time constant $1/B$ is small compared to the time required for a wave to travel out to the far end b and back again, then, when ΔV_a as given by equation [6·10] returns to a at $t = t'$, the current i_1 will have reached a steady value given by $i = -2e_{\max}/Z$. See Fig. 6·4(c). The returning voltage wave impinges upon the source inductance L and gives rise to a current that is the integral of equation 6·12 divided by L. This current is

$$i' = \frac{-4e_{\max}}{Z}[1 - \epsilon^{-Bt} - Bt\epsilon^{-Bt}] \qquad [6\cdot 13]$$

If this be added to the steady current already existing in L at $t = t'$, then the resultant current in L following $t = t'$ is

$$i_1 = i + i' = \frac{-2e_{\max}}{Z} + \frac{4e_{\max}}{Z}[1 - \epsilon^{-Bt} - Bt\epsilon^{-Bt}]$$

$$= \frac{4e_{\max}}{Z}\left[\frac{1}{2} - \epsilon^{-Bt} - Bt\epsilon^{-Bt}\right] \qquad [6\cdot 14]$$

This current reaches a value of zero at $Bt \cong 1.68$. This is after ΔV_R given by equation [6·12] has reached its negative maximum value of $-1.47e_{\max}$ at $Bt = 1.0$. At this instant of current zero, $t = t_2$, if interruption is assumed, the voltage on the line at this instant is essentially 3.0 throughout its length. At point a, the voltage reaches this value of $-3e_{\max}$ shortly after the interruption of current at $t = t_2$. Actually, there will be a small oscillation on the line due to the redistribution of charge on the transmission line subsequent to the interruption at $t = t_2$, but this is omitted in Fig. 6·4.

Accompanying the initial voltage wave given by equation [6·10] as it traveled away from a was the current wave given by equation [6·9]. When it reached the open end b it was totally reflected negatively and, as the voltage and current waves travel back towards a, the line is left uniformly charged to a voltage of $-3e_{\max}$ and the current in the line is zero. In other words, in the line, the returning current wave cancels the initial outgoing current wave as time increases.

It follows without detailed elaboration that a second closing of the switch at t_3 will result in a reversal of line potential, again leaving it uniformly charged to a value of essentially $+5e_{\max}$ simultaneously with the occurrence of a current zero at t_4. Again, if the switch is opened at this instant, the voltages are as shown in Fig. 6·4.

It appears therefore that, although there are basic differences between the lumped-constant and distributed-constant phenomena, the results are essentially the same. Thus, for the maximum switch voltages with lumped constants, the series was 2, 4, 6, etc. For distributed constants, the series is the same for the switch voltages. Similarly for lumped constants, the capacitor maximum voltage series was 1, 3, 5, 7, etc., and for distributed constants, the maximum line voltages follow this same series.

6·6 PREVENTIVE MEASURES—EFFECT OF BREAKER SHUNTING

Resistors. Although it is not to be inferred that all interrupting devices will produce high overvoltages when interrupting capacitive circuits, there are conditions under which it may be desirable to eliminate the *possibility* of building up high voltages due to restriking.

One scheme that has been found highly successful in accomplishing this very desirable objective is to provide resistors across the breaker interrupting contacts which will remain in parallel with these contacts for a few half-cycles following initial contact parting. After these few half-cycles have passed, the resistors can be switched out as part of the normal breaker operation. The effectiveness of the method is quite complete.

The transient analyzer was found most useful in studying the effectiveness of such shunting resistors. Figure 6·6 shows a circuit arrangement which was studied in detail with a resistor *R* across the switch in phase *a* between *a* and *a′*. In this miniature system interruption with controlled restriking of Type 4 (Chapter 5) was made to take place in phase *a*. Phases *b* and *c* were opened or closed as desired. The ground-fault neutralizer was in the circuit or shorted out as desired. Furthermore, many other system parameters were varied at will. The source reactances are based on 82,500 kva.

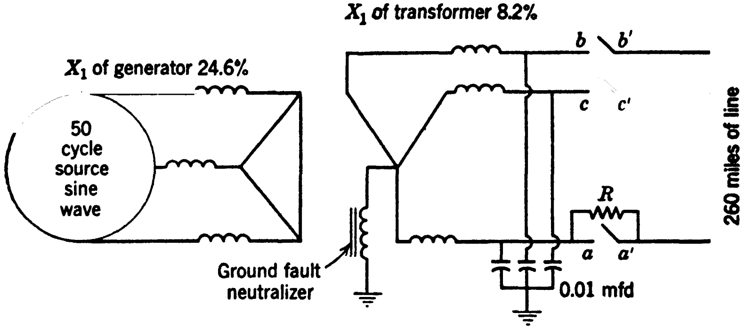


FIG. 6·6. Miniature system studied with the transient analyzer the results of which are shown in Figs. 6·7 and 6·8.

Figure 6·7 shows some typical transient analyzer oscillograms illustrating the phenomena of line charging current interruption with and without the shunting resistor *R* in the miniature system of Fig. 6·6. This figure is for the first phase to clear (phases *b* and *c* closed) with the system neutral solidly grounded. Voltages across the switch contacts are shown to reach values between four and five times normal line-to-neutral crest voltage with two restrikes and without the resistor. When a resistor *R* of 4,600 ohms is inserted as shown in Fig. 6·6, these voltages are substantially reduced, as shown by other oscillograms in Fig. 6·7. Line voltages and bus voltages to ground are also shown.

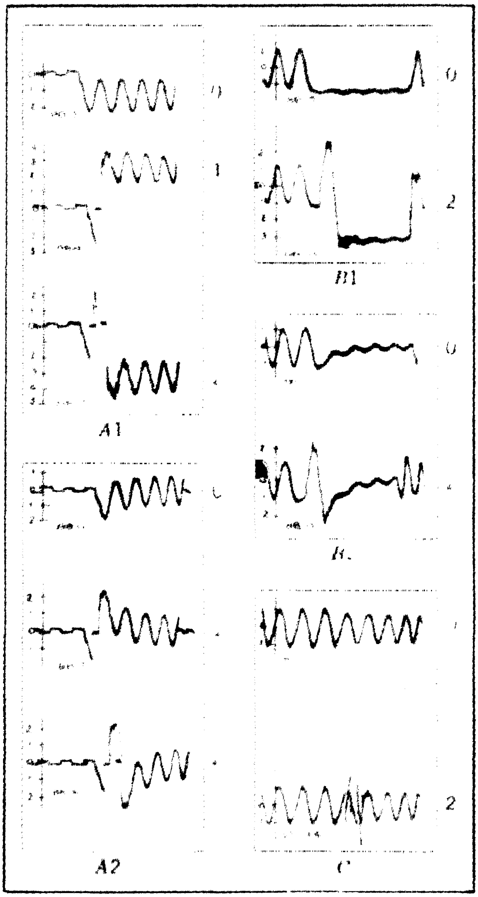


FIG. 6·7. Transient analyzer oscillograms illustrating the phenomena of line charging current interruption with and without shunting resistor in the miniature system of Fig. 6·6. The ordinate is times normal line-to-neutral crest voltage. Numbers on oscillograms show number of restrikes in each instance. *A*, voltage across the switch in phase *a*; (*A*1) no shunting resistor; (*A*2) 4,600 ohms shunting resistor; *B*, line-to-ground voltage on line side of switch in phase *a*; (*B*1) no shunting resistor; (*B*2) 4,600 ohms shunting resistor; *C*, line-to-ground voltage on source side of switch in phase *a*, no shunting resistor.

It is evident from these oscillograms that the shunt resistors are effective in (1) reducing the a-c voltage on a line being switched out before the final break is completed and (2) reducing the d-c voltage left on the line.

With no restriking, the bus voltage at the source is seen to drop noticeably because of the interruption of leading current. However, if restriking takes place, rapid changes in bus voltage can occur which may reach relatively high magnitudes as shown. Voltages on the bus side may be considerably higher than those shown if the neutral impedance is high.

A representative indication of the severity of overvoltages is the voltage across the breaker contacts during interruption. Therefore, the generalized curves of Fig. 6·8 show this voltage as a function of the amount of shunting resistance R for various system conditions. It appears from these curves that there is no particularly critical value of shunt resistance required, but that any value of the order of 2,000 to 4,000 ohms would accomplish a very substantial reduction in overvoltage severity, regardless of the method of system grounding and for any phase to clear, that is, whether it is the first, second, or last phase to clear.

Although even lower values of resistance would appear more effective in controlling these overvoltage magnitudes based upon Fig. 6·8, it must be emphasized that the resistor R must be switched out also to complete the interruption. If R is made very low, then this last interruption becomes essentially the same as if there were no resistance at all. Therefore, based upon this fact and other considerations it seems most desirable to conclude that a resistor of from 2,000 to 4,000 ohms would give the best overall results for the system shown in Fig. 6·6.

It has been shown by this study and numerous similar studies that the effectiveness of a given resistor used in this manner to control switching surge overvoltages depends on the length of line being switched in that particular phase. More generally speaking, in power systems it depends on the microfarads of capacitance being switched off. The ratio R/X_c, where R is the resistor shown in Fig. 6·6 and X_c is the capacitive reactance in ohms at generated voltage frequency of the capacitance being interrupted, is the real criterion of effectiveness. For the case under discussion X_{c1} was 800 ohms. The range of R values from 2,000 to 4,000 ohms would correspond to a range of R/X_{c1} values of from 2.5 to 5.0.

As long as the ratio of R/X_c is kept within this range, resistors will be effective in substantially reducing the severity of transient overvoltages due to delayed restriking in capacitive circuit interruption. This statement holds for switching (1) transmission lines, (2) cables, (3) shunt capacitors, and (4) ground faults in isolated-neutral systems.

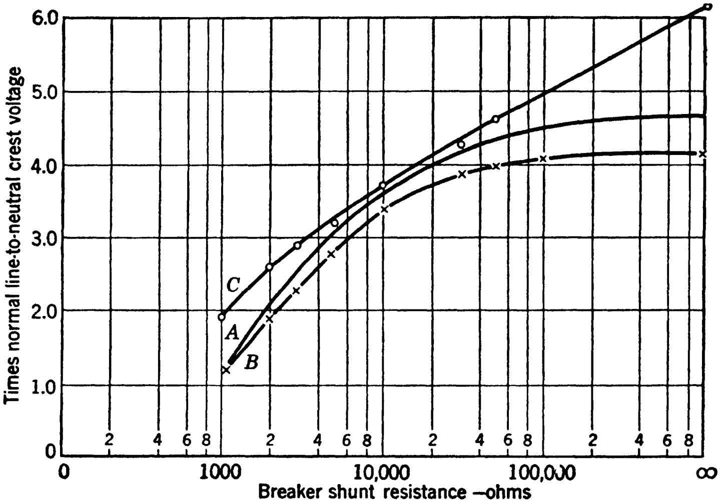


FIG. 6·8. Effect of breaker resistors in reducing transient overvoltages during switching. Voltages shown are the maximum possible across the breaker for two restrikes in the phase being switched for any phase to clear, that is, first, second, or last. *A*, system neutral solidly grounded; *B*, system neutral grounded through ground-fault neutralizer tuned to 260 miles of line; *C*, system neutral grounded through ground-fault neutralizer tuned to half (130 miles) of the total of 260 miles being switched.

In the isolated-neutral system, the ratio R/X_c as used above is not concerned with positive-sequence capacitance but is concerned with zero-sequence capacitance. The X_c used for determining the effectiveness of the resistor R in a given instance should be $3X_{c0}$ where X_{c0} is the total system zero-sequence capacitive reactance.

If a resistor is selected on the basis of sufficiently reducing transient overvoltages for the longest line that a given breaker is likely to be called upon to interrupt, then that resistor should be even more effective in reducing transient switching voltages for shorter lines, although the surge associated with the final interruption of resistor current is likely to be slightly increased for switching very short line lengths.

Figures 6·9 and 6·10 show how such resistors are actually used in a high-voltage circuit breaker. Laboratory tests using this breaker have shown that the presence of the resistors is also effective in reducing the arcing time when the breaker is interrupting inductive (short-circuit) currents. Results of such a series of tests are shown in Fig. 6·11.

In order to test the effectiveness of the breaker shown in Figs. 6·9 and 6·10 when equipped with these resistors (3,000 ohms in each phase) under actual system operating conditions, a series of field tests was made during the week end of March 27 to 30, 1942, on the 230-kva No. 8 oil-circuit breaker in the switchyard of the Southern California Edison Company, at Boulder Power Plant.[10] This breaker is a tank-type breaker which contains multi-break interrupters and has an interrupting time rating of five cycles.

All tests were circuit opening in which only the charging current of the connected line (far end open) was interrupted—commonly termed line-dropping tests. In this type of interruption, leading currents involved are small. Thus, within reasonable limits, arcing time is no important consideration. On the other hand, unless precautions are taken, breaker restriking quickly generates overvoltages which may be harmful to the breaker itself and to other connected equipment. Control of these overvoltages was therefore the primary concern in these tests,

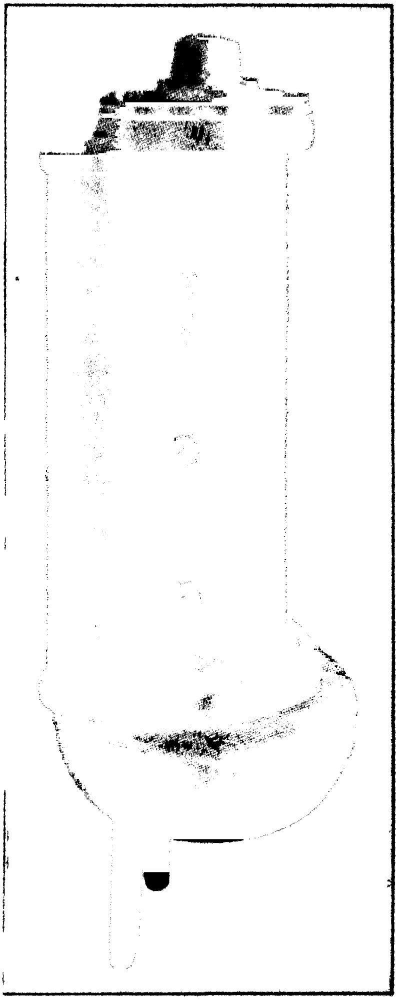


FIG. 6·9. General Electric multi-break interrupter unit for 230-kv oil-circuit breaker with resistor attached. Two such interrupters are used per pole as shown in Fig. 6·10.

and the purpose of the test program was to determine (1) the nature and magnitude of the overvoltages under various system conditions and

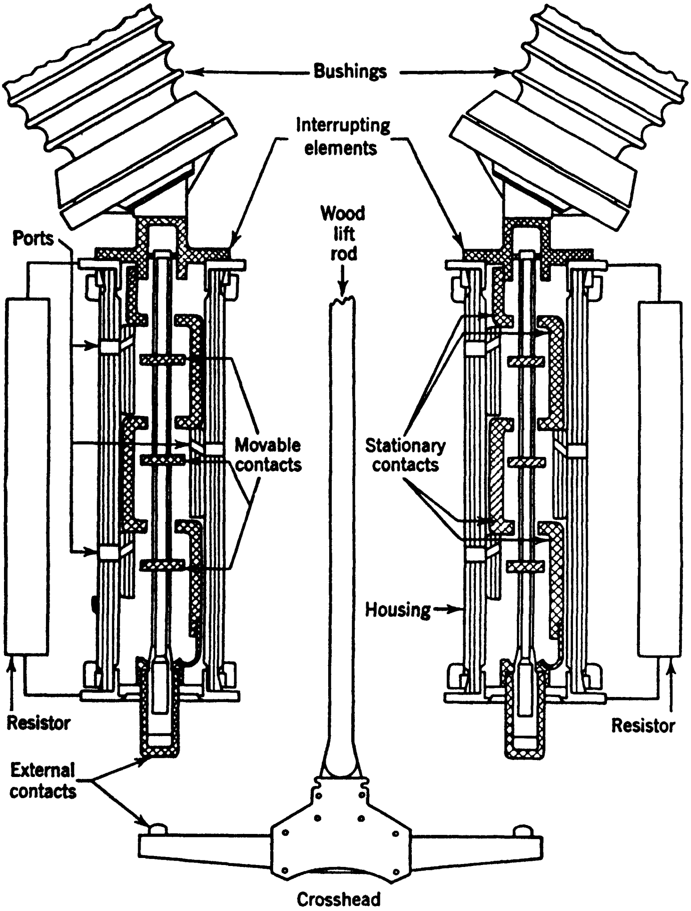


FIG. 6·10. Schematic diagram showing the multi-break interrupters with resistors attached. The resistor current is interrupted in the gap formed by the external contacts shown above.

(2) the effectiveness of resistors assembled in the breaker, in parallel with the interrupters, in reducing these overvoltages to harmless values.

This approach naturally required that the line-dropping test program should include various practical system conditions with and without resistors in parallel with the interrupters. The following conditions were subjected to change: (1) the length of line being disconnected, (2) the neutral connection, and (3) the nature of the power source.

CIRCUIT CONDITIONS. 1. In some tests the entire 233-mile line to Chino substation was disconnected at Boulder Power Plant; in others only the 132-mile section to Pisgah substation.

2. In some tests the neutral was solidly grounded; in others it was grounded through reactance. Three different values of reactance were used; one of these values was that required for tuning as a ground-fault neutralizer to the line length being disconnected; the other two values, corresponding to other neutralizer tap settings, have been used to tune the line in conjunction with additional reactors at the far end of the line and consequently correspond to reactances above the tuned value.

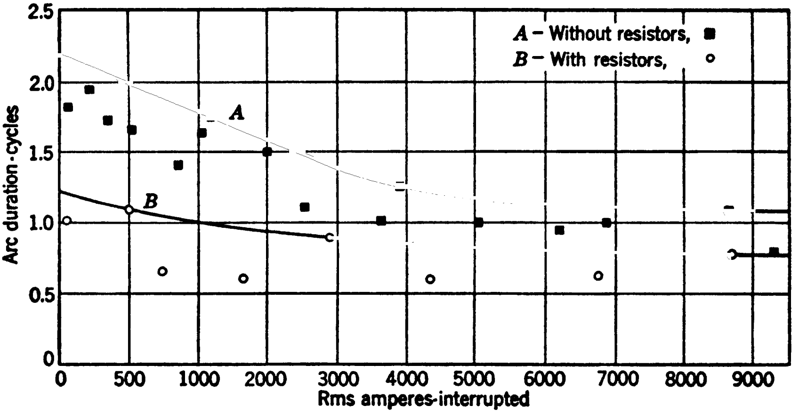


FIG. 6·11. Breaker arcing time during the interruption of inductive short circuits, showing the improvement as a result of reduced recovery rates and better internal voltage distribution brought about by resistors in the breaker.

3. In some tests two local generator-transformer units were connected to the bus, and the remainder of the system was tied in through the second line to Chino substation. In the others only one generator-transformer unit was connected to the bus, and there was no tie with the rest of the system.

TEST RESULTS. A total of ten groups of tests was made at Boulder Power Plant covering practical combinations of circuit and breaker arrangements. Table 6·1 summarizes the results of these tests with respect to the maximum voltages developed and with respect to the

TABLE 6·1

BOULDER POWER PLANT LINE-DROPPING TESTS—230 KV

Test No	Miles Line	Resistors	Neutral Reactor	Circuit Connection	Maximum Voltages *—Times Normal Crest									Crosshead Travel in Inches from Contact Separation to Interruption of Last Phase	
					Bus-to-Ground			Line-to-Ground			Across Breaker				
					A	B	C	A	B	C	A	B	C	Inside Inter-rupters	Final Break (Stroke = 36 Inches)
1*A*	132	Out	Short-circuited	2 generators 2d line	1 6	1 1	1 1	2 0	1 0	1 4	3 0	1 5	2 6		2¾
1*B*					1 1	1 1	1 2	1 3	1 3	1 3	2 2	2 0	2 2		2½
1*C*					1 0	1 0	1 0	1 0	1 0	1 2	2 0	2 0	2 1		1
1*D*					1 8	1 1	1 1	2 0	1 4	1 1	2 4	2 4	2 0		1⅝
1*E*					1 1	1 1	1 1	1 6	1 6	1 4	2 7	2 0	2 6		2¼
2*A*	233	Out	Short-circuited	2 generators 2d line	1 1	1 1	1 0	1 6	1 3	1 6	2 4	2 2	2 3		2⅝
2*B*					1 1	1 1	1 1	1 3	1 2	1 7	2 2	2 4	2 5		2⅝
2*C*					1 3	1 1	1 6	1 2	1 4	1 9	1 8	2 4	3 0		2¾
2*D*					1 6	1 2	1 5	2 2	1 2	1 8	3 0	2 0	3 0		3⅜
2*E*					1 1	1 2	1 2	1 4	1 7	1 7	2 4	2 4	2 2		2½
3*A*	132	In	Short-circuited	2 generators 2d line	1 0	1 0	1 2	1 0	1 0	1 0	1 0	1 0	1 0	1⅛	10⅛
3*B*					1 0	1 0	1 1	1 0	1 0	1 0	1 0	1 0	1 0	1	9½
3*C*					1 0	1 0	1 2	1 0	1 0	1 0	1 0	1 0	1 0	1⅛	9
3*D*					1 0	1 0	1 2	1 0	1 0	1 0	1 0	1 0	1 0	1	9¼
3*E*					1 0	1 0	1 2	1 0	1 0	1 0	1 0	1 0	1 0	1	11½
4*A*	233	In	Short-circuited	2 generators 2d line	1 3	1 0	1 1	1 5	1 1	1 2	1 5	1 4	1 0	1¾	9⅞
4*B*					1 0	1 1	1 1	1 1	1 1	1 1	1 1	1 2	1 3	1⅛	9¼
4*C*					1 1	1 1	1 6	1 3	1 1	1 6	1 4	1 1	1 5	1¼	10⅞
4*D*					1 0	1 1	1 1	1 1	1 0	1 0	1 1	1 4	1 0	⅞	8¾
4*E*					1 0	1 1	1 1	1 0	1 1	1 0	1 5	1 3	1 1	1	8½

5A	132	In	Tap 7	1 generator	1.6	1.6 *	2.0 *	1.5	1.2	1.0	1.4	1.2	1.0	1¾		14
5B					1.7 *	1.4 *	1.5 *	1.0	1.3	1.1	1.0	1.0	1.0	1⅜		11⅞
5C					1.7	1.7	1.7 *	1.7	1.0	1.0	1.6	1.1	1.1	2½		13⅞
5D					1.6 *	2.0 *	1.5 *	1.2	1.0	1.2	1.1	1.0	1.0	1¼		14¾
5E					1.9 *	1.5	1.5 *	1.7	1.1	1.0	1.6	1.2	1.0	2½		14
6A	233	In	Tap 7	1 generator	1.6	1.6 *	1.5	1.5	1.5	1.2	1.8	1.6	1.3	2¾		12¾
6B					1.5	1.5	1.4 *	1.2	1.5	1.3	1.3	1.5	1.2	2¾		12¾
6C					1.4	1.7	1.4	1.3	1.4	1.3	1.2	1.5	1.2	2¾		11⅞
6D					2.0	2.0 *	1.4 *	1.6	1.0	1.0	1.5	1.4	1.0	2¾		12⅝
6E					1.4 *	2.0	1.6	1.1	1.7	1.0	1.3	1.6	1.2	2¾		10½
6F	233	In	Tap 3	1 generator	1.1	1.6	1.4 *	1.2	1.4	1.3	1.5	1.4	1.3	2⅝		11¼
6G					1.2 *	1.6	1.7	1.1	1.6	1.5	1.4	1.5	1.4	2⅞		11⅞
7-1-A	132	Out	Short-circuited	1 generator	1.1	1.1	2.0	1.1	1.1	1.7	1.1	1.9	2.7		2⅞	
7-1-B					1.1	2.0	1.9	1.1	2.0	1.9	1.5	2.6	2.4		3⅛	
7-1-C					1.2	1.3	2.1	1.2	1.5	1.7	2.0	2.3	2.6		3⅛	
7-1-D					1.2	1.4	1.5	1.1	1.2	1.1	1.5	2.1	1.8		3	
7-1-E					1.3	1.2	1.2	1.3	1.1	1.3	1.6	1.9	2.3		2⅝	
7-2-A	132	Out	Tap 9 † Tap 10 † (Tuned system)	1 generator	2.4	2.2	2.2	1.8	2.0	2.7 ‡	2.1	2.9	3.2 ‡		4¼	
7-2-B					3.6	2.8	2.7	2.4 ‡	1.6	1.6 ‡	3.5 ‡	3.2	4.3 ‡		5	
7-2-C					2.6	2.3	2.2	1.6	1.6	2.1 ‡	2.2	3.1	2.4 ‡		4¾	
7A	132	Out	Tap 7	1 generator	2.9	4.4 ‡	3.2	2.1 ‡	4.4 ‡	3.2 ‡	3.3	...	4.6 ‡		6	

* Maximum voltages to ground recorded for both the bus and the line side of the breaker include voltages encountered either in the course of the primary interruption within the interrupters or in the course of the interruption of resistor current by the external break (see Fig. 6·10). Voltages encountered in the interruption of resistor current are marked by an asterisk. For maximum voltage across the breaker only voltages occurring in the course of the primary interruption are recorded, since voltages across the breaker during the interruption of resistor current involve no stress on solid insulating material other than that occasioned by voltages to ground on either side of the breaker. Voltage across the breaker during the interruption of resistor current never exceeded twice normal line-to-neutral crest.

† Two reactors were connected in parallel; one on tap 9 and one on tap 10.

‡ Reading affected by flashover in voltage divider. Approximate correction applied.

breaker crosshead travel necessary for interruption. The overvoltage record, with and without resistors, is presented graphically in Fig. 6·12 in such a way that the effect of the resistors, of the length of line, and of the ground-fault-neutralizer tap setting are readily apparent.

In the course of these field tests, three arrester operations were observed, one on phase *a* and two on phase *c*. These arresters are

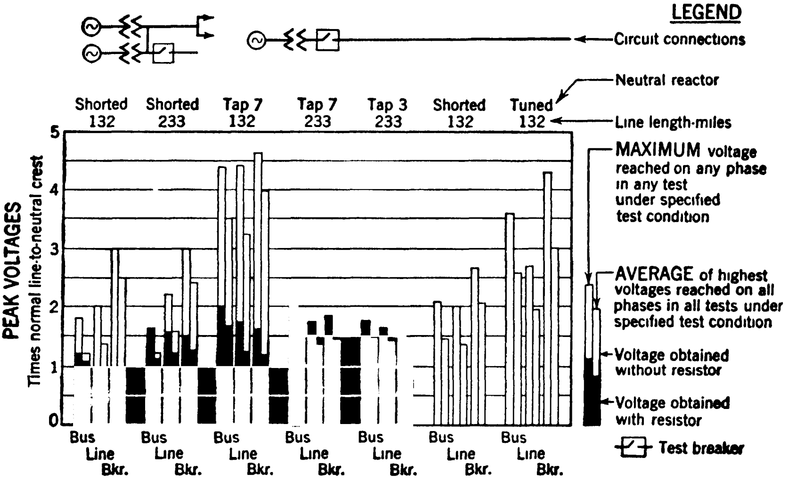


FIG. 6·12. Pictorial summary of results of oil-circuit-breaker tests made at Boulder Power Plant March 28 and 29, 1942. Comparison of voltages caused by interruption of transmission-line charging current. Here the overvoltages, obtained with and without breaker resistors, under various line and ground conditions are shown.

located near the terminals of the 82,500-kva transformer bank at Boulder Power Plant which is about one mile from the Southern California Edison Company switchyard. All three arrester operations occurred during the tests which used the ground-fault neutralizer and no resistors in the breaker. During an earlier series of tests at Boulder Power Plant, at which time the above problems were being diagnosed, under similar system conditions (with the neutralizer untuned and without resistors in the breaker) six lightning-arrester operations were observed. No arrester operations were observed, however, for ten similar tests made at that time with the neutral solidly grounded.

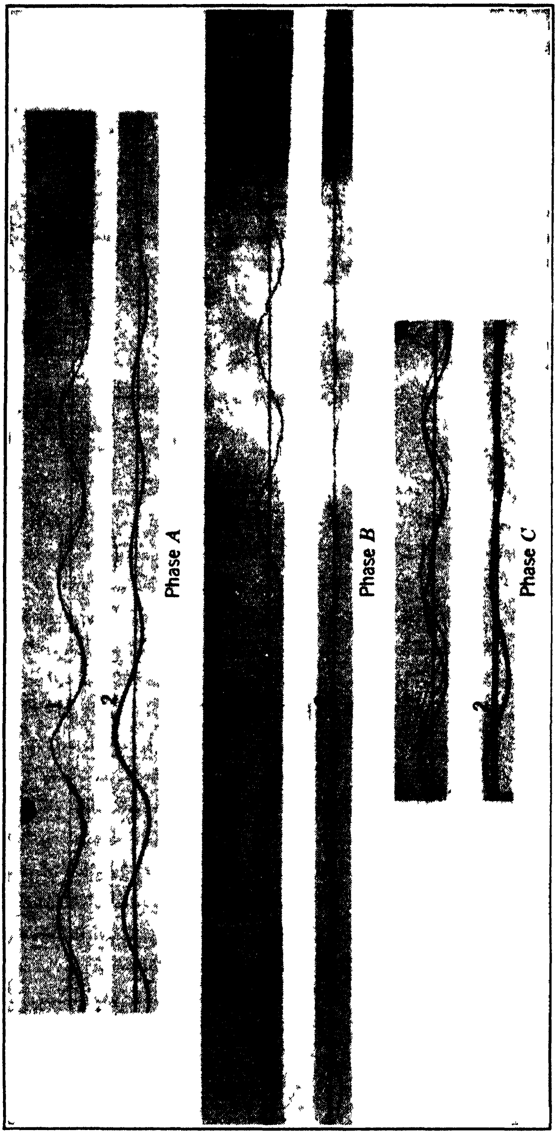
The interrupters were completely examined at four points in the test program. There was no damage to the contact or insulation parts as a result of any of the tests made when the resistors were connected.

On the last inspection after tests with the ground-fault neutralizer and without resistors (during one of which tests with the neutralizer untuned the breaker flashed over externally), there was some damage to the insulation parts of the interrupters but not of sufficient magnitude to prevent the successful interruption of the circuit.

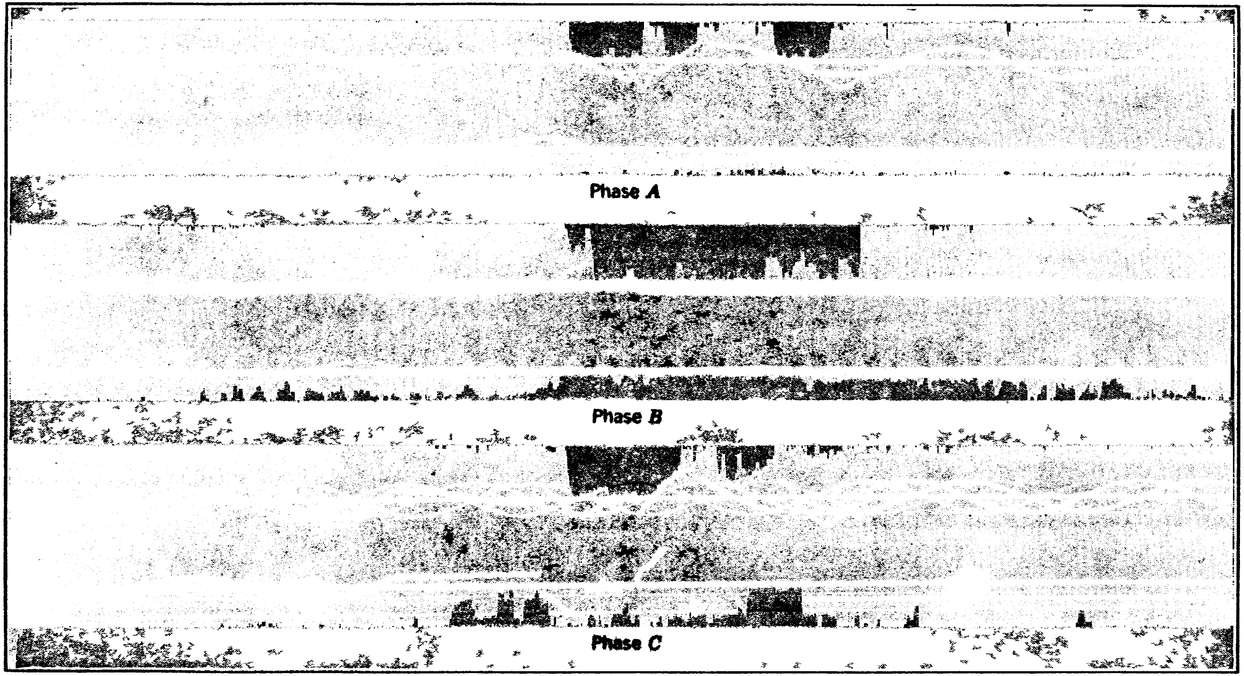
OSCILLOGRAPH MEASUREMENTS AND RECORDS. A good illustration of the effectiveness of internal resistors in reducing transient switching overvoltages is shown in the cathode-ray oscillograms of Fig. 6·13 which places together for easy comparison a test with a 3,000-ohm resistor in the breaker (Fig. 6·13(*a*)) and a duplicate test without resistors (6·13(*b*)). These tests are numbers 5*A* and 7*A*, respectively (Table 6·1), in which 132 miles of line were dropped with the ground-fault neutralizer on tap 7. Figure 6·13(*a*) shows no overvoltages of any consequence although there is restriking during the clearing period. In contrast the restriking associated with Fig. 6·13(*b*) (no resistors) resulted in violent transient disturbances which were so high as to cause sparkover of a protective gap in the cathode-ray-oscillograph measuring circuit which made it impossible to determine accurately the actual crest voltage obtained. The voltages reached for this test, however, were high enough to spark over the 220-kv bushing on the source side of the breaker. This would require a maximum of 1,050-kv crest, depending on several factors, the most important of which are the wave shape of the transient voltage and the lower humidity, both of which would lower the actual flashover.

The oscillograms of Figs. 6·14(*a*) and 6·14(*b*) are the magnetic oscillograph records of the same field tests as presented in Fig. 6·13, namely tests 5*A* and 7*A*, respectively, of Table 6·1. In Fig. 6·14(*a*) the duration of the unity-power-factor resistor current is clearly shown. Characteristic restrike current surges are shown in Fig. 6·14(*b*). Here four separate restrikes took place, all of which interrupted with the arrival of the first current zero of the high-frequency oscillation. Figure 6·14 also exhibits oscillograph measurements of phase current, trip coil current, and breaker travel.

Voltage-to-ground on each side of each pole of the test breaker was indicated on the cathode-ray oscillograph.[9] The deflection plates of this oscillograph were energized by means of capacitance potentiometers, the upper section of which was furnished by the carrier-current coupling capacitors on the two transmission lines. This provided an arrangement capable of reproducing satisfactorily voltages of all types from the high-speed transients obtained upon a restrike in the circuit breaker to the slowly leaking charge left on the lines upon

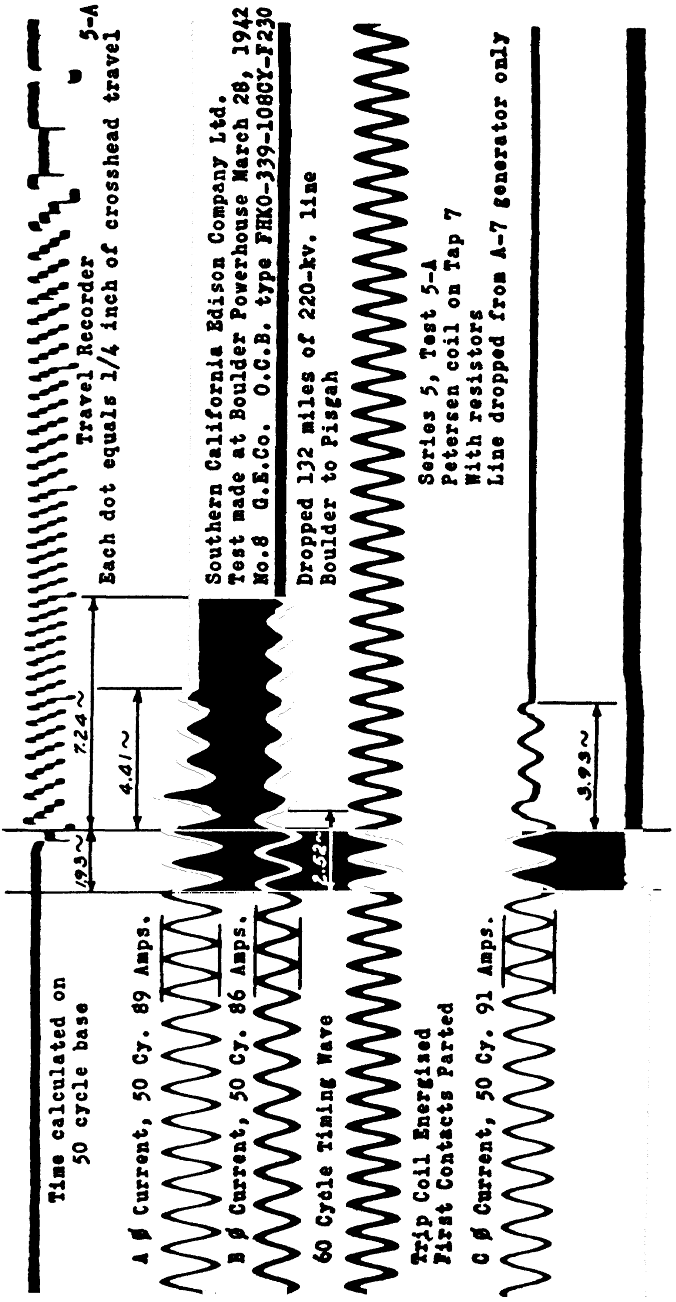


(a)

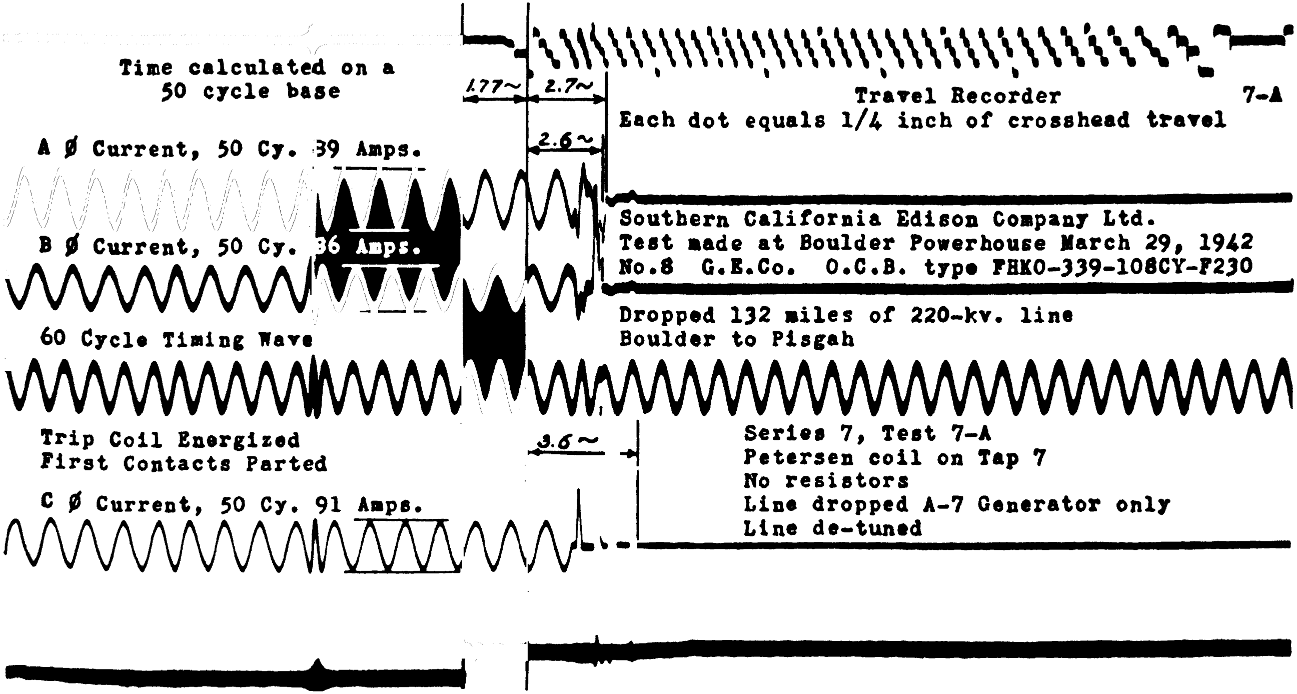


(*b*)

Fig 6·13 Comparative cathode-ray oscillograms of tests made at Boulder Power Plant when dropping 132 miles of 230-kv line with ground-fault neutralizer on tap 7 (*a*) With 3,000 ohms in breaker (test 5*A* in Table 6 1) (*b*) Without benefit of breaker resistors (test 7*A* in Table 6 1) During this test a 220-kv bushing flashed over 1 Voltage to ground on bus side of breaker. 2 Voltage to ground on line side of breaker


Time calculated on
50 cycle base
A ∅ Current, 50 Cy. 89 Amps.
B ∅ Current, 50 Cy. 86 Amps.
60 Cycle Timing Wave
Trip Coil Energized
First Contacts Parted
C ∅ Current, 50 Cy. 91 Amps.
1.95~
7.24~
4.41~
2.52~
3.93~
Travel Recorder
Each dot equals 1/4 inch of crosshead travel
5-A
Southern California Edison Company Ltd.
Test made at Boulder Powerhouse March 28, 1942
No.8 G.E.Co. O.C.B. type FHKO-339-108CY-F230
Dropped 132 miles of 220-kv. line
Boulder to Pisgah
Series 5, Test 5-A
Petersen coil on Tap 7
With resistors
Line dropped from A-7 generator only

(*a*)



(b)

FIG. 6·14. Magnetic oscillograms associated with the voltage records of Fig. 6·13. (a) With breaker resistors. Note resistor current. (b) Without resistors. Four separate restrikes occurred on this test.

final interruption. The two voltages for each pole of the breaker were recorded on parallel traces of the same film, so that the voltage across the breaker pole could be determined readily by graphical subtraction.

DISCUSSION OF RESULTS. Figure 6·12 shows the effect of the resistors upon the overvoltages attending switching. As is indicated by this figure, three pairs of test groups were taken in such a manner that direct comparison can be made to show the effect of the resistors. In these cases, and also in the other groups taken with the resistors connected, the voltage was limited to twice normal line-to-ground peak or less—both when the resistor was being switched on and when the resistor current was being interrupted. When the resistors were not connected, on the other hand, the voltage across the breaker reached three times normal with the neutral solidly grounded and more than four times normal with a neutral reactor connected.

Correspondingly, as shown in Table 6·1, the resistors bring about a considerable reduction of arc length inside the interrupters, as a result of the improved internal voltage distribution and the improvement in the rate of rise of recovery voltage. The residual current which remains in the resistors is then interrupted by the disconnecting break (formed by the crosshead, Fig. 6·10) in about 15 per cent to approximately 35 per cent of the available break distance. Here also the resistors are effective in preventing any build-up of voltage beyond the values shown in Table 6·1 and Fig. 6·12.

The fact that the resistors produce such a substantial reduction in overvoltage makes them preferable to other measures, since they afford protection thereby to other terminal apparatus. Unless the breaker is sufficiently positive in action to prevent restriking and its associated overvoltages, therefore, resistors appear desirable whenever the length and voltage of the lines are such as to give rise to trouble of this type. Because of the dependence of this restriking phenomenon on so many factors, the most important of which is the breaker characteristic, it is impossible to indicate here the capacitance kilovolt-amperes where restriking can be expected to occur. Tests at 15 kv on one manufacturer's breakers,[5] for example, indicated a limit of 10,000 capacitive kilovolt-amperes above which overvoltages could be expected without the aid of internal resistors. The 132 miles of line dropped, as exhibited in Figs. 6·13 and 6·14, represents about 35,000 capacitive kilovolt-amperes. These values cannot be compared directly with those shown at 15 kv, however, because of the wide difference in voltage levels. The foregoing makes clear that, when small capacitive kilovolt-amperes are to be interrupted, with standard power breakers (short lines or cables, protective capacitors, and so forth), no trouble should be expected.

The operations in all the tests presented in this chapter were confined to dropping a length of open transmission line, but, since breaker operation upon all except three-phase faults involves dropping at least one unfaulted phase conductor, the phenomenon may be present in the majority of breaker operations.

The results of these tests are in good qualitative agreement with the results of miniature-system tests as regards the principal features:

1. With the untuned neutral reactor, voltages obtained without the resistor were much higher than those obtained with the neutral solidly grounded.
2. The resistors brought about a substantial reduction of voltage in all cases, and the higher the initial overvoltages, the greater was the percentage of reduction.

A comparison of the overvoltages obtained for the tuned ground-fault neutralizer with those obtained for the solidly grounded system indicates the desirability of supplementing miniature tests with full-scale tests under actual conditions. This is emphasized by the fact that the voltages associated with test series 7–2 are very high, relative to those obtained for test series 1, which was the solidly grounded case with the same line length. From earlier studies, on the other hand, it had been concluded on the basis of the transient-analyzer results that interrupting line-charging current in a ground-fault-neutralizer system is no more difficult from a switching standpoint than in the same system with neutral solidly grounded. The reason for this disagreement apparently lies in the fact that surges communicated to one phase by clearing or restriking in another were of much more importance in the tests with the ground-fault neutralizer than in the tests with the neutral solidly grounded. Not only did this have a direct bearing on voltage magnitudes, but in several instances it appeared to be directly responsible for additional restrikes which contributed still more to the overvoltages. In the grounded-neutral tests there was never more than one restrike that contributed appreciably to the building up of voltage, but several times there were two in the ground-fault-neutralizer tests. Since the miniature-system tests were based on the assumption of two restrikes with the worst timing, the full-scale grounded-neutral tests produced overvoltages considerably lower than predicted. The ground-fault-neutralizer overvoltages were about equal to predicted values, the loss caused by the fact that the worst timing was not realized being approximately balanced by the increase arising from interphase reaction.

6·7 EFFECT OF LOW-SIDE SWITCHING

Occasionally it is possible to take a line out of service by switching on the low-voltage side. In fact, there are systems in which no high-voltage circuit breakers are provided so that line dropping, clearing of faults, and all switching are done at low voltage. For such systems, even though the current being interrupted on the low-voltage side is capacitive for line-dropping operations, no overvoltages of serious

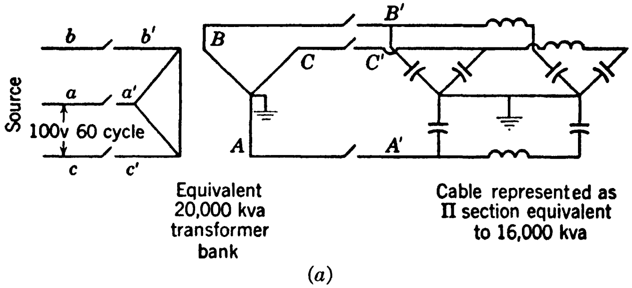
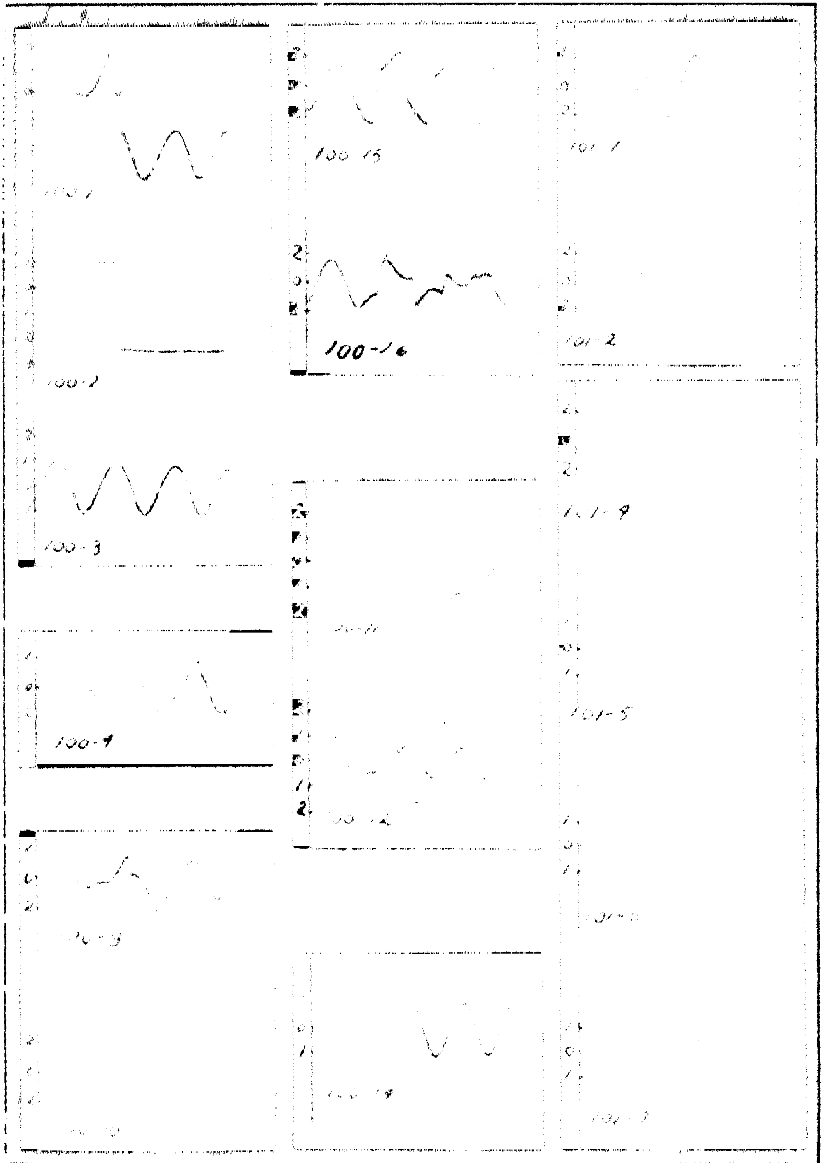

Source
b
b'
a
a'
100v 60 cycle
c
c'
B
C
A
B'
C'
A'
Equivalent 20,000 kva transformer bank
Cable represented as Π section equivalent to 16,000 kva
(a)

FIG. 6·15. Comparison of low-voltage and high-voltage switching of capacitance circuits. (a) Miniature system studied. The actual system was 66 kv. (b) Oscillograms of voltages in the miniature system.

TABLE FOR FIG. 6·15 (b)

	Oscillogram	*Voltage*	*Restrikes*
High-voltage switching (First phase to clear)	100–1	A–A'	1 in Phase A
	100–2	A' to ground	" " " "
	100–3	A to ground	" " " "
Low-voltage switching (First phase to clear)	100–4	a–a'	None
	100–9	a–a'	1 in Phase a
	100–10	a' to ground	" " " "
	100–15	a'–c	" " " "
	100–16	a'–b	" " " "
	100–11	C to ground	" " " "
	100–12	B to ground	" " " "
	100–14	A to ground	" " " "
(Second phase to clear—a open)	101–1	c–c'	None
	101–2	c–c'	1 in Phase C
	101–4	a–a'	" " " "
	101–5	B to ground	" " " "
	101–6	A to ground	" " " "
	101–7	C to ground	" " " "



(*b*)

Fig. 6·15 (*Continued*).

magnitude can occur due to restriking of the type under discussion as long as the kva rating of the transformer energizing the capacitance involved is equal to or greater than the capacitive kva being supplied. This criterion is one that is normally met, of course, in any practical case.

Figure 6·15 shows some results of a transient analyzer study from which the foregoing conclusion is drawn. It is clearly indicated by these oscillograms that the transformer high-voltage winding, connected in wye with neutral solidly grounded, affords an effective discharge path for the line capacitance so that overvoltages cannot be produced as a consequence of restriking when switching is done on the low-voltage delta-connected side of the transformer.

6·8 EFFECT OF NEUTRAL GROUNDING

As far as the production of overvoltages is concerned, the isolated-neutral system is under suspicion every time a ground fault occurs. The arcing fault itself may lead to overvoltages but more likely the final clearing of the faulted portion by breaker operation may lead to undesired overvoltages. Fortunately, the capacitive kva involved during such interruptions is generally quite small and at low voltage so that difficulties due to restriking are not so likely to be encountered as would be true for extensive high-voltage isolated-neutral systems. There are at present very few isolated-neutral systems above 33 kv in this country. One of the best justifications for extinction of the high-voltage isolated-neutral system is the overvoltage problem associated with interruption of ground faults in accordance with the fundamental theory outlined in this chapter.

So far as overvoltages produced by line dropping are concerned, the effect of neutral grounding impedance is not critical. That is, there is no particular critical value that eliminates the possibility of overvoltages associated with restriking during line dropping. It is clear, however, from numerous studies made on miniature systems and from field tests such as those described in this chapter, that the solidly grounded system is least productive of overvoltages. As the amount of neutral impedance is increased, systems become more prolific as far as the production of overvoltages during line dropping is concerned. Systems that permit full neutral displacement (or more) during a single-line-to-ground fault are generally more likely to give rise to overvoltage trouble during faults and switching than those that are solidly grounded.

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7 SWITCHING SURGES AND OVERVOLTAGE ARRESTER DISCHARGE REQUIREMENTS

7·1 INTRODUCTION

In the foregoing chapters, circuit arrangements and switching with or without faults have been discussed, largely from the point of view of the possibilities of producing overvoltages in power systems. It is logical at this point to bring into the discussion the effect of overvoltage protective devices and to inquire first with regard to the ability of the arresters to limit transient overvoltages in power systems and second with regard to the discharge capacity that an arrester must have if it is to limit such transient overvoltages without damage to itself.

The history of the "lightning" arrester goes back through several decades.[1, 2] The need for a device for discharging lightning currents without permitting damage to the electrical apparatus connected to a power system was apparent early in the history of the development of power systems. The need for protection of insulation against the severe lightning discharges was very real. A tremendous amount of well-directed effort was expended in developing suitable devices which, for instance, could be connected between one terminal of a transformer and ground, and, which, when so connected, would limit the maximum voltage that could exist between the terminal and ground. There was also much coordinated effort directed to the determination of magnitudes of lightning voltages and lightning stroke currents. Simultaneously investigations were made of the problem of shielding transmission lines so that lightning discharges could not produce voltage surges on transmission lines to the extent to which they would otherwise be produced if no shielding in the form of overhead ground wires were present. The theory of the counterpoise was developed and put into practice. The need for keeping tower footing resistances to low values was apparent. Methods of system neutral grounding entered into the picture, and finally the ground-fault neutralizer found much favor as an economically sound means for reducing outages on overhead lines due to lightning discharges.

Years of effort, cooperation, and coordination of theory and practice went into the development of now-accepted practices with regard to overvoltage protection of power systems. The entire philosophy of protection was to minimize the consequences of lightning discharges to an economically justifiable extent as far as they might affect continuity of service. Arresters were therefore designed to withstand all but the most severe lightning surges that nature could produce. It was entirely logical that such devices came to be known as "lightning arresters."

Since about 1930, it has become increasingly evident that lightning arresters were serving a more general purpose, namely, that they were and are serving as *overvoltage* arresters. They serve to limit overvoltages regardless of origin. It has become apparent also that certain switching surges may be of sufficient magnitude and duration to exceed the discharge capacity of arresters designed to withstand the discharging of all but the most severe lightning discharges. Therefore, switching surges become an important factor for consideration in the proper design and application of arresters. In a number of installations, it became apparent that lightning arresters were functioning more often on switching surge overvoltages than on lightning discharge overvoltages. Consequently there was a very real need for studying switching surges and the subsequent arrester discharges to see what duty was imposed upon the arrester each time its series gap was sparked over by a system switching overvoltage surge.

In view of the six preceding chapters, it is apparent that the possible combinations of system, switching, and fault conditions are very great in number. The complications of arrester location in the system and arrester voltage rating, likewise reduced to miniature-system scale, added also to the number of possibilities. Numerous studies were made of various arrangements, and from these studies certain salient facts were disclosed. It is the object of this chapter to bring out those salient facts.

7·2 FUNDAMENTAL CONSIDERATIONS

At the outset, it is desirable to recognize that, *exclusive of lightning currents,* the sources of energy that an arrester may be called upon to dissipate may be (1) energy generated appearing as power-follow current, (2) energy stored in inductance, and (3) energy stored in capacitance. There are cases in which all three of these may be encountered in varying proportions. On the other hand, there are occasions on which one of these sources of energy is so predominant that the others may be neglected.

The first-named source of energy (item 1) requires a valve element to limit the power-follow current which, when used with an appropriate gap structure, affords the basis for rating arresters in terms of maximum kv rms. Sparking over an arrester gap will result in the flow of power-follow current when an arrester is connected to the usual power system. This is true even for the isolated-neutral system although the power-follow current may be capacitive rather than inductive. Although a half-cycle approximately of power-follow current is normally expected subsequent to gap operation, this requirement of passing follow current in addition to either (2) or (3) or both should not be overlooked. Power-follow currents associated with rms voltages within the maximum kv rating of an arrester are within the overall design rating of the complete arrester. Item 1, therefore, is simply associated with recognizing the maximum dynamic rms voltage that can occur for any possible fault and system condition (including all *momentary* conditions resulting from switching operations during transient disturbances) and selecting an arrester with a kv maximum rating corresponding to this with a suitable small margin of safety. The rules for selecting and applying arresters properly on this basis are included in published literature and accepted by the industry. Therefore, this phase of the problem will not be given further consideration here.

An illustration of the second source of energy (item 2, energy stored in inductance) being predominant is the case of a current limiting fuse giving rise to a sufficiently high voltage surge so as to cause an arrester gap to spark over, thus abruptly shifting the inductive current from the fuse path to the arrester. Correct coordination between design and application of such fuses and arresters is necessary in order to ensure satisfactory overall performance. Interruption of magnetizing current off normal current zero is also an example. The latter example in most instances does not result in excessive arrester discharge duty, as will be shown in a subsequent section.

An illustration of the third source of energy (item 3) being predominant is the interruption of line-charging current with breaker restriking. Here restriking across the contacts of the interrupter may leave the line charged to a potential several times normal line-to-neutral crest. Field tests have shown conclusively that when the arresters are not present, such switching surge voltages may reach values up to six times normal line-to-neutral crest, as was shown in Chapter 6. This is undesirable from the standpoint of possible line flashover or damage to terminal equipment. When arresters are connected to the line, they are required to discharge the energy stored on the line since they are voltage sensitive and do not distinguish between lightning and any other overvoltage. This type of surge appears to impose the most severe discharge duty upon arresters from the standpoint of frequency of occurrence and, in extreme cases, may impose discharge requirements severe enough to damage the valve elements. Consequently, this item will receive the most consideration in this chapter. It attains greater significance in view of the fact that, generally speaking, any unbalanced fault condition on a cable or overhead line results in the final interruption of only line-charging current in either one or two phases. The problem attains even further significance in isolated-neutral systems since the interruption of *every ground fault* presents a similar hazard of overvoltage possibilities, as has already been mentioned in Chapter 6.

There is evidence to indicate that the arrester discharges from switching surges of this last type are similar in nature to the long-duration low-current lightning discharge. It follows that the arrester's discharge capacity required for the one will also be equally effective for the other.

7·3 INDUCTIVE ENERGY TRANSIENTS

For purposes of analysis, consider the simple three-phase circuit in Fig. 7·1(*a*). The current limiting fuses are devices with very high rupturing capacity so that they will interrupt a fault current before it reaches the peak value available based upon calculations using normal system voltage and impedance viewed from the fault. When a fault occurs as shown, the fault current builds up on the first cycle to some value at which the fuse suddenly ruptures and becomes an open circuit. The current in the inductance L at this instant is some value i_0 which cannot be stopped instantly. The action of the fuse causes a high voltage to appear because it *tends* to cause the current to stop. This merely causes the arrester gap to spark over so that all that happens is that the current i_0 is suddenly transferred from the fuse path to the arrester. This current continues to flow through the arrester as some function of time until a current zero is reached at which time the arrester seals off and the system is then normal again except for the fact that the faulted portion has been separated (amputated often seems to be a more appropriate word).

The transition state on the part of the fuse from a good conductor to a good insulator is a complicated function of a number of variables.

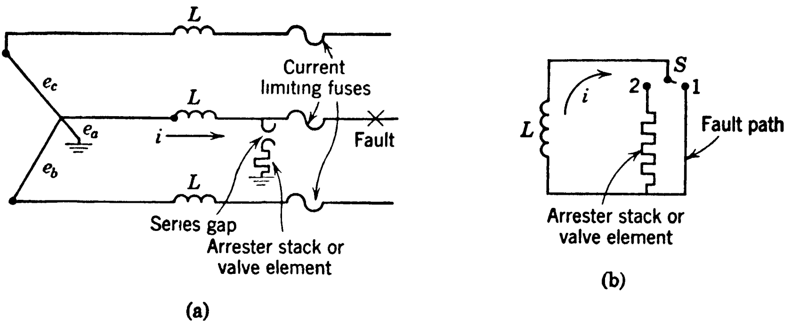


FIG. 7·1. Elements of inductive energy transients and arrester discharge duty (*a*) Three-phase system. (*b*) Simple equivalent circuit (approximate).

The transition does not take place instantaneously but requires some time so that overvoltages are controlled satisfactorily. However, for purposes of this analysis, only the limiting case of instantaneous transition from conductor to insulator will be considered.

As far as the arrester's discharge current is concerned, it depends on the initial value i_0 and on the system voltage tending to cause power-follow current to flow. In the simple circuit shown, e_a is the system voltage that gives rise to the flow of power-follow current. The differential equation of fault-current flow immediately following fuse operation is

$$e_a = L\frac{di}{dt} + Ri \qquad [7\cdot 1]$$

where R is a function of i only, and not a function of t. As a first approximation, suppose that the effect of e_a is ignored, even though it is recognized that the circuit is a non-linear one and therefore this is not strictly permissible. It is apparent that the discharge duty of the arrester is partly associated with power-follow current and partly with the energy stored in the inductance L at the time of fuse rupturing. If the arrester is properly selected for the system, the power-follow current alone should not be of special interest or concern. It is the initial value of current $i = i_0$ and the dissipation of energy $\frac{1}{2}Li_0^2$ stored in the inductance that is of primary concern.

With e_a being omitted for convenience, the system becomes as shown in Fig. 7·1(b). The initial value of current $i = i_0$ is flowing through the switch in position 1 when the switch is instantaneously thrown to position 2. Let the resistance of the arrester stack be denoted by

$$R = K_1 i^{-a} \qquad [7\cdot 2]$$

The differential equation for the circuit then is

$$Ri = K_1 i^{(1-a)} = -L\frac{di}{dt} \qquad [7\cdot 3]$$

For representative arrester materials a is a constant for purposes of practical analysis equal to between 0.7 and 0.8. K_1 is most simply defined as the resistance of the arrester valve element when the current through it $i = 1$ ampere.

Equation [7·2] can be solved by separating variables, thus

$$i^{(a-1)}\,di = -\frac{K_1}{L}\,dt$$

which yields

$$\frac{1}{a}\,i^a\Big]_{i_0}^{i} = -\frac{K_1}{L}\,t\Big]_0^{t}$$

or

$$\frac{1}{a}\,(i^a - i_0{}^a) = -\frac{K_1}{L}\,t \qquad [7\cdot 4]$$

from which

$$i = \left(i_0{}^a - \frac{aK_1t}{L}\right)^{1/a} \qquad [7\cdot 5]$$

Arresters are commonly applied and thought of in terms of units each rated 12 kv rms. If n such units are connected in series and each has a constant K, then equation [7·5] becomes

$$i = \left(i_0{}^a - \frac{anKt}{L}\right)^{1/a} \qquad [7\cdot 6]$$

It is apparent from this expression that the duration of discharge for a given L is decreased by adding more units in series (n increased) as would be required in higher voltage systems. It will be shown in the next section that just the reverse is true for the capacitance discharge.

An increase in L increases the duration of discharge. However, as L increases, the initial current i_0 necessarily becomes small so that the current through the arrester is necessarily very small if L is high. Consequently, it appears that the discharge of transformer magnetizing current alone as ordinarily encountered in de-energizing a transformer results in relatively light arrester discharge duty.

If equation [7·6] is rewritten so as to give the current at any time as a fraction of the original, then

$$\frac{i}{i_0} = \left(1 - \frac{anKt}{Li_0{}^a}\right)^{1/a} = (1 - K_0t)^{1/a} \qquad [7\cdot7]$$

where $K_0 = anK/Li_0{}^a$.

This expression is of interest by comparison with the conventional exponential expression for a decaying current in a simple resistance and inductance series circuit for which

$$\frac{i}{i_0} = \epsilon^{-(K/L)t} \qquad [7\cdot8]$$

The current in such a circuit never completely disappears, but the current in equation [7·7] becomes zero at a definite time given by

$$t_0 = \frac{1}{K_0}$$

It is also interesting to note that, if $a = 1$ (an ideal condition not obtained in present-day lightning arresters), then the fractional current i/i_0 is a linear function of time.

$$\frac{i}{i_0} = 1 - K_0t \qquad [7\cdot9]$$

All these facts are illustrated in the curves of Fig. 7·2. Curves of i/i_0 as a function of K_0t are plotted showing the effect of a as a parameter. As a is made smaller, the departure from a straight-line decay becomes greater. If $a = 0$ (constant resistance) then the i/i_0 curve, if plotted on this figure, would coincide with a vertical line through the origin and a horizontal line through the origin.

As an illustration of the significance of these curves, refer to Fig. 7·1(*a*). If the system shown is a 15-kv system, it would require a 12-kv arrester. Therefore $n = 1$. The value of K for a single 12-kv unit may be approximately 4,650 volts. For a typical modern arrester,

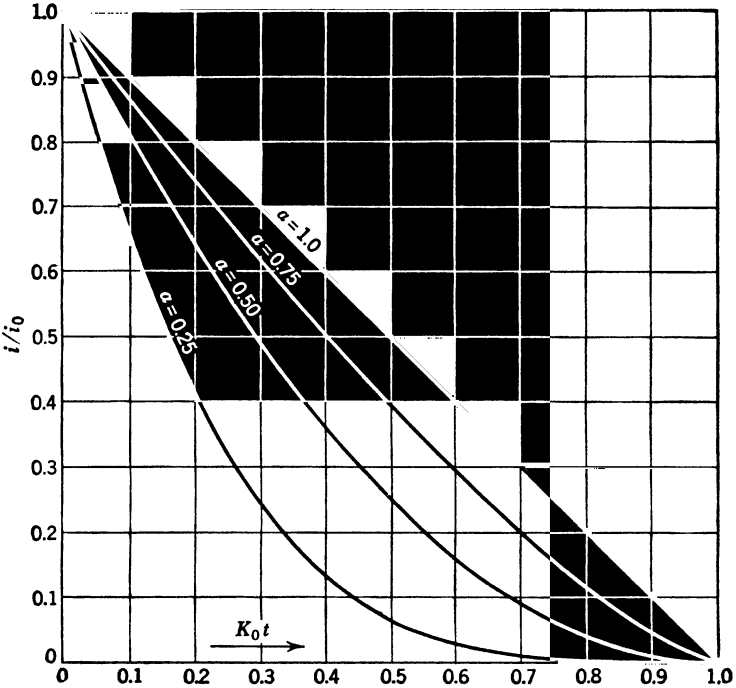


FIG. 7·2. Generalized curves of transient current decay in an inductive circuit with series non-linear resistor. $i/i_0 = (1 - K_0t)^{1/a}$ where $K_0 = anK/Li_0^a$, i_0 = initial value of current at $t = 0$, and i = current at any time.

$a = 0.75$. If $L = 0.01$ henry and if the fuse limits the current to 260 amperes, then

$$K_0 = \frac{0.75(1)(4{,}650)}{0.01(260)^{0.75}} = 5{,}390$$

The time required for the current to change from the initial value of 260 amperes to zero is then

$$t_0 = \frac{1}{K_0} = \frac{1}{5{,}390} = 0.000185 \text{ sec., or 185 microseconds}$$

This time value corresponds to $K_0t = 1.0$ in Fig. 7·2. It is an easy matter to convert this time scale to Fig. 7·2, and the ordinate for $a = 0.75$ can be translated to amperes readily by multiplying the numerical values read from the curve by 260, the initial value of current.

This transient decay for the case cited is over within 185 microseconds, and the current through the arrester does not exceed 260 amperes. At $t = 92.5$ microseconds the current value is 104 amperes. In terms of the duty imposed upon arresters by lightning discharges this is rather small by comparison. It is also small in comparison with certain capacitance discharge current transients, as will be shown in the next section. The transient decay for the illustration cited is less severe, as far as the duty imposed on the arrester is concerned, than the normal arrester power-follow current associated with the arrester rms kv rating. Even if the two effects were combined in practical cases the resultant duty appears to be not excessive. Although cases of inductive energy transients more severe than the one used as an illustration are not likely to be encountered in normal system operation, it should not be concluded, however, that it is *impossible* for more severe cases to occur.

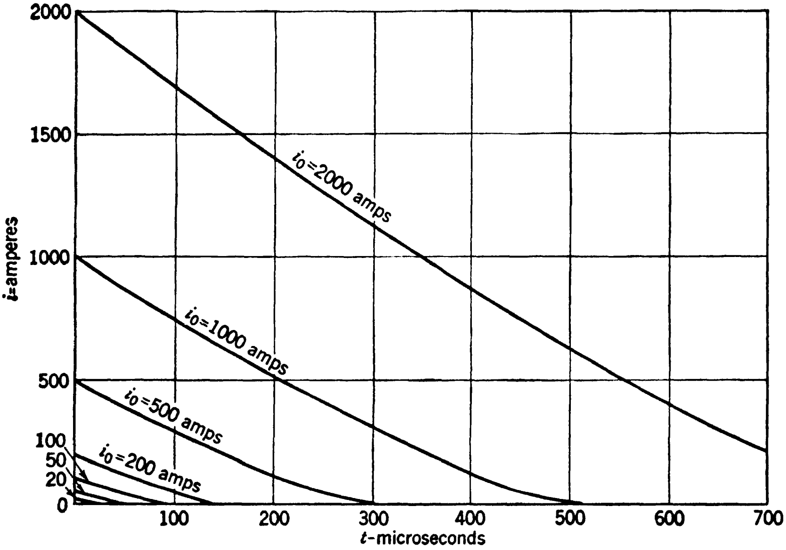


FIG. 7·3. Transient current decay in an inductive circuit with typical arrester. $i = i_0(1 - K_0t)^{1/a}$ where $K_0 = anK/Li_0{}^a$, i_0 = initial current, $K = 4{,}650$ for each arrester unit, $a = 0.75$, $n/L = 100$, n = number of arrester units, L = inductance in henries. Note: For any other value of n/L, divide abscissa scale by n/L.

In order to facilitate the interpretation of the generalized curves of Fig. 7·2, the curves of Fig. 7·3 were prepared for representative modern arresters.

7·4 CAPACITIVE ENERGY TRANSIENTS

Consider the simple three-phase system shown in Fig. 7·4(*a*). If the capacitor bank is being de-energized, then restriking in the interrupter represented by the switch in phase *a* may cause a building up of voltage on the capacitor *C* which may be sufficiently high to cause the series gap in the arrester to spark over. If the switch is actually open at the time of sparkover or if interruption is completed at about that time, then the arrester is left to discharge the capacitor *C* alone. The circuit that represents this condition is shown in Fig. 7·4(*b*). The differential equation for current flow in this circuit is

$$0 = Ri + \frac{1}{C}\int i\,dt \tag{7·10}$$

Substitution of equation [7·2] for R gives

$$K_1 i^{(1-a)} = -\frac{1}{C}\int i\,dt \tag{7·11}$$

where i is the current through the stack. Differentiation yields

$$K_1(1-a)i^{-a}\frac{di}{dt} = -\frac{1}{C}i$$

and separation of variables yields

$$i^{-(1+a)}\,di = -\frac{1}{CK_1(1-a)}\,dt \tag{7·12}$$

Integration gives

$$\left[\frac{i^{-a}}{-a}\right]_{i_0}^{i} = -\left[\frac{t}{CK_1(1-a)}\right]_0^t$$

from which

$$i = \left[\frac{t}{C}\frac{a}{K_1(1-a)} + i_0^{-a}\right]^{-1/a} \tag{7·13}$$

If the fractional current is desired, then

$$\begin{aligned}\frac{i}{i_0} &= \left[1 + \frac{a i_0^{a}}{CnK(1-a)}t\right]^{-1/a}\\ &= [1 + K't]^{-1/a}\end{aligned} \tag{7·14}$$

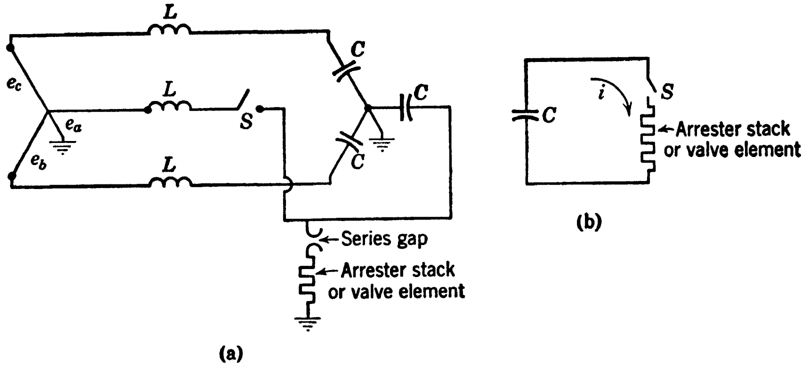

L
C
e_c
L
S
C
e_a
e_b
L
C
Series gap
Arrester stack or valve element
(a)
C
i
S
Arrester stack or valve element
(b)

FIG. 7·4. Elements of capacitive energy transients and arrester discharge duty. (*a*) Three-phase system. (*b*) Simple equivalent circuit.

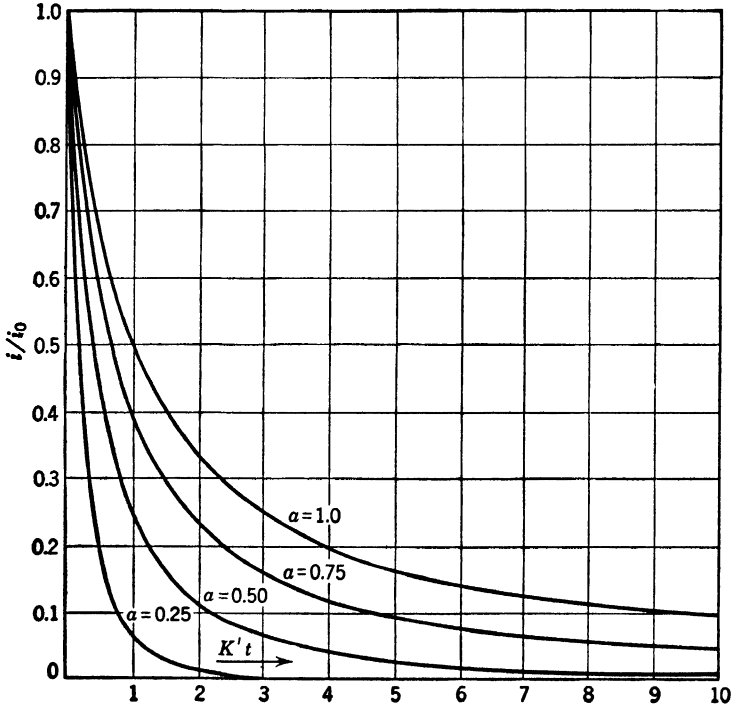


FIG. 7·5. Generalized curves of transient current decay in a capacitive circuit with series non-linear resistor. $i/i_0 = (1 + K't)^{-1/a}$ where $K' = ai_0^a/CnK(1 - a)$, i_0 = initial value of current at $t = 0$, and i = current at any time.

where, as in equation [7·6], the value of K_1 is replaced by nK, and $K' = ai_0^a/CnK(1 - a)$. Equation [7·14] is plotted in Fig. 7·5 with $K't$ as abscissa and values of a as a parameter. It will be observed that these generalized curves do not come to zero for any finite value of $K't$. In this respect, they differ from the generalized curves of Fig. 7·2 for the inductive circuit. If $a = 0$ (the limiting case for constant resistance), the curve of i/i_0 as a function of time would approach a vertical line through the origin and a horizontal line through the origin, just as the limiting case did for the inductive circuit.

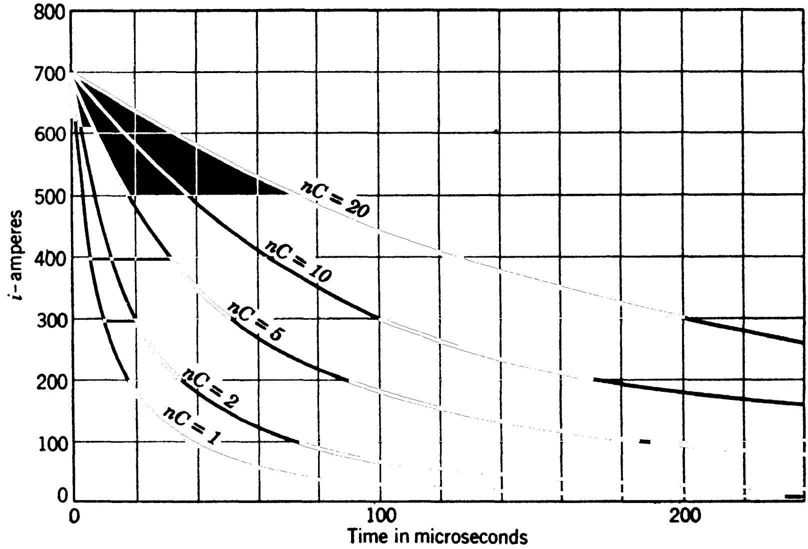


FIG. 7·6. Transient current decay in a capacitive circuit with typical arrester. $i = i_0(1 + K't)^{-1/a}$ where $K' = ai_0^a/CnK(1 - a)$, $i_0 = 700$ amperes, $K = 4{,}650$ for each arrester unit, $a = 0.75$, n = number of arrester units in series, and C = capacitance in microfarads.

Since the practical cases of interest are those representative of modern arrester performance, the curves of Fig. 7·6 have been prepared. All these curves are for an initial current of 700 amperes because this current is approximately the arrester current corresponding to typical arrester series gap sparkover voltage. For the simple circuit under discussion, this initial value of current is therefore fixed at 700 amperes. It is independent of the number of series units since series gap sparkover increases linearly with the number of arrester units in series. With K and a equal to 4,650 and 0.75, respectively, for a typical station-type arrester, the only other parameter is nC. This product is the parameter chosen in plotting the curves of Fig. 7·6.

The most significant fact disclosed by these curves is that, although the initial value of current is not high, the time duration of discharge is quite long. In a low-voltage system, this time duration may not be long enough to be particularly significant. In a 15-kv circuit (refer to Fig. 7·4(*a*)) a 12-kv arrester would normally be used, and therefore $n = 1$. If the capacitance C being switched off corresponds to that of a cable circuit of even two miles in length, C would be, approximately, one microfarad. Therefore $nC = 1.0$, and the curve of Fig. 7·6 for $nC = 1.0$ gives the current time curve for the discharge of the capacitor through the arrester. After 20 microseconds, there is a current of about 15 amperes flowing. This would not seem to be an especially significant case.

If, on the other hand, the system happened to be a 138-kv system for which, everything considered, ten arrester units were required, then $n = 10$. If a circuit is being de-energized for which the capacitance is one microfarad (this would be approximately the equivalent zero-sequence capacitance of 100 miles of typical overhead line), then $nC = 10$ and the curve of Fig. 7·6 for which $nC = 10$ will apply, distributed constant effects being neglected. After 20 microseconds, a current of 180 amperes is flowing. After 200 microseconds a current of about 15 amperes is flowing, and after 1,000 microseconds there is still a current of about two amperes flowing. As the current decreases, the increase in resistance tends to cause the current to die away very slowly. It is this persistence of the capacitor-arrester combination that gives the capacitive energy switching surge unique distinction. The severity of the discharge is increased in general, with the product nC. Therefore, in the higher voltage systems where long lines or cable circuits may be switched with restriking in the interrupter, switching surges and the arrester discharge capacity required become a very real engineering problem of fundamental importance.

7·5 DISTRIBUTED-CONSTANT EFFECTS

In Fig. 7·7 the fundamental circuit leading to the building up of capacitance stored energy is shown. The source of voltage consisting of generator and transformer is intended to represent any equivalent system from which the line (or cable, or shunt capacitor) is to be disconnected by means of the circuit breaker. The detailed description of how overvoltages may be generated as a result of restriking in the breaker and the effects of lumped and distributed constants has been given in Chapter 6. Similarly in Fig. 7·8 the condition for possible building up of overvoltages in an isolated-neutral system is indicated. This also was discussed in Chapter 6. In view of the discussion in the preceding section, it follows that these two cases are probably of greatest concern from the standpoint of switching surges and arrester discharge capacity. Because of the similarity between the two circuits (Figs. 7·7 and 7·8) as outlined in Chapter 6, it will be considered adequate to discuss in detail here only the system of Fig. 7·7. Transient analyzer results with semi-distributed constants and miniature scale models of arresters will be presented and compared with results of theoretical analysis where possible.

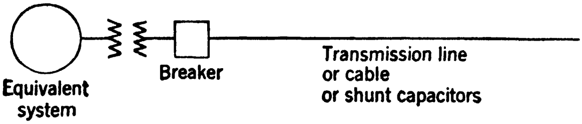


Fig. 7·7. Frequently encountered circuit which may give rise to serious overvoltages if restriking occurs in the breaker during opening.

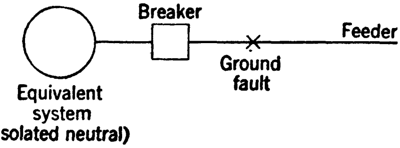


Fig. 7·8. Frequently encountered isolated-neutral circuit which may give rise to serious overvoltages if restriking occurs in the breaker during opening to remove the faulted feeder.

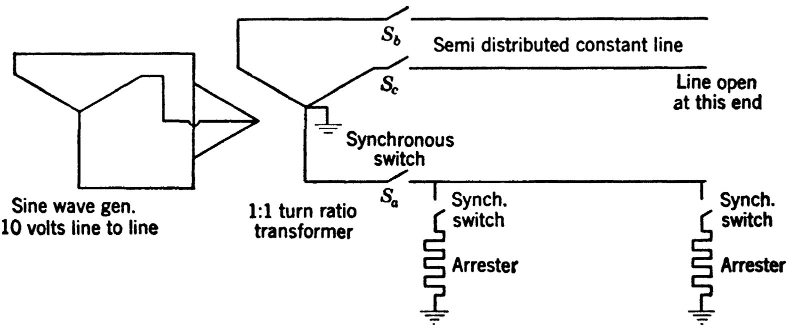
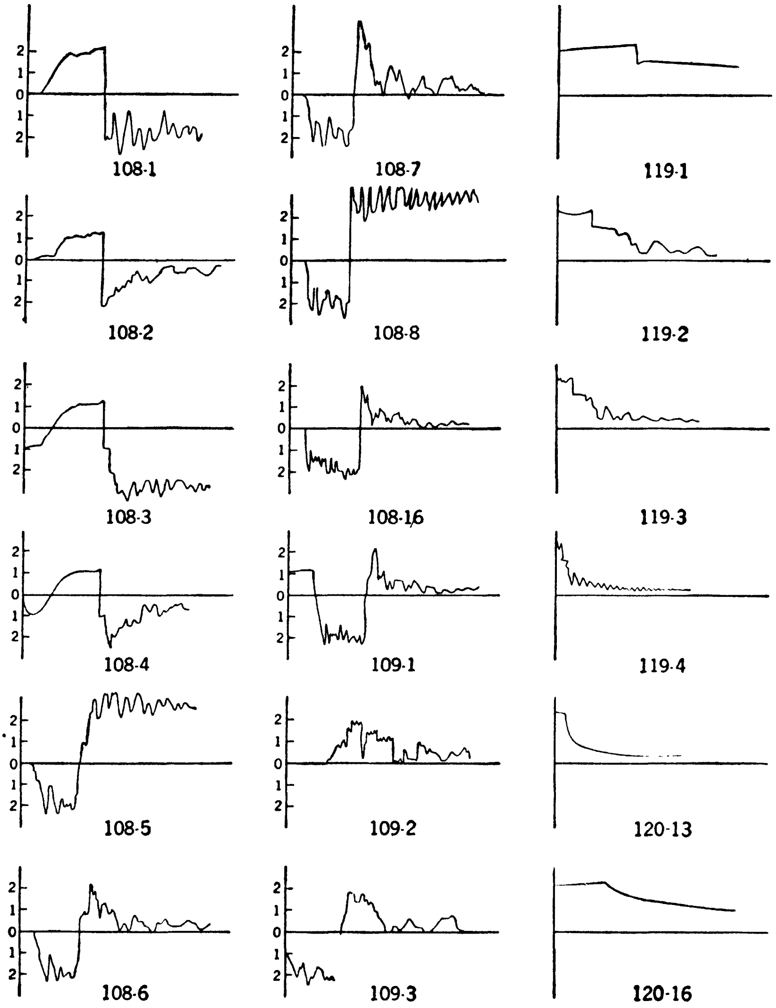


Fig. 7·9. System studied on transient analyzer. The miniature arresters used were small Thyrite disks which could be stacked in various combinations to simulate the desired arrester maximum kv rating.

A representative system studied on the transient analyzer is shown in Fig. 7·9. Typical transient analyzer oscillograms for a variety of conditions are shown in Fig. 7·10, and the details describing each oscillogram are shown in Table 7·1. From these records, a general idea of the nature and shapes of switching surges associated with the dropping of long lines is obtainable, with and without arresters.



108·1 108·7 119·1
108·2 108·8 119·2
108·3 108·16 119·3
108·4 109·1 119·4
108·5 109·2 120·13
108·6 109·3 120·16

FIG. 7·10. Typical oscillograms obtained from the transient analyzer study of the system shown in Fig. 7·9. See Table 7·1 for identification.

TABLE 7·1

Identification of Oscillograms

(Time scale on all oscillograms is 1,000 microseconds per small division except as noted.)

Oscillogram Number	Phase to Clear	Arrester	Voltage	Miles of Line	Restrikes	Minimum Sparkover Voltage	12-kv Arrester Units
108–1	1st	None	Far end	200	1	2.3	12
–2	1st	Far end	Far end	200	1	2.3	12
–3	1st	None	Near end	200	1	2.3	12
–4	1st	Near end	Near end	200	1	2.3	12
–5	1st	None	Near end	200	2	2.3	12
–6	1st	Near end	Near end	200	2	2.3	12
–7	1st	Near end	Far end	200	2	2.3	12
–8	1st	None	Far end	200	2	2.3	12
–16	1st	Far end	Far end	100	2	2.3	12
109–1	1st	Near end	Near end	100	2	2.3	12
–2	1st	Near end	Near end	300	2	2.3	12
–3	1st	Far end	Far end	300	2	2.3	12
119–1	...	....	Across arrester *	200	..	2.3	12
–2	...	....	Across arrester	200	..	2.3	12
–3	...	....	Across arrester	200	..	2.3	12
–4	...	....	Across arrester	200	..	2.3	12
120–13	...	....	Across arrester	2.6 microfarads	..	2.3	12
–16	...	....	Across arrester	2.6 microfarads	..	2.3	12

* Time scale on 119–1 is 80 microseconds per small division.
–2 is 400 microseconds per small division.
–3 is 800 microseconds per small division.
–4 is 2,000 microseconds per small division.
120–13 is 2,000 microseconds per small division.
–16 is 80 microseconds per small division.

The technique of using synchronous switches to perform the desired sequence of events has been described in Chapter 1. The use of small Thyrite disks to simulate the actual arrester characteristic has been described in the literature, but for completeness the procedure will be outlined here. Once the desired arrester volt-ampere characteristic has been reproduced to scale, either synchronous switch control or thyratrons with adjustable grid bias voltage can be used to insert the miniature-system arrester into the circuit at the proper instant.

The procedure for proper scale representation of arresters in the miniature system is best described in the following way. The miniature system of Fig. 7·9 is an exact duplication in *ohms* of a large system. The miniature-system transmission line has constants per mile of inductance, capacitance, and resistance which are representative of a typical overhead transmission line, as was pointed out in Chapter 1. Therefore the only difference, except for the semidistributed-constant effects in the line, between the miniature system and the actual system being represented is associated with the fact that the miniature system is operated at a much lower voltage than any actual power system would be. In Fig. 7·9, the normal system voltage on the line is 110 volts line-to-neutral or $110\sqrt{3}$ volts line-to-line. Since Thyrite can be used in very small disks without changing the exponent a in equation [7·2], then in representing any actual system with an arrester of given rms kv rating in that system, it is only necessary to meet the requirement that the *resistance* in ohms of the actual arrester in the actual system at some voltage, e.g., normal line-to-neutral voltage, is *duplicated* in ohms in the miniature system at the corresponding miniature-system normal line-to-neutral voltage. Then in terms of per-unit voltages in either system the arrester resistances are identical. It is only necessary that this correspondence be determined at *one* voltage (normal line-to-neutral in the case cited); the exponent a remaining unchanged assures duplication at *all* corresponding per-unit voltages between the actual and the miniature systems. The per-unit sparkover voltage of the arrester in the miniature system should be controlled by means of the synchronous switches to correspond to the per-unit sparkover voltage of the arrester in the actual system to complete the representation.

The system represented by the cases summarized in Table 7·1 are for a 161-kv solidly grounded system. In this system it was desired to study the effects of a 12-unit (145-kv) arrester. The minimum sparkover voltage of the arrester series gap was taken as 2.3 times normal line-to-neutral crest voltage as shown in column 7 of this table. The first column identifies the oscillogram number; the second, the phase to clear; the third indicates whether or not the arrester was in the system and where it was located; the fourth indicates the location at which the voltage trace on the oscillogram was obtained; the fifth gives the number of miles of equivalent three-phase line being switched; and the sixth indicates the number of controlled restrikes in the switch.

Oscillogram 108–1 shows the nature of the voltage surge associated with one restrike in the switch with reclearing at the first current zero. This oscillogram shows the voltage-to-ground at the far end with no arrester connected. Oscillogram 108–3 shows for the same condition, the voltage-to-ground at the near end.

With an arrester in the circuit at the far end, the voltage there is as shown in oscillogram 108–2. With an arrester in the circuit at the near end, the voltage there is as shown in oscillogram 108–4. These oscillograms 108–1 to 108–4 inclusive illustrate clearly the nature of the surges under discussion and the effect of the presence of the arrester. If no arresters are present (oscillograms 108–1 and 108–3) a charge is left on the line which stays on until drained off due to leakage or failure of line insulation. Thus, these are extremely long duration voltages due to the charge trapped on the line. If an arrester is connected to the line and the voltage is high enough, then gap spark-over will occur and the charge on the line finds a path through the arrester stack to ground. However, it is important to recognize the fact that here the arrester affords the only path to ground, neglecting leakage paths. Therefore the entire burden of discharging the line falls upon the arrester. In terms of energy ($\frac{1}{2}Ce^2$) stored on the line, or coulombs (*it*) the burden is small as compared to lightning discharges. However, lightning discharges are generally of short duration, but high in current magnitude. In contrast, the switching surge discharge current through an arrester is rarely expected to reach a magnitude of 500 amperes but may result in a *unidirectional* current flow lasting for several cycles of fundamental frequency as indicated in these oscillograms.

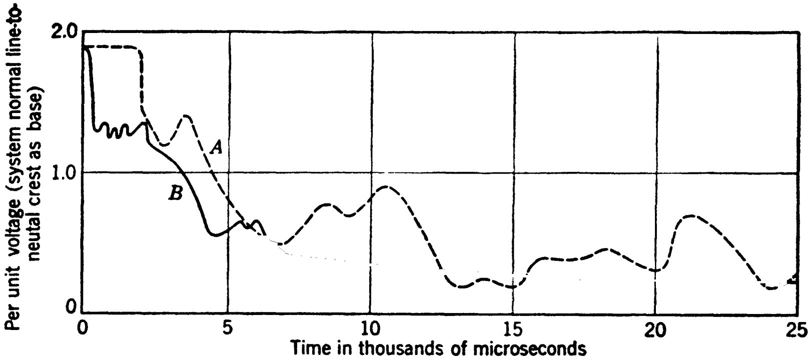


FIG. 7·11. Comparison of arrester voltage time curves during discharge. Arrester at far end of line. *A*, miniature representation of 161-kv system, 200 miles of line. *B*, actual 161-kv system, 175 miles of line.

In Fig. 7·11 the voltage-time curve *A* is taken from oscillogram 108–2; curve *B* in this figure is taken from an actual test oscillogram in a 161-kv system under the same system conditions except that there were 175 miles of line instead of 200. It is apparent that the time duration of discharge is comparable for the two cases. The miniature-system curve appears to last slightly longer (i.e., in general it lies above curve *B*), but this is partly due to the fact that 200 miles of line were in the circuit for curve *A*, whereas 175 miles of line were in the circuit for curve *B*. There are other minor variations, such as the more violent oscillations in curve *A* and the longer duration of the initial discharge voltage for curve *A* than for curve *B*. Both of these are probably attributable to the fact that restriking in the miniature system was controlled and a more violent restriking condition leading to arrester gap sparkover was set up. The exact conditions preceding arrester gap sparkover for curve *B* are not known. This is of little consequence, however, as the important basis for comparing curves *A* and *B* for the present purpose is the overall time duration of the discharge.

Oscillograms 108–5 to 108–8 shows similar phenomena for the same system and conditions but for two restrikes. The overvoltages shown with no arrester in are not so high as they could be with two restrikes. In order for the arrester to spark over on the second restrike and not on the first, it was necessary for the first restrike to occur at such an instant that the resulting overvoltage was just under arrester gap sparkover. Then the second restrike could take place at the instant that gives maximum overvoltage, and this in turn would impose upon the arrester the most severe discharge duty possible for this system. However, it will be observed that the case of two restrikes is only very slightly worse than that of one restrike as far as the arrester volt-time discharge curve is concerned. The difference is most apparent at the beginning of the discharge. The tail remains essentially as long as for the case of one restrike. Furthermore, the location of the arrester, whether at the near or far end of the line, makes no significant difference.

Oscillograms 108–16 and 109–1 to 109–3, inclusive, show similar results for the same conditions except that the first two are for 100 miles of line, whereas the second two are for 300 miles of line. A careful study of these records reveals that, for the given number of arrester units (12) corresponding to a 145-kv arrester in a 161-kv system, the tail is of duration essentially in proportion to the miles of line. This fact suggests that for a given number of arrester units, the important factor is the amount of capacitance that must be discharged through the arrester as was indicated in Section 7·4.

Since it was shown that the voltage distribution on the line was of negligible consequence as far as the length of tail was concerned, it seemed desirable to compare lumped capacitance with distributed capacitance on the basis of uniform voltage distribution on the line preceding arrester gap sparkover. Oscillograms 119–1 to 119–4 inclusive are for the distributed-constant line, and oscillograms 120–13 and 120–16 are for the same microfarads of lumped capacitance. It becomes clear that, as far as the tail or duration of discharge is concerned, the lumped capacitance equivalent gives essentially the same results as the distributed constants. However, there are differences near the beginning of discharge which may be significant as far as arrester discharge capabilities are concerned. To bring these differences out more clearly, Fig. 7·12 was prepared. It is clear that, starting with uniformally distributed voltage along the line, the current at the start is less for distributed constants. After about 200 microseconds the current is greater for the distributed-constant case. This is because the surge impedance of the line is in series with the arrester. From there on, the relative current magnitudes shift with respect to each other several times but the overall duration remains essentially

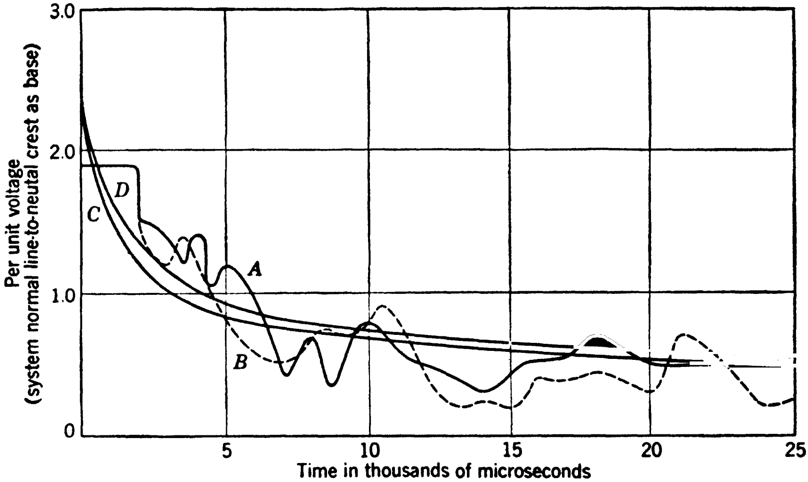


FIG. 7·12. Arrester volt-time characteristic during capacitance stored energy discharge. 161-kv system—12-unit arrester (145-kv maximum rating), 200 miles of line. *A*, 1 restrike, arrester at near end of line. *B*, 1 restrike, arrester at far end of line. *C* and *D*, lumped capacitance discharge; *C*, capacitance corresponding to C_0 of 200 miles of line; *D*, capacitance corresponding to total capacitance to ground of one conductor.

the same in either event. Since the agreement between results for the lumped-constant case and the semidistributed-constant case becomes even better for shorter line lengths and cable, it is believed justifiable as a first approach to evaluate the severity of surges from the standpoint of arrester discharge duty simply on the basis of microfarads to be discharged.

Acceptance of this leads to approval of the arrangement in Fig. 7·4(*b*) as the basic circuit of interest. The capacitor C charged to a voltage e_0, corresponding to minimum gap sparkover voltage of the arrester, discharges through the arrester when the switch S is closed. The differential equation for this circuit was solved in Section 7·4. It is shown there that the severity of discharge duty is proportional to the product nC where n is the number of 12-kv arrester units in series and C is the capacitance that is to be discharged. This is essentially equivalent to saying that the severity of arrester duty is proportional to system voltage and line capacitance. Since there is some basis for thinking of line lengths being proportional to kv, the foregoing criteria can be summarized by saying that the severity of arrester discharge duty may be considered very roughly proportional to the square of system voltage. Obviously there will be numerous exceptions to this. Cable circuits, shunt capacitor banks, and various other factors will give rise to exceptions.

Studies of various arrester materials have shown that some of these materials may possess high discharge capacity for short-duration surges (5 × 10 microsecond waves) but have relatively low discharge capacity on the low-ampere long-duration discharges. Therefore such surges become a factor of importance to both designers and users of arresters. Since improvements in arrester design are constantly being sought, it will be well to avoid being specific. It seems entirely in order to point out, however, that the station-type (heavy duty) arrester has about twice the discharge capacity, for long-duration discharges representative of these switching surge phenomena, that the line or distribution-type (lighter duty) arrester has.

7·6 FIELD TESTS

Field tests have been made under several different sets of system conditions to substantiate the foregoing information. The results have afforded bench marks with which miniature-systems tests, laboratory, and analytical results could be coordinated. One system on which tests were made involved a 161-kv source from which 175 miles of line were dropped by means of opening a circuit breaker, with a 145-kv arrester at the far end. Curve B of Fig. 7·11 was obtained in one such test. Another case involved a 115-kv source from which 75 miles of line were dropped with a ten-unit arrester at the near end.

A summary of the maximum arrester currents obtained in the 115-kv tests is shown in Fig. 7·13. A total of 26 three-phase openings were

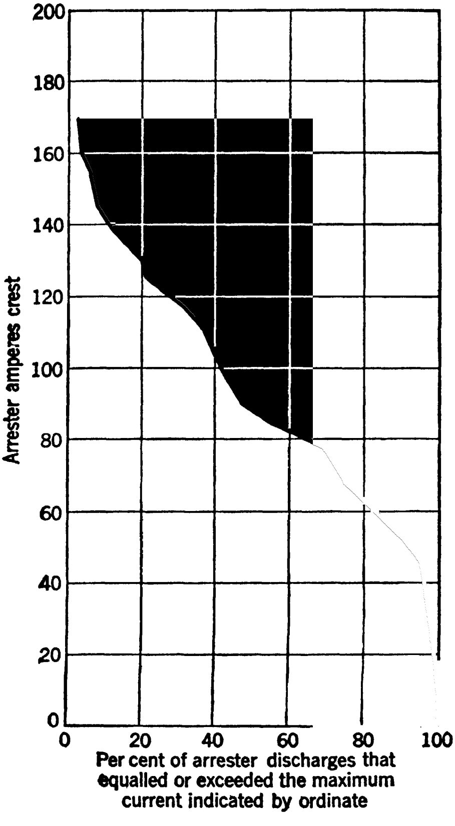


FIG. 7·13. Distribution of peak arrester currents measured during line-dropping tests in 115-kv system. Line length, 75 miles. 50 measured currents.

made. This caused 34 arrester operations giving a percentage curve as shown. Actually, as far as *frequency of occurrence* is concerned, there is no favoring of either positive or negative discharges. The maximum current measured was 170 amperes crest. Theoretical studies indicate that the maximum current possible under the conditions of test is by calculation approximately 350 amperes crest. The probability of getting 350 amperes, however, is very small. Any attempt to evaluate probabilities of this nature rapidly becomes engulfed in difficulties. Obviously, the actual currents depend intimately on the breaker restriking characteristics, and these are difficult to predetermine.

Figure 7·14 shows some further results of the 115-kv tests. This curve shows that frequently the arrester was required to discharge the line several times in quick succession during opening. In one case, it was required to discharge the line five times on successive half-cycles during one opening.

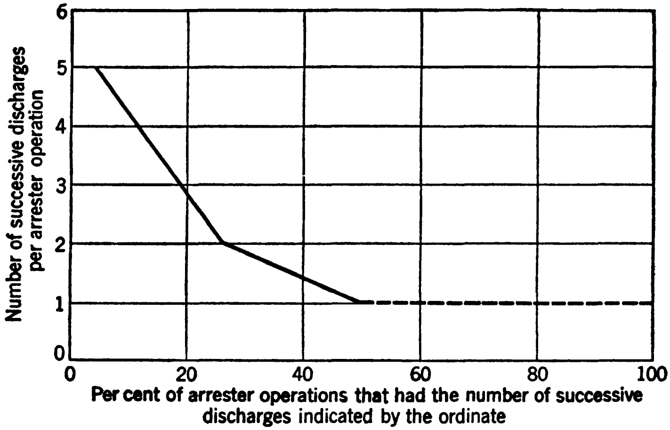


FIG. 7·14. Number of successive discharges as a function of per cent of number of operations. Results of 115-kv system-switching tests dropping 75 miles of line.

7·7 CONCLUSIONS

It must be emphasized that the entire problem of system overvoltages associated with breaker restriking such as has been discussed here is intimately associated with breaker performance characteristics. There are breakers that do not permit voltages of the type discussed here of sufficient magnitude to cause arrester operation. On the other hand, there are numerous installations in operation now that do give rise to this problem. This chapter applies directly to these cases.

Although there are other good reasons for sectionalizing long lines by means of intermediate switching stations, the facts presented in this chapter indicate one additional advantage to be gained by planning to avoid switching extremely long lines or cables of exceptional length at high voltage. Some of the extreme cases in this regard may require special attention from the arrester application standpoint.

Other conclusions are:

1. The severity of discharge duty is a function of surge impedance and increases with the amount of capacitance (length of line) being switched and the number of arrester units in series.

2. Cable circuits increase the discharge duty imposed upon arresters because of low surge impedance and increased capacitance per unit length.

3. It is rarely to be expected that arrester currents resulting from switching surges will exceed 500 amperes. The long duration of unidirectional current flow, rather than current magnitude, is the chief reason for their having significance from an arrester design and application standpoint.

4. Discharge duty imposed upon arresters in isolated-neutral systems may be particularly severe. In effect, an arrester operating as a result of switching surges must discharge the zero-sequence capacitance of all three phases. The duty is relatively severe for arresters connected either at the neutrals of wye-connected transformers or connected at the terminals line to ground.

5. In low-voltage systems, or in systems with directly connected generation, where the capacitance is small there is little chance of arrester failure due to switching surges of the type discussed here.

6. The discharge capacity that is incorporated in the design of an arrester to enable it to withstand long-duration switching surges will be equally effective in discharging long-duration lightning surges.

7. Any impedance to ground in parallel with an arrester will tend to relieve the arrester. Transformers connected from line to ground may be particularly helpful in this regard. Because of the long time duration of discharge, even the magnetizing impedance of a transformer affords finally, because of saturation, a low impedance path in parallel with the arrester.

References

1. *Traveling Waves on Transmission Systems,* by L. V. Bewley, John Wiley & Sons, New York, 1951, Second Edition. Dover reprint, 1963.

2. *AIEE Lightning Reference Book,* published by the American Institute of Electrical Engineers, New York, July 1937.

3. "The Calculation of Circuits Containing Thyrite," by Theodore Brownlee, *General Electric Review,* Part I, April 1934, pages 175–179; Part II, May 1934, pages 218–223.

8 OVERVOLTAGES CAUSED BY SUDDEN LOSS OF LOAD

8·1 INTRODUCTION

System disturbances resulting in switching may cause some machines on a system to pick up more load and other machines to reduce their load. Certain machines may in fact be caused suddenly to drop full load under some conditions of operation and faults when subsequent switching operations are performed. That the terminal voltage on a machine suddenly rises when a typical load is rejected is fairly well known. The consequences are not always fully appreciated, however, especially when capacitance (cables and transmission line) is left on the terminals of machines after load is dropped. The consequences are especially significant from the standpoint of the overvoltages produced, arrester applications, and circuit breaker applications.

It is the purpose of this chapter to outline the fundamental nature of the phenomenon and extend the analysis to practical cases, including the effects of overspeeding, long transmission lines, transformer saturation, and corona losses.

8·2 LOAD LOST AT GENERATOR TERMINALS

Consider the one-line diagram of a system consisting of only a generator and load as shown in Fig. 8·1(*a*). The generator is assumed to be of round-rotor construction for simplicity so that the effects of saliency need not be brought in. Resistance of the armature windings is assumed to be negligible, and there is no rotor winding other than the main field winding. Assume that the machine is carrying a load per phase represented in the vector diagram in Fig. 8·1(*b*) by a current I and terminal voltage V_t which is normal voltage for the machine. The excitation voltage is then $E_d = V_t + I(jX_d)$ vectorially. Similarly, $E_d' = V_t + I(jX_d')$.

If the breaker shown is suddenly opened, the current I immediately drops to zero. Consequently the synchronously rotating armature mmf disappeared. In order for the flux linkages in the field circuit to remain constant an instantaneous change in field current takes place which, when losses are included, finally disappears exponentially at a rate depending on the field time constant. The effects of sudden changes of load, or terminal conditions, can be analyzed, as is well known, by considering that the voltage E_d' remains constant for a short period of time immediately after the sudden change in terminal conditions takes place. This is not merely an assumption but can be

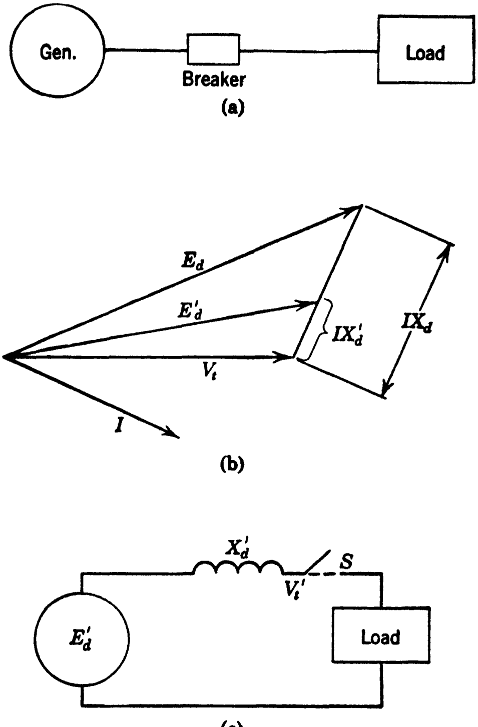


FIG. 8·1. Elements of sudden load dropping. V_t is terminal voltage to neutral; I is load current per phase; E_d' is voltage back of transient reactance; E_d is excitation voltage; X_d' is transient reactance; X_d is synchronous reactance. (*a*) One-line diagram. (*b*) Vector diagram for system in (*a*). (*c*) Equivalent circuit for system in (*a*).

shown to be rigorously correct as a direct consequence of the constant flux linkage theorem which is basic in synchronous machine theory.[13, 14, 15] If a long interval of time is to be considered, then the decay of flux linkages with time must be considered; but, for a matter of several cycles after a change in terminal conditions, it is fundamental to all system stability studies that E_d' can be considered constant in a synchronous machine.

As a consequence of the foregoing, it is possible to represent the system of Fig. 8·1(*a*) by the simple equivalent circuit of Fig. 8·1(*c*). If the switch *S*, corresponding to the breaker, is then opened, the terminal voltage, which was normal with the switch closed, suddenly rises to the value

$$V_t' = E_d' \tag{8·1}$$

which is higher for the normal lagging power factor loads than V_t. Thus there is normally a sudden overvoltage at the terminals of a synchronous machine whenever a typical load is suddenly dropped. If nothing were done to the excitation of the machine, the armature voltage would increase as time goes on, ultimately reaching a value of E_d, neglecting saturation effects.

If the machine were initially operating at unity power factor, then the sudden increase in terminal voltage when load is dropped would be very small; if the load were at very low lagging power factor, then the sudden increase might be from approximately 10 per cent to 30 per cent or more, depending on the load, the load power factor, and the value of transient reactance X_d'.

8·3 LOAD LOST AT THE RECEIVING END OF A TRANSMISSION LINE

Consider the system shown in Fig. 8·2(*a*) consisting of a generator, step-up transformer, transmission line, breaker, and a receiving-end load. If the load is suddenly dropped at the receiving end, it is similar to opening the switch *S* in the simplified equivalent circuit of Fig. 8·2(*b*). Here V_t' does not become E_d' as in equation [8·1] when the switch is suddenly opened but becomes

$$V_t' = E_d' \frac{X_{c1}}{X_{c1} - (X_t + X_d')} \tag{8·2}$$

Thus there is a further amplification of the terminal voltage if capacitance is left on the bus when load is rejected. In the diagram of Fig. 8·2(*b*), C_1 is the equivalent positive-sequence capacitance of the line. It should be recognized also that, if the line is long, then there is a higher voltage at the receiving end of the line when the breaker is opened than at the sending end, in accordance with well-known transmission-line theory. It is clear from the foregoing analysis that, if an unloaded transmission line is ever left to be energized by a machine or machines so that the normal voltage line charging kva and machine ratings are comparable in value, then the voltage on the line, and at the machine terminals, may reach values far above normal voltage. Corona losses and transformer saturation may be important factors in limiting these overvoltages. Arresters (if present) would also be expected to function and limit such overvoltages, possibly not without damage to themselves if the sustained voltages are above the arrester ratings.

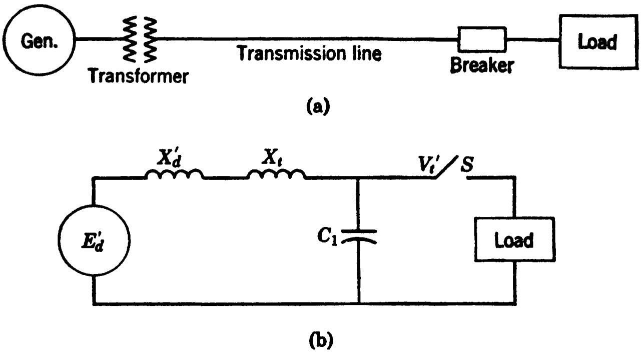


FIG. 8·2. Elements of sudden load dropping at the receiving end of a transmission line. (*a*) One-line diagram. (*b*) Simplified equivalent circuit for (*a*).

8·4 EFFECT OF OVERSPEEDING

It was stated that the constant flux linkage theorem was basic to the foregoing analysis. It follows that, if the field flux linkages remain constant, then the generated voltage should vary directly with speed. Consequently, E_d' varies directly with speed under the conditions outlined in Sections 8·2 and 8·3. When load is suddenly dropped on a machine there is an immediate net accelerating torque applied to the rotor of that machine so that inevitably there is an ultimate increase of speed. The amount of this overspeeding varies considerably, depending on the type of machine. Water-wheel generators generally overspeed considerably more than steam turbines for the main reason that it is not possible for the speed governor to actuate the butterfly valve of a water-wheel generator so quickly without damage to the installation as it can the valves of a steam turbine, and thereby reduce the mechanical torque on the rotor. Steam turbines can be kept down to overspeeds of 10 or 15 per cent. For the water wheel, the large confined column of water involved cannot be stopped immediately but must be slowed up gradually so that considerably greater overspeeds are reached before input and output torque equilibrium is reached. This represents an additional source of overvoltage in that E_d' in equations [8·1] and [8·2] should be multiplied by the instantaneous per-unit speed. Furthermore, this fact has additional significance in that the inductive reactances should be multiplied by this per-unit speed and the capacitive reactances should be divided by the per-unit speed. If S is the per-unit speed, then equation [8·2] becomes

$$\begin{aligned} V_t' &= E_d'S \frac{X_{c1}/S}{(X_{c1}/S) - S(X_t + X_d')} \\ &= \frac{SE_d'X_{c1}}{X_{c1} - S^2(X_t + X_d')} \qquad [8\cdot 3] \end{aligned}$$

Since water wheels are ordinarily associated with the longer transmission lines, equation [8·3] is very significant in view of the foregoing discussion. Furthermore, as mentioned previously, the overvoltage at the receiving end of a given transmission line as in Fig. 8·2(*b*) is further amplified owing to overspeeding. In effect, the line is increased in length in proportion to the speed S.

8·5 EFFECT OF TRANSFORMER SATURATION

That transformer saturation does not always act to reduce overvoltages during loss of load phenomena is not an immediately obvious fact. Under some conditions it does serve to reduce overvoltages, under certain other system conditions it actually serves to increase overvoltages to approximately double the values disclosed by the foregoing analysis, neglecting corona losses and the presence of overvoltage protective equipment. Such additional overvoltages may occur for certain combinations of transmission-line length and system reactance when a lightly loaded or unloaded transformer is connected to a transmission line. It will be shown in this section that such a system is most susceptible to overvoltage phenomena when the transformer is connected to the receiving end of that transmission line, although, theoretically, similar phenomena may occur for any transformer location, including the transformer at the sending end. These overvoltages are in addition to the normal fundamental-frequency voltage rise and are of a combination subharmonic and harmonic nature. The conditions would probably be most serious from a practical point of view, when accompanied by generator overspeeding immediately following sudden loss of load caused by low-side switching which would simultaneously remove any receiving-end synchronous condenser capacity otherwise normally present. It should be borne in mind that either end of a long transmission line may be considered the receiving end in the sense that the term is employed in this section.

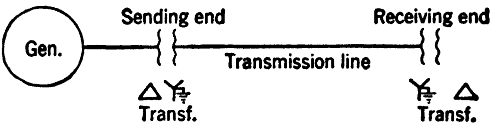


Fig. 8·3. One-line diagram of system studied.

Figure 8·3 shows the basic circuit under discussion. In a system such as this, the receiving-end transformer would become considerably overexcited if its load were suddenly lost during a system disturbance severe enough to cause the receiving-end low-voltage circuit breakers to open. This would leave the transformer energized with no load and thus with only exciting current flowing. However, the magnetizing impedance of a transformer, although fundamentally reactive, is a variable quantity because of saturation of the iron core. During periods of saturation, relatively high peak currents may flow momentarily.

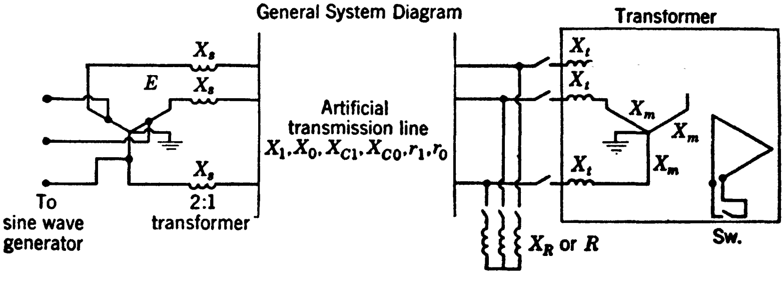


Fig. 8·4. Three-phase miniature system equivalent of the system shown in Fig. 8·3.

In order to study appropriately, without simplifying assumptions, the problem involving a distributed-constant line and a saturable reactance, the method of miniature-system representation was employed. For this purpose, the system of Fig. 8·4 was set up as a three-phase representation of the system of Fig. 8·3. The artificial transmission line was made up of ten-mile sections as previously described. In this particular study, the abruptness of transformer saturation could be varied simply by varying X_t as shown in Fig. 8·5.

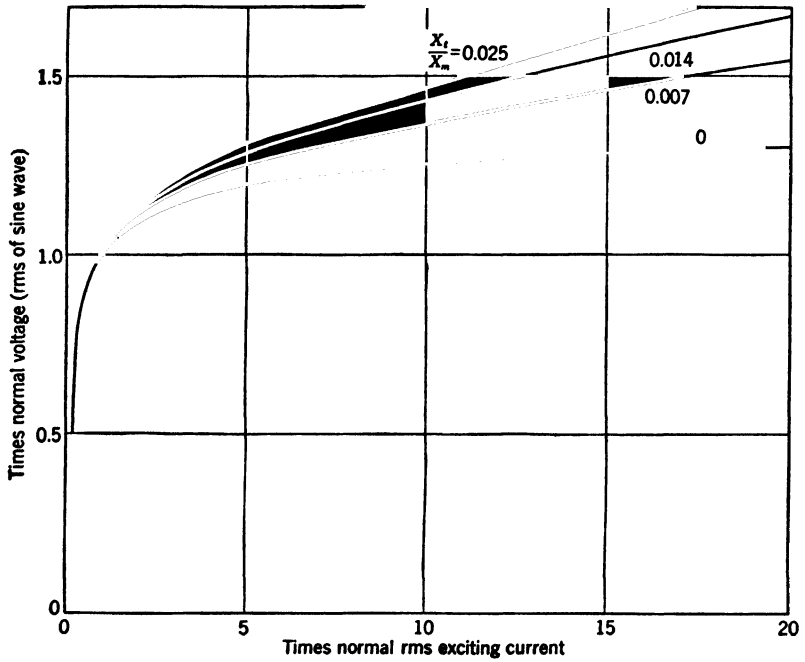


FIG. 8·5. Saturation curves used in this study. D-c resistance of winding expressed as a ratio to X_m is 0.0003. X_m = 13,000 ohms at 110 volts, 60 cycles, for one transformer.

The values of X_t used were always small relative to X_m so that X_m remained substantially independent of X_t/X_m. Consequently the connected receiving transformer kilovolt amperes remained essentially independent of X_t/X_m, and only the saturation characteristic of the receiving-end transformer was modified by varying X_t/X_m as shown in Fig. 8·5.

Each single-phase transformer used at the receiving end had a magnetizing impedance of $X_m = 13{,}000$ ohms at normal voltage of 110 volts. Two of these units could be put in each phase in parallel, the value of receiving-end magnetizing reactance thus being halved, or two units could be put in series in each phase (provided that the value of E in volts was doubled) the value of receiving-end transformer magnetizing reactance thus being doubled. In any event, the receiving-end transformer kilovolt-amperes to be associated with any value of X_m is given by the following relation:

$$X_m = \frac{(\text{kv})^2 \times 1{,}000}{\text{kva} \times I_m}$$

where X_m is the receiving-end transformer magnetizing reactance in ohms per phase at normal voltage; kv is normal system line-to-line voltage; kva is the receiving-end transformer bank rating; I_m is the rms exciting current in per-unit per phase at normal voltage. Thus if $X_m = 13{,}000$ ohms and if a three per cent exciting current and a 230-kv system are assumed, the receiver-end transformer kva rating represented by the miniature system would be

$$\text{kva} = \frac{(230)^2(1{,}000)}{13{,}000(0.03)} = 135{,}000 \text{ kva}$$

Provision was made for operating the receiving-end transformer with or without the delta open. Shunt reactors or resistors of variable kilovolt-ampere rating, as well as resonant shunts and synchronous condensers, could be switched in at the receiving end. Series capacitors for line-reactance compensation could be inserted as desired. Provision was made for varying the resistance of the transmission line and of the receiving-end transformer and for varying the sine-wave generator voltage, thus changing the normal operating flux density in the receiving-end transformer as desired. Consequently numerous factors separately or in combination could be carefully studied.

The effect of lightning arresters in limiting these overvoltages and thus protecting terminal equipment was not included in this study. It follows from the results revealing the nature of the phenomenon, however, that lightning arresters may be damaged by such sustained overvoltages.

Nomenclature

E = voltage back of X_s in per-unit (base voltage is 1.0 in Fig. 8·5).

X_s = source reactance.

X_t = transformer series reactance (for saturation adjustment).

$X_0/X_1 = \dfrac{\text{zero-sequence line reactance}}{\text{positive-sequence line reactance}}$.

$r_1/X_1 = \dfrac{\text{line resistance}}{\text{line reactance}}$ (positive sequence).

$$V_R = \frac{\text{crest voltage at receiving end with transformer}}{\text{crest voltage at receiving end without transformer}}.$$

$$X_{c1}/X_R = \frac{\text{positive-sequence capacitive reactance}}{\text{shunt reactance}}.$$

$$r_0/X_0 = \frac{\text{line resistance}}{\text{line reactance}} \text{ (zero sequence).}$$

f = cycles per second.

X_m = magnetizing reactance in ohms at normal voltage and frequency.

X_{c1}, X_{c0} = positive- and zero-sequence shunt capacitive reactance of line, respectively.

X_{sc} = series-capacitor reactance.

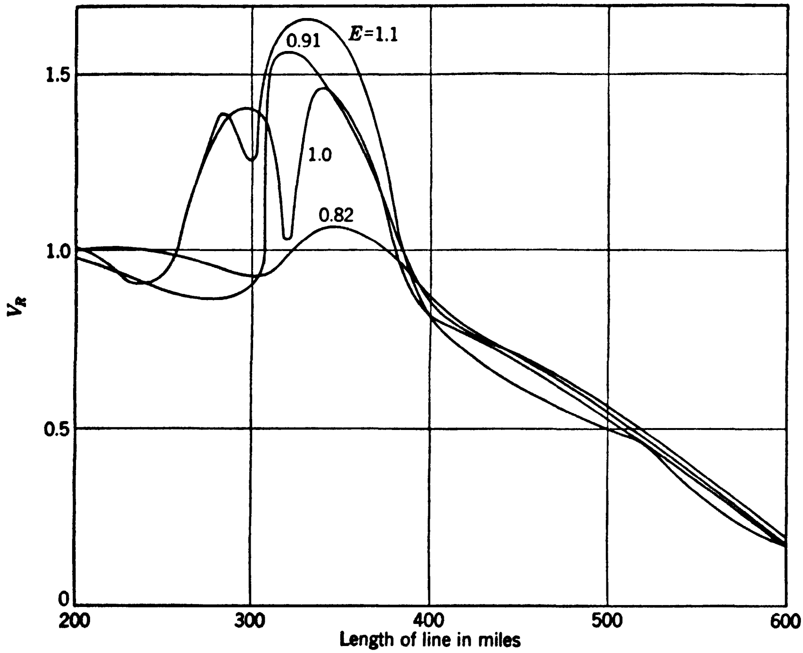
X_L = non-saturable reactance in parallel with X_{sc}.

R = resistance in parallel with X_{sc}.

X_R = non-saturable shunt reactance at receiving-end transformer terminals.

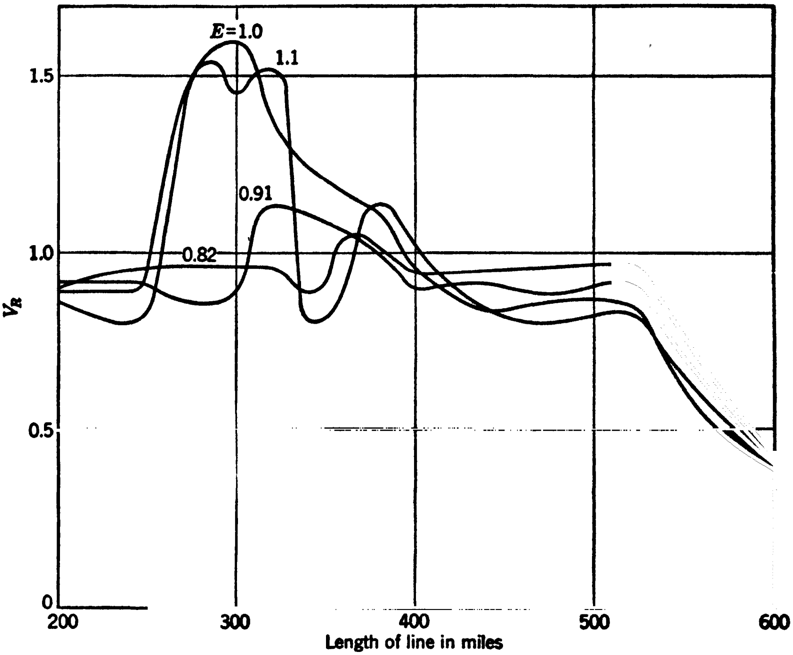
R_R = shunt resistance at receiving-end transformer terminals.

Discussion of Results. With the system set up as in Fig. 8·4, curves as shown in Figs. 8·6(*a*) and 8·6(*b*) were obtained. It should be



(*a*)

Fig. 8·6. Voltages V_R obtained in the system of Fig. 8·4. $X_s = 86$ ohms; $f = 60$ cycles per second; $X_0/X_1 = 4.0$; $r_1/X_1 = 0.1$; $X_t/X_m = 0.014$; $X_m = 13{,}000$ ohms; no series-capacitor compensation; no load on system; no shunt reactors. (*a*) Delta closed. (*b*) Delta open.



(*b*)

FIG. 8·6 (*Continued*).

emphasized that the voltage V_R is a ratio of crest voltage-to-ground at the receiver end with the transformer connected, to the crest voltage at the same location with the transformer disconnected. Thus the normal rise in the line is eliminated from the results, and the effect of the transformer alone is shown by these figures.

The oscillograms shown in Fig. 8·7 are typical illustrations. No two of these oscillograms were taken simultaneously, and so it is not possible to correlate instantaneous values of voltage and current. However, the variation in voltage and current wave shapes with time is indicated. The voltage is seen to be made up largely of a second harmonic and subharmonics. The current traces clearly show evidence of subharmonics and even harmonics. The time required for this condition to be reached following sudden loss of load varies from a few cycles to a second or two, depending on the value of E and the system constants.

The values of E maintained back of X_s range from a maximum of 1.1 down to values such that the effect of the transformer is negligible from the standpoint of producing overvoltages. It is felt that this covers the practical range, including the case of sudden loss of load at the receiving end. Unit voltage $E = 1.0$ defines the normal operating point on the saturations curves of Fig. 8·5 when applied directly to the terminals of the transformer.

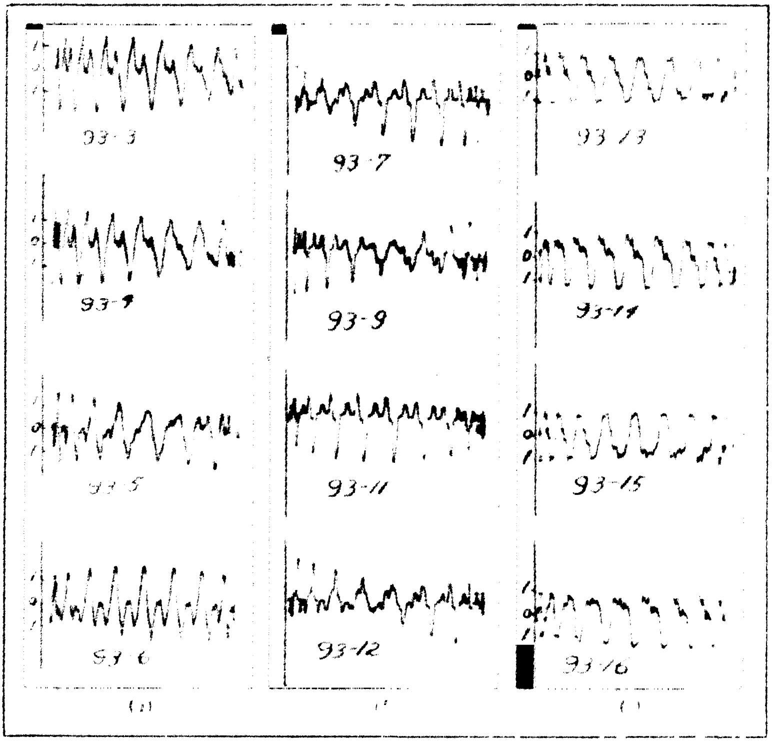


FIG. 8·7. Oscillograms showing the nature of the phenomenon. System constants as in Fig. 8·6(*a*), with 320 miles of line. (*a*) V_R, phase *a* voltage to ground at receiving-end transformer terminals. (*b*) Receiving-end phase *a* transformer magnetizing current. (*c*) Voltage-to-ground at sending-end times that which would exist without the transformer.

It is to be noted that, if E is reduced approximately to values such that the receiving-end transformer is not overexcited, no overvoltages will occur. It is significant also that the region of abnormality is not greatly affected by the delta being opened or closed. Furthermore, it was found during the study that the phenomenon is essentially independent of the method of grounding the receiving-end transformer neutral when a closed delta winding is present.

It should be noted that, whereas the system studied is characterized by a "sending end" and a "receiving end," in the actual case either end may become the receiving end in the sense employed here. That is, during a system disturbance, either end of the line may be switched out on the low-voltage side, leaving an unloaded system of the type illustrated in Fig. 8·4.

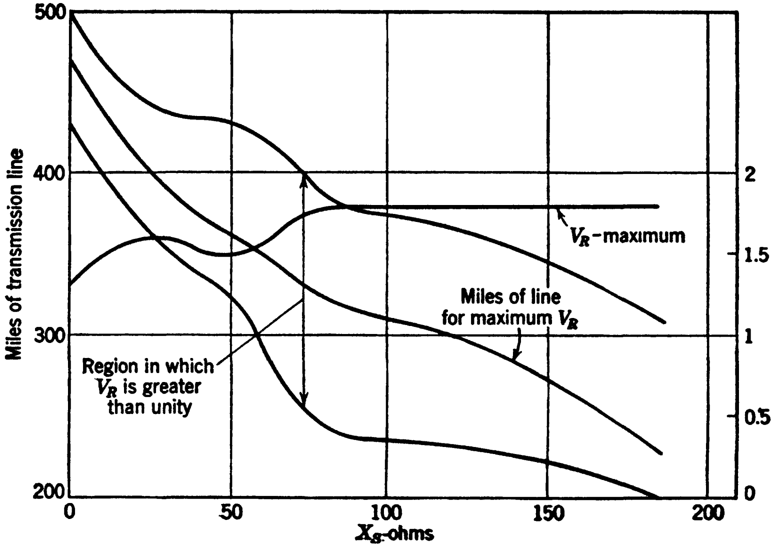


FIG. 8·8. Boundaries of overvoltage region as affected by line length and source reactance with receiving-end transformer delta closed. Frequency, 60 cycles per second.

Similar tests made under numerous miniature-system conditions have made it possible to summarize the effect of source reactance X_s as shown in Fig. 8·8. Since the case of open delta is of little practical significance, only the closed-delta condition is shown. This figure shows the regions in which V_R is greater than unity in terms of miles of transmission line and source reactance in ohms. Since the delta being opened has little effect on the region of abnormality in general, it is to be concluded that the reactance of the tertiary winding is of little consequence. Furthermore, the negligible effect of the delta being opened has some significance from the standpoint of the theory involved, as will be shown later.

The curves of Fig. 8·8 indicate that overvoltages are to be expected over a range of line lengths from 200 miles on up to slightly over 400 miles when the system frequency is 60 cycles per second, and source reactances are in the practical range. With lower frequencies the line lengths could be increased inversely as the frequency before encountering the difficulty. The maximum value of V_R obtained was 1.8 with the delta closed, and the length of line producing the maximum overvoltage is shown to increase as the source reactance decreases. Thus, under system conditions corresponding to these, voltages of the order of twice those normally expected may exist at the terminals of the receiving-end transformer if the line length and system source reactance are in the right combination, and if corona and lightning arresters are not voltage-limiting factors.

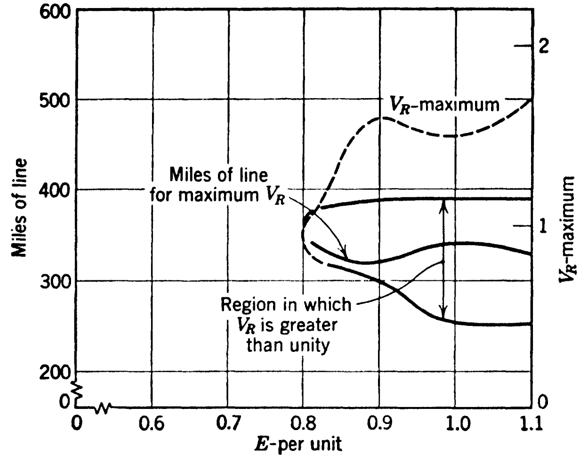


FIG. 8·9. Effect of sending-end voltage. System constants as in Fig. 8·6(*a*).

Although the curves of Fig. 8·8 are for the transformer located at the receiving end of the transmission line, it can be shown that the critical combination of length of line and source reactance that is susceptible to these phenomena is independent of the transformer location. Even the sending-end transformer can produce the same phenomena for the same range of constants shown in Fig. 8·8. However, a somewhat higher range of values of E (voltage back of equivalent system reactance, X_s) would be required to give rise to the phenomenon here. The receiving-end transformer location renders a given system more vulnerable to such disturbances because of the normal-frequency voltage rise in the line.

The effect of the voltage E maintained within the system is shown in Fig. 8·9 for the same case as in Fig. 8·6. As the system voltage is decreased, the range of system constants capable of giving rise to this abnormality decreases, and at $E = 0.8$ the phenomenon disappears entirely. This suggests the theoretical possibility of avoiding such overvoltage phenomena by using low flux density transformers, although other means may generally be more desirable.

The effect of varying the magnetizing reactance X_m is illustrated in Fig. 8·10. It is believed that this covers the practical range of X_m

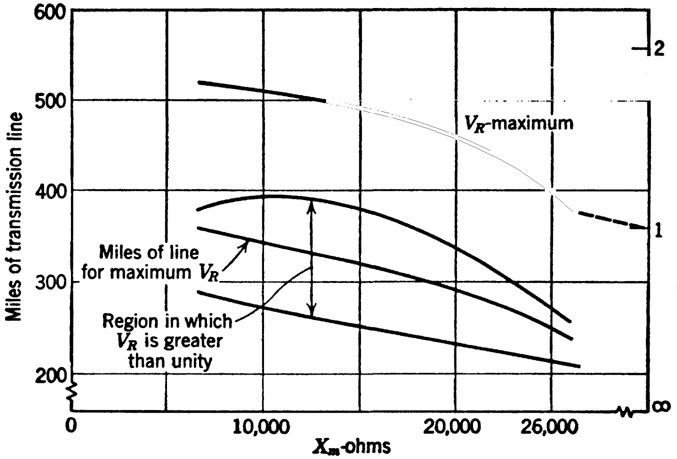


Fig. 8·10. Effect of varying the receiving-end transformer magnetizing reactance X_m. System constants as in Fig. 8·6(*a*).

and shows that the phenomenon is not critical to a particular value of X_m. Obviously, if X_m is ∞, the value of $V_R = 1$ as shown, and, if very high values of X_m are connected to the transmission line, little disturbance of the type under discussion would be expected. In order to make the results more general, values of X_m are given in ohms, but they can readily be converted to specific transformer kilovolt-ampere ratings in systems of given voltage rating, if the magnetizing current in per cent is known or assumed. The formula given in the preceding section can be used for this purpose.

The effect of line resistance is illustrated in Fig. 8·11. It appears that, if r_1/X_1 could be as high as 0.4 or higher, the phenomena would not occur. However, practical values of r_1/X_1 in lines such as these under discussion would fall considerably below this value—on the order of 0.2 or less. Therefore, line resistance would not be expected to eliminate the overvoltages in a practical case. Furthermore, any attempt to increase the circuit resistance may increase the tendency for hunting to take place.

The effect of steepness of saturation is illustrated in Fig. 8·12. It is evident from this illustration that the phenomenon is not critical to the abruptness of saturation over a practical range, although the more abrupt saturation characteristic in general appears to lead to higher voltages and broaden the range of constants for which they may be obtained.

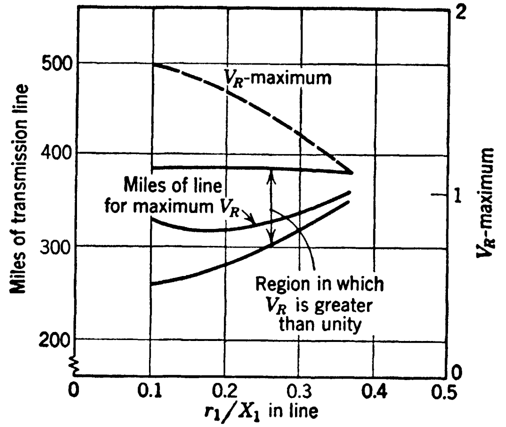


FIG. 8·11. Effect of line resistance. System constants as in Fig. 8·6(*a*).

The effect of generator overspeeding is illustrated in Fig. 8·13. A system normally not in the region of overvoltages may be brought into the region by generator overspeeding as a result of loss of load.

After observing the effects of various factors in producing these overvoltages, means were investigated for controlling them. Among the first means investigated were shunt reactors placed at the terminals of the receiving-end transformer. The ability of reactors to accomplish the desired purpose is illustrated in Fig. 8·14. It is interesting to note that shunt reactor kilovolt-amperes as high as 65 per cent of normal line-charging kilovolt-amperes may be required to keep V_R down to unity. This amount of reactance does not include compensation for the normal rise in the line, since V_R as defined is independent of the normal rise in voltage in the line. If generator overspeeding is an appreciable factor, shunt reactors to take care of the normal fundamental-frequency rise in the line may not prevent the occurrence of this phenomenon because of the increase in applied frequency.

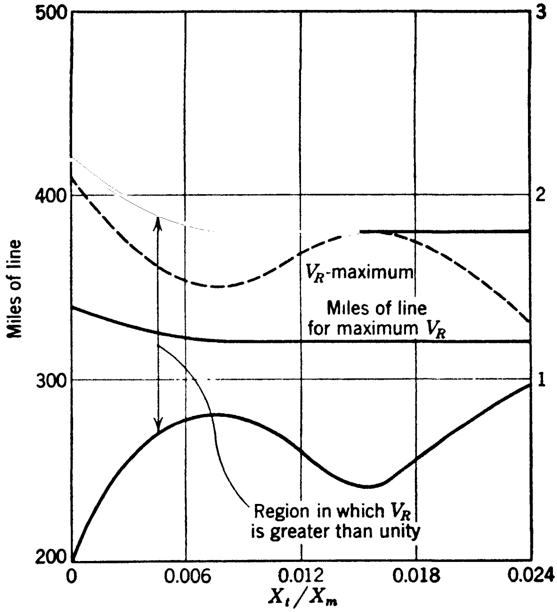


FIG. 8·12. Effect of abruptness of transformer saturation at receiving end. System constants as in Fig. 8·6(a).

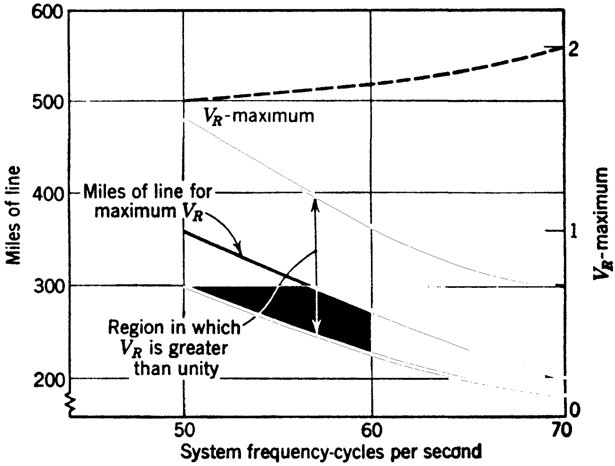


FIG. 8·13. Effect of generator overspeeding. System constants as in Fig. 8·6(a).

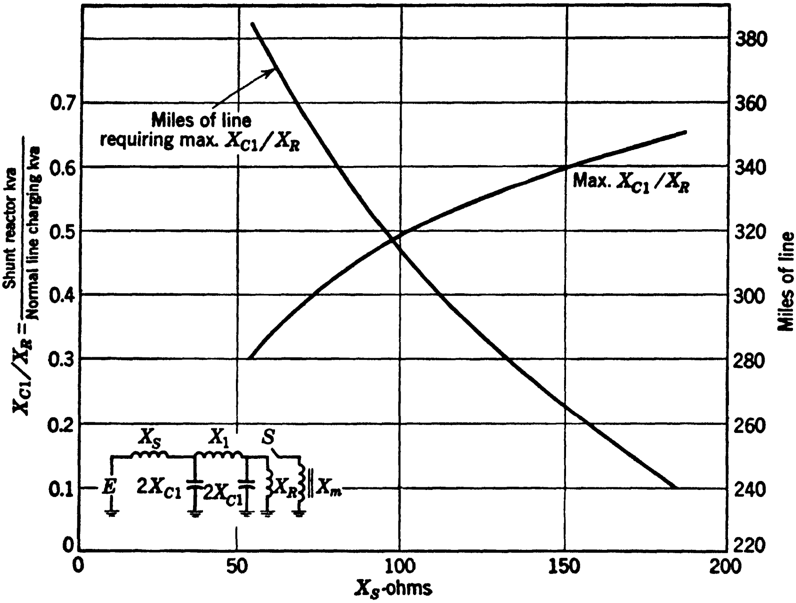


FIG. 8·14. Maximum shunt reactance kilovolt-amperes at receiving-end transformer terminals required to reduce V_R to unity. Shunt reactor kilovolt-amperes expressed as a ratio to normal line-charging kilovolt-amperes in accordance with the insert. System constants as in Fig. 8·6(*a*).

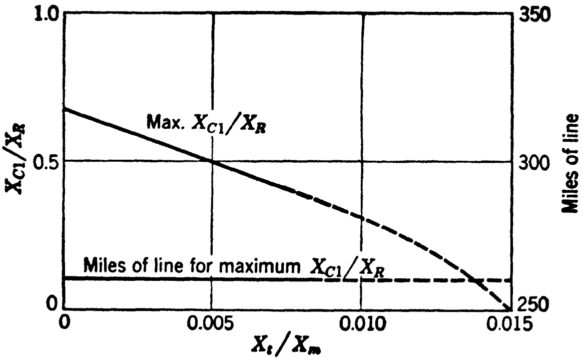


FIG. 8·15. Effect of abruptness of saturation on the required shunt reactor kilovolt-amperes to reduce V_R to unity. System constants as in Fig. 8·6(*a*).

Figure 8·15 indicates that if the receiving-end transformer saturates more abruptly, more reactor kilovolt-amperes may be required to stabilize the voltages.

Figure 8·16 suggests that the maximum required X_{c1}/X_R is likely to increase as the value of X_m decreases. This is representative for the practical range of X_m.

Figure 8·17 shows that a synchronous condenser is very effective in eliminating the overvoltage hazard. In most actual instances, a condenser at the receiver end of a size greater than the minimum required for overvoltage stability is likely to be present. Consequently this is one very practical and effective means of overcoming the difficulty. However, provision must be made for keeping the condenser on the system, even though the load is lost. Thus, in addition to controlling the fundamental-frequency voltage at the receiver end during normal load conditions, a condenser is expected to be effective in eliminating abnormal harmonic and subharmonic voltages such as these during system disturbances.

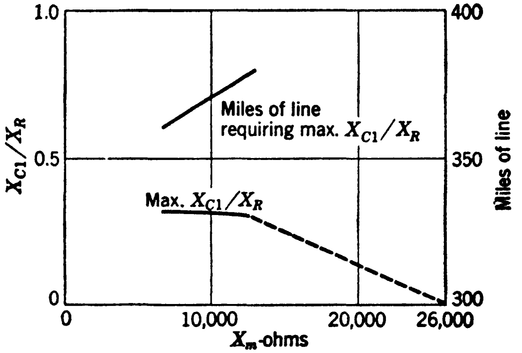


FIG. 8·16. Effect of magnitude of X_m on the required shunt reactor kilovolt-amperes to reduce V_R to unity. System constants as in Fig. 8·6(a).

The curves of Fig. 8·17 require some discussion, because they show a minimum condenser requirement for the length of line giving rise to the maximum values of V_R. Practically, however, the necessity for reducing V_R at all is greatest when V_R is greatest. Therefore, for practical purposes, the maximum condenser requirements here are more nearly the minimum values shown in Fig. 8·17. In other words, a condenser of 15,000 kva would probably be adequate for keeping V_R from becoming destructive in magnitude for this case.

One other very effective means that has been found for eliminating the overvoltages is the second-harmonic resonant shunt located at the receiver-end transformer. The shunt may be either delta- or wye-connected. If wye-connected, it makes no difference whether the neutral point is grounded or not. Its effectiveness is illustrated in Fig. 8·18. The reason for the effectiveness of the second-harmonic shunt is discussed in the following section.

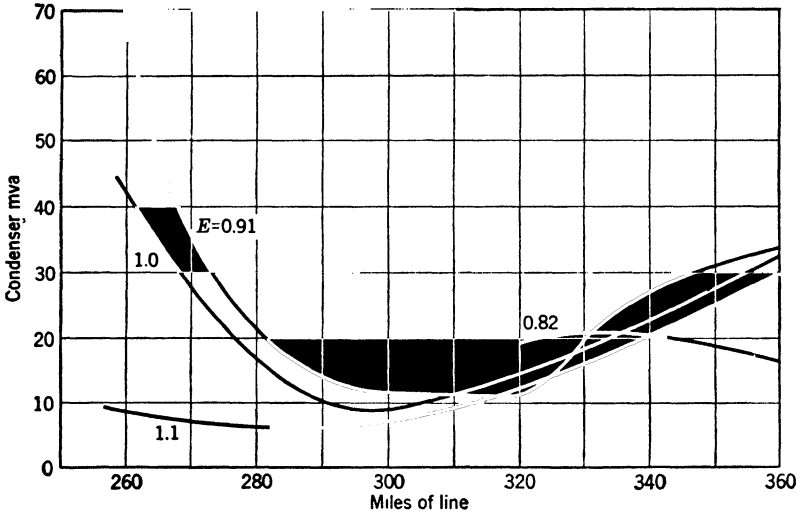


FIG. 8·17. Synchronous-condenser kilovolt-amperes capacity required at the receiving end to reduce V_R to unity. System as in Fig. 8·6(a), with $X_t/X_m = 0$, transient reactance of condenser assumed to be 30 per cent on rating in 230-kv system. Voltage back of transient reactance of condenser, 133 per cent.

The effect of series capacitors at the mid-point of the transmission line is illustrated in Fig. 8·19. Series capacitors at the mid-point of the line for 50 per cent compensation of the line drop were used here. The region of abnormality was broadened considerably by the presence of series capacitors as can readily be seen by comparing this figure with Fig. 8·6(a) or 8·6(b). The reason for this broadening of the region of abnormality is that an additional phenomenon, different in nature, is introduced by the presence of series capacitors. This is discussed further in the following section (Section 8·6).

It is possible to eliminate the phenomenon entirely by using shunt reactors located at the terminals of the receiving-end transformer.

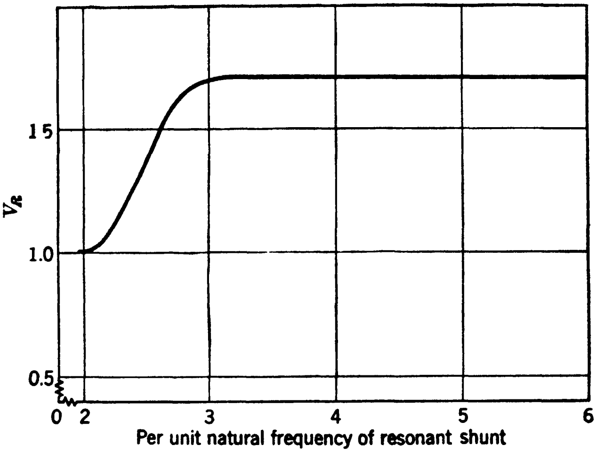


FIG. 8·18. Effect of resonant shunt on V_R. Shunt consists of reactor and capacitor in series located at the terminals of the receiving-end transformer. Reactor, 6.0 henries. Capacitance varied. System constants as in Fig. 8·6(*a*) with 320 miles of line.

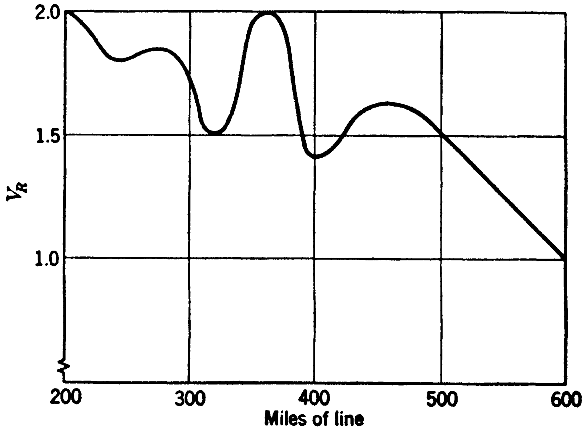


FIG. 8·19. Relationship between V_R and miles of line with series capacitors. Series-capacitor kilovolt-amperes at mid-point of line correcting for 50 per cent of line-reactance drop. System constants as in Fig. 8·6(*a*). $E \leqq 1.1$

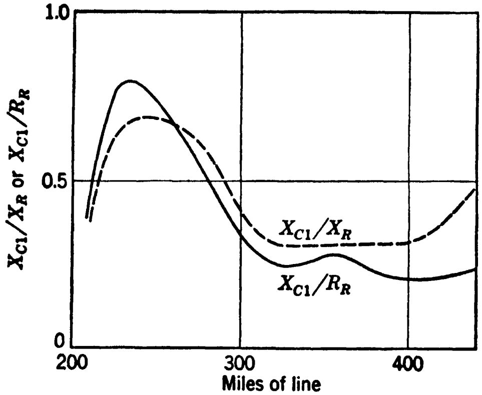


FIG. 8·20. Shunt reactance or resistance required at the receiving-end transformer terminals to hold V_R down to unity for $E \leq 1.1$ System constants as in Fig. 8·6(*a*) with 50 per cent series-capacitor line-reactance drop compensation at the mid-point of the line.

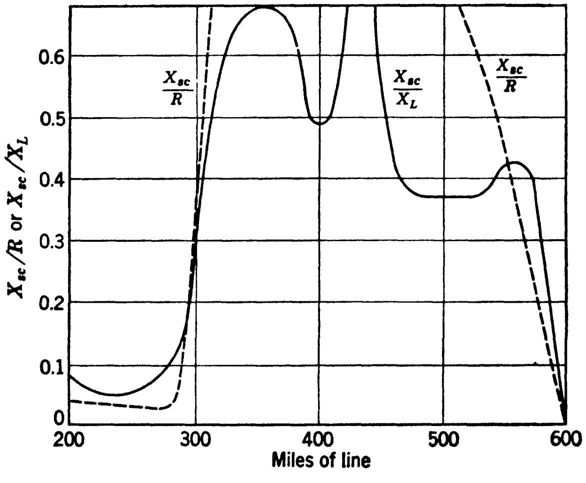


FIG. 8·21. Resistance or reactance required in parallel with series capacitor to hold V_R down to unity for $E \leq 1.1$. X_{sc} = reactance of series capacitors; X_L = reactance in parallel with X_{sc}; R = resistance in parallel with X_{sc}. Other system constants as in Fig. 8·6(*a*) with 50 per cent series-capacitor line-reactance drop compensation at the mid-point of the line.

The amount of reactance required is shown for one case in Fig. 8·20. In this figure the amount of shunt resistance required for the same case is also shown.

The possibility of using a resistor in shunt with the series capacitor has previously been given some consideration. A reactor in parallel with the series capacitor also is effective as shown in Fig. 8·21, although, for this case, the effectiveness of either a resistor or reactor so placed decreases rapidly for lengths of line above 280 miles.

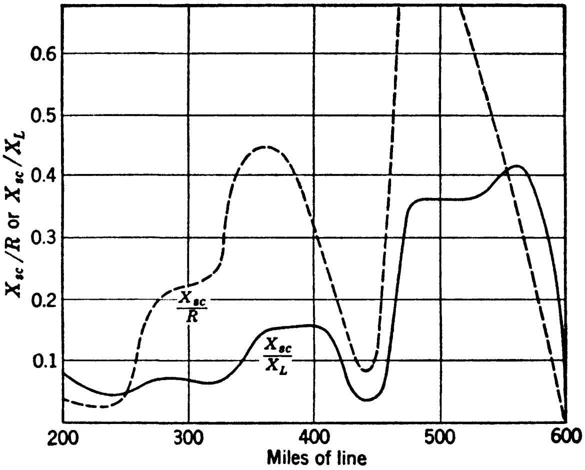


FIG. 8·22. Resistance or reactance required in parallel with series capacitors to hold V_R down to unity for $E \leq 1.1$. X_{sc} = reactance of series capacitors; X_L = reactance in parallel with X_{sc}; R = resistance in parallel with X_{sc}. Second-harmonic shunt at receiving-end transformer terminals. Shunt made up of 2,000-ohm inductance in series with 0.32-microfarad capacitance wye-connected and ungrounded. Other system constants as in Fig. 8·6(a) with 50 per cent series-capacitor line-reactance drop compensation at the mid-point of the line.

In Fig. 8·22, the curves show the amount of reactance or resistance required in parallel with the series capacitor for the same case, the only difference being that a second-harmonic resonant shunt has been connected to the terminals of the receiving-end transformer. Reactance is seen to be quite effective here as a means of eliminating the phenomenon for line lengths up to well over 400 miles, whereas resistance is not nearly so effective. Furthermore, resistance is undesirable since it introduces losses during normal load conditions which in the proportions required would be intolerable in most cases.

A condenser of practical size at the receiving end is very effective in eliminating the voltage instability, either with or without series capacitors.

8·6 THEORY OF EFFECT OF SATURATION

The cause of the phenomena discussed in the preceding section can best be understood by first considering a single-phase circuit as shown in Fig. 8·23. Here the essential elements of the three-phase circuit of Fig. 8·4 are present, the transmission line being represented by a capacitor X_c, and the source reactance by X_L. The transformer X_m, which can be connected or disconnected by the switch, corresponds to the receiving-end transformer of Fig. 8·8 where X_t and X_m define the transformer characteristics.

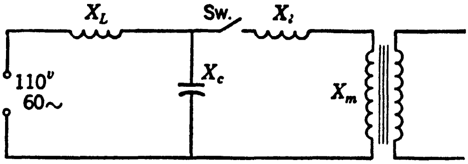


FIG. 8·23. Single-phase lumped-constant circuit studied.

With V_R defined as in the preceding discussion the results obtained by varying X_c/X_L and X_m/X_L are shown in Fig. 8·24. Here two peaks of overvoltage are shown, one largely of second-harmonic and subharmonic nature for X_c/X_L just below four and the other largely of third-harmonic nature for X_c/X_L just below nine. The higher values of X_m/X_L cause the peaks of overvoltage to move closer to four and nine, respectively. Now when $X_c/X_L = 4$, the circuit viewed from the switch location with the switch open is tuned to the second harmonic, and, when $X_c/X_L = 9$, it is tuned to the third harmonic.

For the case of X_c/X_L just below nine, it is not difficult to see why overvoltages will occur. If $X_c/X_L = 9$, the impedance of the circuit viewed from the switch with the switch open is infinite to the third harmonic, neglecting resistance. Consequently, in trying to supply the required third-harmonic current to the transformer, a large third-harmonic drop is encountered, and this shows up at the transformer terminals as indicated in the curves. If X_c/X_L is lower than about 7.5 and higher than about 12, the impedance to the third harmonic required by the transformer is low enough essentially to eliminate the overvoltages of the third-harmonic type.

It is to be noted that the overvoltages of this type are a maximum when X_c/X_L is slightly less than nine. This means that the system impedance is capacitive and of a high value to the third harmonic. This is to be expected since, with the switch closed, the path for third harmonics includes X_m also, which, although difficult to evaluate under these saturated conditions, nevertheless is fundamentally reactive to the third harmonic. This probably compensates for the resultant capacitive reactance of the system to bring the overall system more nearly in tune to the third harmonic at the point giving rise to the maximum V_R shown in Fig. 8·24.

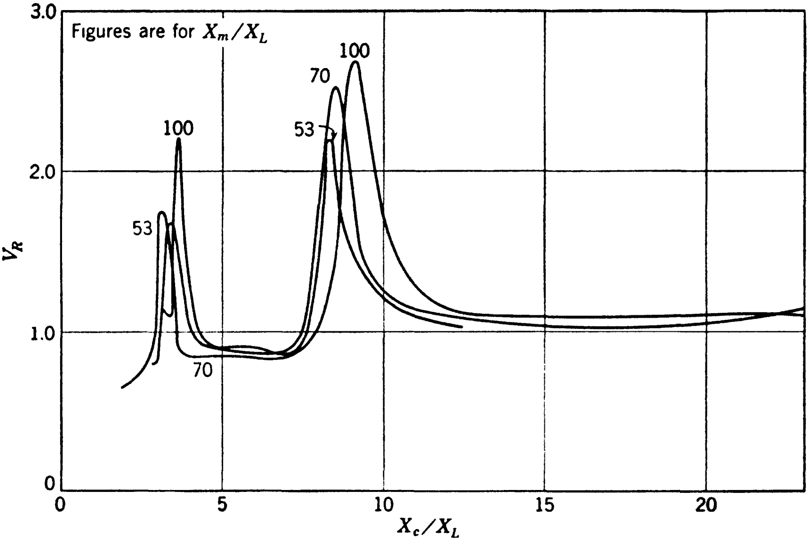


FIG. 8·24. Results obtained in the study of the circuit shown in Fig. 8·23.

With regard to the peak values of V_R for X_c/X_L slightly less than four, the explanation is more difficult, because, although the system impedance with the switch open is infinite (neglecting resistance) to the second harmonic when $X_c/X_L = 4$, the transformer in normal steady-state operation with sinusoidal voltage applied does not require a second harmonic to flow. Consequently the preceding explanation, although satisfactory for the region of third-harmonic overvoltages, is not satisfactory for the region of second-harmonic overvoltages.

In order to visualize the cause of the second-harmonic overvoltages, it is desirable to substitute the equivalent circuit of Fig. 8·25 for the circuit of Fig. 8·23. In this figure, the two reactances X_L and X_c correspond to the reactances so labeled in Fig. 8·23. The reactances X_m and X_s are constant (not saturable) and represent the transformer of Fig. 8·23.

In this equivalent circuit, the saturation of the transformer is represented by two straight lines as shown in Fig. 8·25(*b*). The effect of saturation of this type is introduced in the equivalent circuit by closing switch S at every instant that the current i into the equivalent transformer exceeds a critical value and opening it at every instant that the current i falls below this critical value. In the illustration of Fig. 8·25(*b*), this critical value has been arbitrarily taken to be 0.9 of the crest value which would be taken by X_m if the switch S were never closed. On this basis the switch S would be opened and closed every half-cycle of fundamental frequency, or the switching

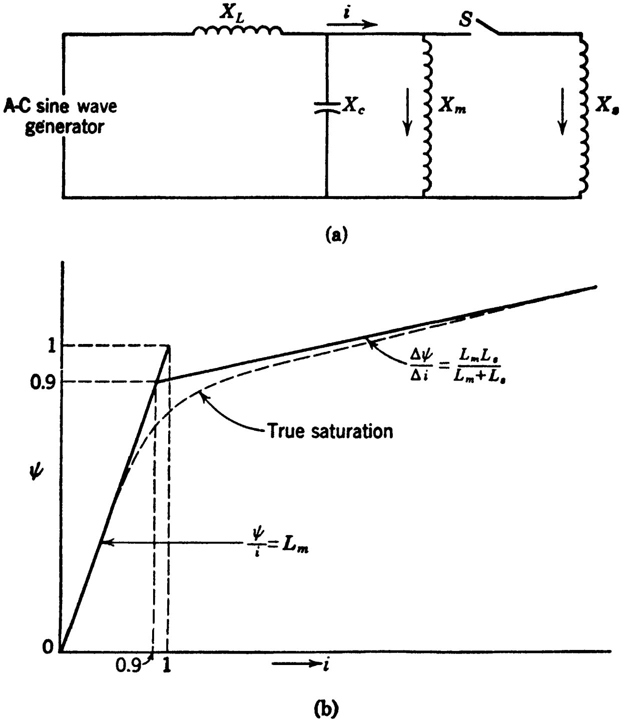


FIG. 8·25. Equivalent circuit (system of Fig. 8·23) for visualizing the underlying cause of the phenomenon.

cycle would be repeated at a frequency corresponding essentially to second-harmonic frequency, and, since the circuit made up of X_c and X_L is approximately tuned to the second harmonic, it is to be expected that the transient oscillations will be those corresponding at least approximately to the natural frequency as dictated by X_c/X_L.

Furthermore, this concept aids greatly in visualizing the production of subharmonics as well as the second harmonics. Consider the equivalent circuit as operating in a steady-state condition with switch S open. Then assume that on some half-cycle, saturation is to be introduced in accordance with the preceding theory. This will mean that switch S must be closed when i reaches 0.9 of the crest value that it had been reaching on previous half-cycles. This instant can also be defined with respect to the fundamental-frequency voltage wave. Following the closing of the switch, a high peak of current will flow momentarily, and, when the total i returns to 0.9, switch S will be opened. This will leave a natural-frequency oscillation in the circuit, and, when the current i has gone through zero and reaches a value of 0.9 again, switch S must be closed. This time, however, the instant of closing is not the same with respect to the fundamental-frequency voltage wave as it was on the first half-cycle because of the transient condition. However, the switch must be opened again when the current decreases to 0.9 in accordance with our criterion. Furthermore, the next half-cycle is likely to require that switch S be closed at still a different point on the voltage wave so that several cycles may elapse before two cycles are identical or nearly identical. Consequently subharmonics must appear, and the lowest subharmonic will be that corresponding to the period in cycles of fundamental frequency required for the exact initial conditions of closing switch S to repeat. It is conceivable that, in a very low loss circuit, this might take a very long time; that is, the subharmonic frequency might be very low.

The preceding analysis of the single-phase circuit can be applied directly to the three-phase circuit by recognizing that the regions of abnormality in the single-phase circuit were associated with circuit constants that offered infinite impedance to the second or third harmonics. In Fig. 8·26, the relationship between miles of line and source reactance for which infinite impedance is offered to harmonics is shown. Also the region in which V_R was found to be greater than unity (from Fig. 8·8) is shown. It is significant that this region is close to the curve giving the relationship between miles of line and source reactance which offers infinite impedance to the second harmonic. Thus it corresponds to the single-phase case which for X_c/X_L

slightly less than four gave rise to overvoltages of a similar nature. Furthermore, the maximum values of overvoltage in the single-phase circuit occurred when X_c/X_L was slightly less than four. This meant that the impedance of the system was actually capacitive. In Fig. 8·26, the region of abnormality lies above the line defining infinite impedance to the second harmonic. This also means that the reactance in this region is capacitive, the similarity between the three-phase and the single-phase case thus being further established.

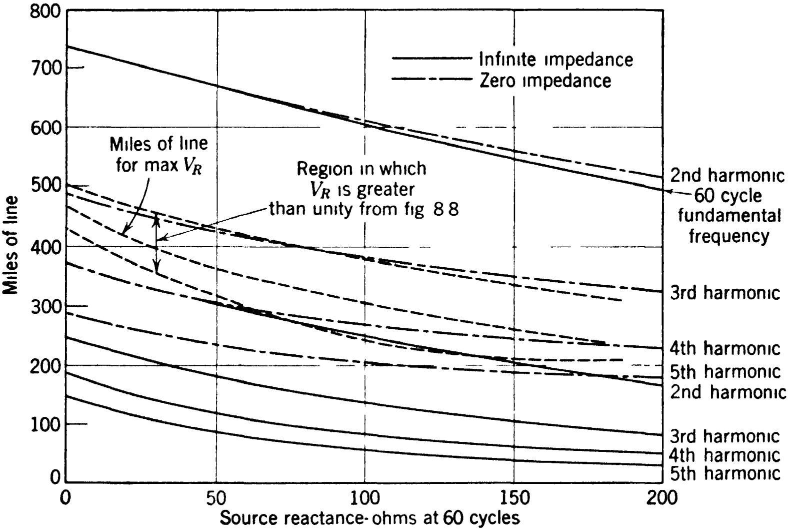


FIG. 8·26. Relationship between miles of line and source reactance which offers infinite or zero impedance to harmonics viewed from any point along the line. Resistance neglected.

In Fig. 8·26, a curve is shown for infinite impedance to third-harmonic current. Infinite impedance to the third harmonic occurs naturally for shorter line lengths than for the second harmonic for a given source reactance. However, with a receiving-end transformer having a closed delta, the third harmonic is not necessarily supplied from the line, and therefore the system impedance has little effect. If the receiving-end transformer had no closed delta, then a region of overvoltages should be expected here of third-harmonic nature corresponding to X_c/X_L slightly less than 9.0 in Fig. 8·24. This region does exist but is considered of little practical significance in connection with this book and so is not included in the curves.

The question as to why the delta being opened or closed made little difference in the phenomenon is answered in Fig. 8·26. Curves showing the relationship between miles of line and source reactance for zero impedance viewed from the receiving end are shown. The curve for zero impedance to the third harmonic is shown to fall partially within the region of abnormality, near its upper extremity. Thus in the abnormal region the system offers a low impedance to third-harmonic currents, and it makes little difference whether the receiving-end transformer has a closed delta or not, since in either event third-harmonic current can easily be supplied.

The preceding discussion would not be complete without some mention of why the second-harmonic shunt is so effective in reducing the abnormal condition in the long lines case. In the single-phase case, the third-harmonic region of overvoltages could be eliminated by a third-harmonic resonant shunt, and the second-harmonic region could be eliminated by a second-harmonic shunt. By analogy between the single-phase and three-phase circuits it is evident that the abnormal region shown in Fig. 8·26 or Fig. 8·8 can be eliminated by a second-harmonic shunt. Furthermore, for reasons just mentioned, with the delta closed, no region of abnormality appears for the third harmonic. Consequently, for the delta-closed condition, which is of greatest practical interest, a second-harmonic shunt is completely effective in eliminating the abnormal condition brought about by exciting power transformers through long lines.

The preceding analysis has dealt with only the second and third harmonics. Higher harmonics, when suppressed by systems offering infinite impedance to them, will produce voltage distortions, but the overvoltages produced will in general be of no practical significance as compared to the phenomena as discussed in this chapter.

Whereas the foregoing discussion has dealt primarily with long lines carrying magnetizing currents, it is possible that the phenomena observed may be encountered in other cases when the load requires harmonic currents to flow through long lines or from sources that tend to suppress them. Such a case may be encountered when, for example, rectifiers are supplied from long transmission lines.

When series capacitors are present, the impedance of the system viewed from the receiving-end transformer terminals remains only slightly changed as far as harmonics are concerned. This is readily understood, because with any practical series-capacitor compensation the reactance of the series capacitor varies inversely as the frequency. Consequently, in the region that caused voltage instability without series capacitors, it is to be expected that instability of the same type will be found with series capacitors. In addition, the presence of series capacitors introduces the possibility of series resonant conditions in a non-linear circuit consisting essentially of source and line reactance, series capacitor, and saturable reactor (transformer), which accounts for the broadening of the region of abnormality shown in Fig. 8·19. This circuit has been discussed in the technical press, and its behavior is generally well understood. It generally requires some form of "shock" excitation for the subharmonics to appear. That is, there are several modes of ultimate behavior, depending on the initial conditions. This is in contrast to the phenomena characteristic of the circuit shown in Fig. 8·23 which is discussed in this chapter and which generally does not require shock excitation for the phenomena to appear. The harmonics and subharmonics appear spontaneously, within a few cycles generally, and represent the only mode of steady-state behavior for the circuit. That is, the circuit can respond only in accordance with the curves of Figs. 8·6(*a*) and 8·6(*b*), whether the system voltage is raised gradually or whether the system is suddenly energized.

8·7 MATHEMATICAL DISCUSSION

In the equivalent circuit of Fig. 8·25, the per-unit current i_S (base current $= 1/L_m$) through the switch following the closing of it at an initial angle θ after a positive voltage crest (assumed to be unity) is

$$i_{S1} = A_1 \sin(t + \theta_1) + B_1 \sin(\omega_C t + \phi_1) + D_1 \qquad [8\cdot 4]$$

where

$$A_1 = \frac{X_m(\omega_0^2 - 1)}{X_s(\omega_C^2 - 1)}$$

$$B_1 = \frac{X_m(\omega_0^2 - \omega_C^2)}{X_s\omega_C^2(1 - \omega_C^2)} \sqrt{\sin^2\theta_1 + \omega_C^2 \cos^2\theta_1}$$

$$D_1 = -\frac{X_m\omega_0^2 \sin\theta_1}{X_s\omega_C^2}$$

$$\omega_0 = \sqrt{\frac{X_c(X_m + X_L)}{X_m X_L}},$$ the circuit natural frequency in per-unit with switch S open

$$\omega_C = \sqrt{\frac{X_c(X_s X_m + X_s X_L + X_m X_L)}{X_m X_L X_s}},$$ the circuit natural frequency in per-unit with the switch S closed

$$\sin\phi_1 = \frac{\sin\theta_1}{\sqrt{\sin^2\theta_1 + \omega_C^2\cos^2\theta_1}}$$

$$\cos\phi_1 = \frac{\omega_C\cos\theta_1}{\sqrt{\sin^2\theta_1 + \omega_C^2\cos^2\theta_1}}$$

The current i flowing into the equivalent transformer is

$$i_{c1} = A_1'\sin(t+\theta_1) + B_1'\sin(\omega_C t + \phi_1) + D_1' \qquad [8\cdot 5]$$

where

$$A_1' = \left(1 + \frac{X_s}{X_m}\right)A_1$$

$$B_1' = \left(1 + \frac{X_s}{X_m}\right)B_1$$

$$D_1' = \sin\theta_1 + \left(1 + \frac{X_s}{X_m}\right)D_1$$

The voltage across the capacitor is

$$e_{c1} = F_1\cos(t+\theta_1) + G_1\cos(\omega_C t + \phi_1) \qquad [8\cdot 6]$$

where

$$F_1 = \frac{(\omega_0^2 - 1)}{(\omega_C^2 - 1)}$$

$$G_1 = \frac{(\omega_0^2 - \omega_C^2)}{\omega_C(1 - \omega_C^2)}\sqrt{\sin^2\theta_1 + \omega_C^2\cos^2\theta_1}$$

Interrupting the current i_S, the voltage across the capacitor is

$$\begin{aligned} e_{01} &= \cos(t+\theta_1') + H_1\sin\omega_0 t + J_1\cos(\omega_0 t + \alpha_1) + K_1\cos(\omega_0 t + \beta_1) \\ &= \cos(t+\theta_1') + M_1\cos(\omega_0 t + \Sigma) \qquad [8\cdot 7] \end{aligned}$$

where

$$H_1 = \frac{(\omega_0^2 - \omega_C^2)\omega_0\sin\theta_1}{\omega_C^2}$$

$$J_1 = \frac{(\omega_0^2 - \omega_C^2)}{\omega_C^2(1 - \omega_C^2)}\sqrt{\sin^2\theta_1 + \omega_C^2\cos^2\theta_1}\sqrt{\omega_C^2\cos^2\delta_1 + \omega_0^2\sin^2\delta_1}$$

$$K_1 = \frac{(\omega_0^2 - \omega_C^2)}{(\omega_C^2 - 1)}\sqrt{\cos^2\theta_1' + \omega_0^2\sin^2\theta_1'}$$

$$\left.\begin{aligned} \delta_1 &= \omega_C t_{c1} + \phi_1 \\ \theta_1' &= t_{c1} + \theta_1 \end{aligned}\right\} t_{c1} \text{ is the interval of time in radians during which the switch was closed}$$

$$\sin \alpha_1 = \frac{\omega_0 \sin \delta_1}{\sqrt{\omega_C^2 \cos^2 \delta_1 + \omega_0^2 \sin^2 \delta_1}}$$

$$\cos \alpha_1 = \frac{\omega_C \cos \delta_1}{\sqrt{\omega_C^2 \cos^2 \delta_1 + \omega_0^2 \sin^2 \delta_1}}$$

$$\sin \beta_1 = \frac{\omega_0 \sin \theta_1'}{\sqrt{\cos^2 \theta_1' + \omega_0^2 \sin^2 \theta_1'}}$$

$$\cos \beta_1 = \frac{\cos \theta_1'}{\sqrt{\cos^2 \theta_1' + \omega_0^2 \sin^2 \theta_1'}}$$

The current i becomes

$$i_{01} = \sin (t + \theta_1) - \frac{H_1}{\omega_0} \cos \omega_0 t + \frac{J_1}{\omega_0} \sin (\omega_0 t + \alpha_1) + \frac{K_1}{\omega_0} \sin (\omega_0 t + \beta_1) \quad [8\cdot8]$$

Closing the switch at the next instant when the current i reaches the critical value

$$i_{S2} = A_1 \sin (t + \theta_2) + B_2 \sin (\omega_C t + \phi_2) + D_2 + M_1 S_2 \sin (\omega_C t + \psi_2) + W_2 M_1 \quad [8\cdot9]$$

A_1 defined in equation [8·4].

$$B_2 = \frac{X_m(\omega_0^2 - \omega_C^2)}{X_s \omega_C^2(1 - \omega_C^2)} \sqrt{\sin^2 \theta_2 + \omega_C^2 \cos^2 \theta_2}$$

$$D_2 = -\frac{X_m \omega_0^2 \sin \theta_2'}{X_s \omega_C^2}$$

$$\theta_2 = t_{01} + t_{c1} + \theta$$

$$\sin \phi_2 = \frac{\sin \theta_2}{\sqrt{\sin^2 \theta_2 + \omega_C^2 \cos^2 \theta_2}}$$

$$\cos \phi_2 = \frac{\omega_C \cos \theta_2}{\sqrt{\sin^2 \theta_2 + \omega_C^2 \cos^2 \theta_2}}$$

$$S_2 = \frac{X_m}{\omega_C^2 X_s} \sqrt{\omega_0^2 \sin^2 \Delta_2 + \omega_C^2 \cos^2 \Delta_2}$$

$$\sin \psi_2 = \frac{\omega_0 \sin \Delta_2}{\sqrt{\omega_0^2 \sin^2 \Delta_2 + \omega_C^2 \cos^2 \Delta_2}}$$

$$\cos \psi_2 = \frac{\omega_C \cos \Delta_2}{\sqrt{\omega_0{}^2 \sin^2 \Delta_2 + \omega_C{}^2 \cos^2 \Delta_2}}$$

Δ_2 = angle of ω_0 frequency voltage past positive voltage crest when switch is closed

$$W_2 = -\frac{X_m \omega_0 \sin \Delta_2}{X_s \omega_C{}^2}$$

The voltage across the capacitor is

$$e_{c2} = \frac{X_s}{X_m} \frac{di_{S2}}{dt}$$

and the current i_{c2} into the equivalent transformer is of the form of equation [8·5].

At the time of interruption, the current i_{02} takes the form of equation [8·8] subject to the new boundary conditions. The voltage e_{02} takes the form of equation [8·7] also subject to the new boundary conditions.

This procedure can go on for as long as desired, each time imposing the new boundary conditions. It can readily be seen that these successive boundary conditions will not in general be the same for adjacent cycles of fundamental frequency. That is $\theta_1 + \pi \neq \theta_2$, $\theta_2 + \pi \neq \theta_3$, $\cdots \theta_n + \pi \neq \theta_{n+1}$. Ultimately, however, $\theta_n + a\pi = \theta_{n+a}$ and a definite sub-harmonic frequency will appear. Furthermore, the voltage oscillation across the capacitor will be of fundamental frequency plus a natural-frequency oscillation which is predominantly that defined as ω_0, the natural frequency of the system with the switch open. This is brought about by the fact that the switch S is closed in general only a short fraction of a fundamental-frequency period and that period is generally in the vicinity of fundamental-frequency voltage zero. Therefore it is to be expected that, when circuit losses are low, the natural frequency of the circuit with the switch open and subharmonics will appear in the voltage across the capacitor in the circuit of Fig. 8·25.

8·8 CALCULATION OF CONDITIONS FOR INFINITE IMPEDANCE TO HARMONICS—EFFECT OF TRANSFORMER LOCATION

Viewed from the receiving end, the system of Fig. 8·3 or Fig. 8·4 neglecting losses offers an impedance

$$Z_r = j\frac{\omega L_s \cos \omega l\sqrt{LC} + \sqrt{\frac{L}{C}} \sin \omega l\sqrt{LC}}{\cos \omega l\sqrt{LC} - \omega L_s \sqrt{\frac{C}{L}} \sin \omega l\sqrt{LC}} \qquad [8\cdot10]$$

where L = inductance per mile in henries.
C = capacitance per mile in farads.
L_s = source inductance in henries.
$\omega = 2\pi f$.
l = length of line in miles.

In the transient analyzer artificial line

$$L = 0.002 \text{ henry per mile}$$

$$C = 0.016 \text{ microfarad per mile}$$

For $Z_r = \infty$, the criterion is

$$\cos \omega l\sqrt{LC} - \omega L_s \sqrt{\frac{C}{L}} \sin \omega l\sqrt{LC} = 0 \qquad [8\cdot 11]$$

or, rewriting,

$$Z_r = \infty \quad \text{when} \quad \tan \omega l\sqrt{LC} = \frac{1}{\omega L_s} \sqrt{\frac{L}{C}} \qquad [8\cdot 12]$$

This equation was used with the above numerical constants for plotting the curves relating source reactance and miles of line to give infinite impedance in Fig. 8·26.

For $Z_r = 0$, the criterion is

$$\omega L_s \cos \omega l\sqrt{LC} + \sqrt{\frac{L}{C}} \sin \omega l\sqrt{LC} = 0 \qquad [8\cdot 13]$$

or, rewriting,

$$Z_r = 0 \quad \text{when} \quad \tan \omega l\sqrt{LC} = -\omega L_s \sqrt{\frac{C}{L}} \qquad [8\cdot 14]$$

This equation was used with the above numerical constants for plotting the curves relating source reactance and miles of line to give zero impedance in Fig. 8·26.

It can also be shown that the relationship between source reactance ωL_s and line length to give infinite impedance is independent of the point considered along the line. If l is the total length of line considered, and d is the distance between the sending end and the point from which the impedance is to be determined, then

$$Z = \frac{-j\sqrt{\frac{L}{C}}\cot \omega(l-d)\sqrt{LC}\cdot j\left[\dfrac{\omega L_s \cos \omega d\sqrt{LC}+\sqrt{\frac{L}{C}}\sin \omega d\sqrt{LC}}{\cos \omega d\sqrt{LC}-\omega L_s\sqrt{\frac{C}{L}}\sin \omega d\sqrt{LD}}\right]}{-j\sqrt{\frac{L}{C}}\cot \omega(l-d)\sqrt{LC}+j\left[\dfrac{\omega L_s \cos \omega d\sqrt{LC}+\sqrt{\frac{L}{C}}\sin \omega d\sqrt{LC}}{\cos \omega d\sqrt{LC}-\omega L_s\sqrt{\frac{C}{L}}\sin \omega d\sqrt{LC}}\right]}$$

$$= \frac{-\sqrt{\frac{L}{C}}\cot \omega(l-d)\sqrt{LC}\left[\omega L_s \cos \omega d\sqrt{LC}+\sqrt{\frac{C}{L}}\sin \omega d\sqrt{LC}\right]}{\left[-\sqrt{\frac{L}{C}}\cot \omega(l-d)\sqrt{LC}\right]\left[\cos \omega d\sqrt{LC}-\omega L_s\sqrt{\frac{C}{L}}\sin \omega d\sqrt{LC}\right]}$$

$$+ \omega L_s \cos \omega d\sqrt{LC} + \sqrt{\frac{L}{C}}\sin \omega d\sqrt{LC} \qquad [8\cdot 15]$$

$Z = \infty$ when the denominator of equation [8·15] is zero. Rewriting this denominator

$$-\sqrt{\frac{L}{C}}\cot \omega(l-d)\sqrt{LC}\cdot\cos \omega d\sqrt{LC}$$

$$+ \omega L_s \cot \omega(l-d)\sqrt{LC}\cdot\sin \omega d\sqrt{LC}$$

$$+ \omega L_s \cos \omega d\sqrt{LC} + \sqrt{\frac{L}{C}}\sin \omega d\sqrt{LC}$$

$$= -\sqrt{\frac{L}{C}}\cos \omega d\sqrt{LC}\,\frac{1+\tan \omega l\sqrt{LC}\tan \omega d\sqrt{LC}}{\tan \omega l\sqrt{LC}-\tan \omega d\sqrt{LC}}$$

$$+ \omega L_s \sin \omega d\sqrt{LC}\,\frac{1+\tan \omega l\sqrt{LC}\tan \omega d\sqrt{LC}}{\tan \omega l\sqrt{LC}-\tan \omega d\sqrt{LC}}$$

$$+ \omega L_s \cos \omega d\sqrt{LC} + \sqrt{\frac{L}{C}}\sin \omega d\sqrt{LC} = 0$$

$$- \sqrt{\frac{L}{C}} \cos \omega d\sqrt{LC} - \sqrt{\frac{L}{C}} \cos \omega d\sqrt{CL} \tan \omega l\sqrt{LC} \tan \omega d\sqrt{LC}$$

$$+ \omega L_s \sin \omega d\sqrt{LC} + \omega L_s \sin \omega d\sqrt{LC} \tan \omega l\sqrt{LC} \tan \omega d\sqrt{LC}$$

$$+ \omega L_s \cos \omega d\sqrt{LC} \tan \omega l\sqrt{LC} - \omega L_s \cos \omega d\sqrt{LC} \tan \omega d\sqrt{LC}$$

$$+ \sqrt{\frac{L}{C}} \sin \omega d\sqrt{LC} \tan \omega l\sqrt{LC} - \sqrt{\frac{L}{C}} \sin \omega d\sqrt{LC} \tan \omega d\sqrt{LC} = 0$$

$$\omega L_s[\sin \omega d\sqrt{LC} + \sin \omega d\sqrt{LC} \tan \omega l\sqrt{LC} \tan \omega d\sqrt{LC}$$

$$+ \cos \omega d\sqrt{LC} \tan \omega l\sqrt{LC} - \cos \omega d\sqrt{LC} \tan \omega d\sqrt{LC}\,]$$

$$= \sqrt{\frac{L}{C}}\,[\cos \omega d\sqrt{LC} + \cos \omega d\sqrt{LC} \tan \omega l\sqrt{LC} \tan \omega d\sqrt{LC}$$

$$- \sin \omega d\sqrt{LC} \tan \omega l\sqrt{LC}$$

$$+ \sin \omega d\sqrt{LC} \tan \omega d\sqrt{LC}\,]\omega L_s \tan \omega l\sqrt{LC}\,[\sin \omega d\sqrt{LC} \tan \omega d\sqrt{LC}$$

$$+ \cos \omega d\sqrt{LC}\,] = \sqrt{\frac{L}{C}}\,[\cos \omega d\sqrt{LC} + \sin \omega d\sqrt{LC} \tan \omega d\sqrt{LC}\,]$$

and finally

$$\tan \omega l\sqrt{LC} = \frac{1}{\omega L_s}\sqrt{\frac{L}{C}}$$

which is identical with equation [8·12], which was to be proved.

Similarly it can be shown that the condition for $Z = 0$ as given by equation [8·13] holds for any location along the line.

8·9 EFFECT OF CORONA LOSS

In a high-voltage system it is to be expected that corona loss on the overhead line would serve to reduce greatly the overvoltages otherwise obtained in accordance with the foregoing procedure and analysis. One attempt to evaluate the effect of corona loss was made in a specific case, the one-line circuit diagram for which is shown in Fig. 8·27.

In this particular case it is to be noticed that the most likely condition of overvoltage and therefore the one requiring most careful consideration from the point of view of this chapter is that occurring when full load is suddenly dropped by the opening of the 230-kv breaker at the load end. The system data are as follows:

1. *Generator.*
   66,700 kva, 13,800 volts, 0.9 p.f.
   $X_d' = 0.25$.
   $X_d = 0.90$.
   $X_l = 0.18$.
2. *Voltages held before disturbance occurs.*
   Sending end: 13.8 kv at generator terminals.
   Receiving end: 211 kv when generator is operating at rated load.
3. *Transmission line.*
   Line resistance (positive-sequence): 0.12 ohm/mile.
   Line reactance (positive-sequence): 0.76 ohm/mile.
   Line capacitance (positive-sequence): 0.015 μf/mile.
4. *Voltage regulator.*
   Voltage regulated at water-wheel generator terminals.
5. *Overspeed characteristics.*
   Increases linearly with time reaching maximum of 130 per cent in 3.5 seconds.
6. *Sending-end transformers.*
   Three 22,250 kva single-phase delta- or wye- connected with grounded neutral.
   Turn ratio: $\dfrac{132}{13.2} = 10.0$.
   Transformation ratio: $\dfrac{230}{13.2} = 17.4$.
   Exciting current: 2.06 per cent at 100 per cent voltage.
7. *Cable capacitance*: 0.517 μf $= C_1 = C_0$.
8. *Time constants.*
   Open-circuit time constant, $T_{d0}' = 6$ seconds.
   Time constant with transmission line in,

$$T_d' = \frac{X_d' + X_e}{X_d + X_e} T_{d0}' = \frac{0.25 - 1.45}{0.90 - 1.45} T_{d0}'$$

$$= 2.18 T_{d0}' = 13.1 \text{ seconds.}$$

Subtransient time constant, $T_d'' = 0.2$ second, where X_e is the positive-sequence capacitive reactance of the transmission line.

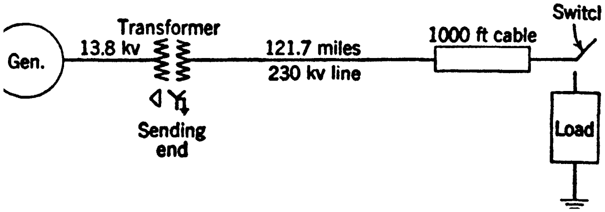


FIG. 8·27. System analyzed for effect of corona loss.

For full load at 0.9 power factor, $E_d' = 1.13$ and $E_d = 1.61$. If certain reasonable assumptions are made concerning the regulator-exciter characteristic, then with no line or transformer in, the terminal voltage due to suddenly dropping load is given with sufficient accuracy, neglecting subtransient effects, saturation, and corona, by

$$E_t = 1.13 + 0.48(1 - \epsilon^{-0.167t}) - K\left[1 + \frac{b\epsilon^{-0.167t} - 0.167\epsilon^{-bt}}{0.167 - b}\right] \qquad [8\cdot16]$$

neglecting overspeeding. If overspeeding is included, then

$$E_t = (1 + 0.0856t)\left[1.13 + 0.48(1 - \epsilon^{-0.167t}) - K\left(1 + \frac{b\epsilon^{-0.167t} - 0.167\epsilon^{-bt}}{0.167 - b}\right)\right] \qquad [8\cdot17]$$

up to $t = 3.5$ seconds, neglecting subtransient effects. For this specific case, the regulator and exciter characteristics were such that values of $K = 2$ and $b = 0.66$ were reasonable so that the regulator restored normal voltage at the machine terminals at $t = 3.5$ seconds when a maximum overspeed of 1.22 times normal was reached under the foregoing assumptions. This is shown in Fig. 8·28, with and without overspeed, curves 1 and 2, respectively.

With this regulator characteristic, and the line in, the expression for receiving-end voltage E_R can be shown to be

$$E_R = (1 + 0.0856t)[-0.87 - 0.11\epsilon^{-5t} + 2.89\epsilon^{-0.0763t} - 0.587\epsilon^{-0.66t}] \qquad [8\cdot18]$$

This neglects certain second-order effects which are expected to be insignificant. This expression is shown plotted in Fig. 8·28 also (curve 3). Voltages at the sending end and at the transformer end are also shown (curves 4 and 5, respectively). Voltage at the far end is seen to reach 1.7 times normal or 408 kv rms line-to-line, which would be impressed on the cable.

These calculations show that the sending-end transformer is greatly overexcited under such overspeeding and overvoltage conditions. Thus, it is to be expected that these overvoltages will be affected by the saturation of the sending-end transformer in accordance with the foregoing theory which pointed out that it made no difference where the transformer is located along the line. The cable capacitance makes the effective length of line relatively long, and, since the source reactance is relatively high (312 ohms referred to 230-kv terminals), the system is in the susceptible region of Fig. 8·8. The system, when studied on the transient analyzer, neglecting corona losses entirely, produced overvoltages in accordance with the foregoing theory of 2.8

per-unit or 670 kv line-to-line under the assumed condition of dropping load.

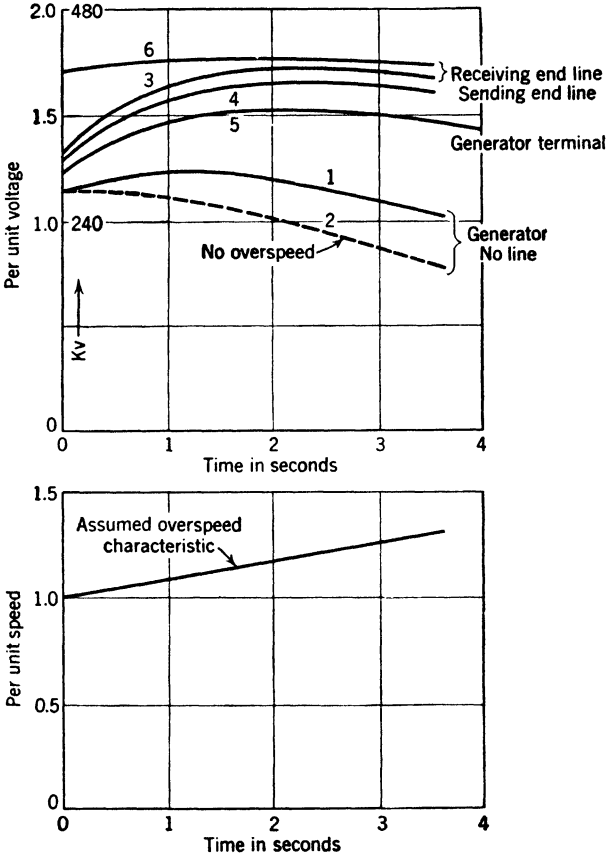


FIG. 8·28. Overvoltages caused by suddenly dropping load, 230-kv line. Curve 1, without line but with overspeed, voltage at generator terminals; curve 2, without line and no overspeed, voltage at generator terminals; curve 3, with line and overspeed, voltage at receiver end of line; curve 4, with line and overspeed, voltage at sending end; curve 5, with line and overspeed, voltage at generator terminals; curve 6, with line and overspeed, including effects of corona and transformer saturation, voltage at receiving end.

Obviously corona losses would limit such overvoltages. In order to evaluate the extent of such limitation, the following expression for corona loss was used to determine the equivalent shunt resistance to put in the miniature system to simulate corona loss.[16]

$$P = 390(85)\sqrt{\frac{r}{s}}\left(\frac{E - E_0}{10^5}\right)^2 \text{ kw/mile/conductor} \qquad [8\cdot 19]$$

Here

$$r = 0.504 \text{ inches (ACSR, 795,000 cm)}$$

$$s = 29 \text{ feet} = 348 \text{ inches}$$

and

$$P = 0.01265(E - E_0)^2 \qquad [8\cdot 20]$$

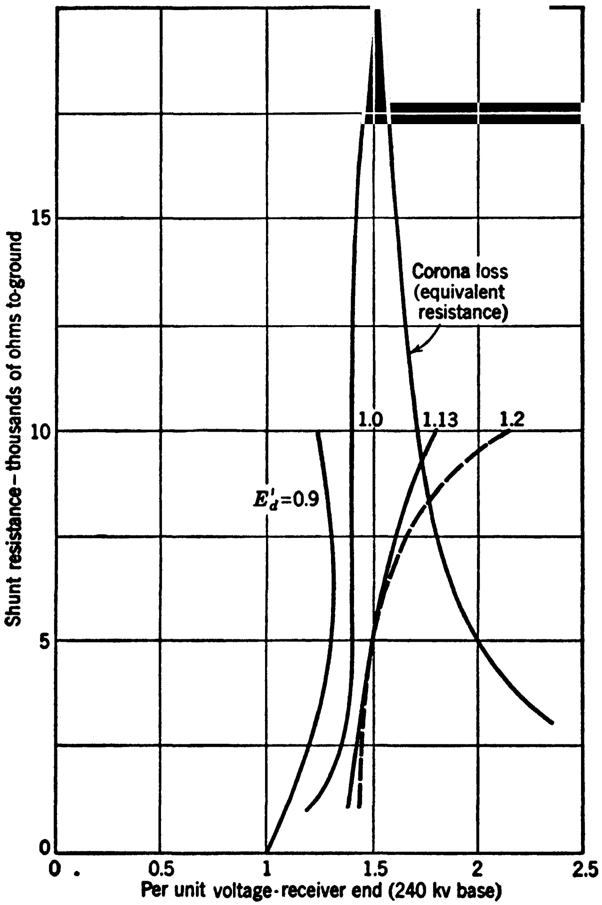


FIG. 8·29. Effect of corona loss. Intersections of corona loss curve with voltage-shunt resistance curves are the resultant per-unit voltages for the values of E_d' associated with each curve. Thus, the voltage to ground at the receiver end does not vary greatly with E_d' until E_d' is reduced considerably.

The critical disruptive voltage is $E_0 = 176$ kv effective volts to neutral for the conductor in this case. Consequently

$$P = 0.01265(E - 176)^2 \qquad [8\cdot21]$$

The shunt resistance corresponding to this loss as a function of E in per-unit is shown in Fig. 8·29. Also shown in this figure are the per-unit voltages obtained in the miniature system with $E_d' =$ constant but varying shunt resistance. The intersections of these latter curves with the corona-loss curve (from equation 8·21) are the equilibrium points due to the combined effects of saturation and corona. From these intersections, and with a knowledge of the machine and exciter-regulator characteristics, it becomes possible to interpolate with satisfactory accuracy and arrive at the resultant voltage-time curve, including both transformer saturation and corona loss. This desired curve is shown in Fig. 8·28 also (curve 6). The effect of sending-end transformer saturation when corona is considered in this way is to increase the maximum voltage only slightly (from 1.7 to 1.75), but the time duration of the overvoltage is considerably increased since this voltage is relatively insensitive to the excitation voltage E_d' until it has been greatly reduced. Thus, for the case cited, it appears that the overvoltage of about 1.7 is expected to last at approximately a constant value for about four seconds.

In conclusion, it appears therefore that overvoltages due to dropping load should be considered in selecting proper arresters and that consideration should be given to the possibility of relaying long lines in such a way that unloaded transformers are not left even momentarily to be energized through long transmission lines.

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9 OVERVOLTAGES CAUSED BY OPEN CONDUCTORS

9·1 INTRODUCTION

It has been recognized for some time that in three-phase circuits high voltages may cause damage to equipment after the blowing of a fuse or the non-simultaneous opening or closing of switch contacts. Motors on such circuits may reverse their direction of rotation. The phenomena may be accompanied by overvoltages. Failures of lightning arresters under system conditions involving blowing of fuses or the opening of switches appear to have occurred because of high sustained overvoltages above the arrester ratings. The energizing of system neutral grounding transformers one phase at a time has been known to cause arrester failure and damage to the transformer. Instances of the breakage of a line conductor with resulting abnormal voltages have been noted. The dangers of using fuses and single-pole disconnecting switches under certain conditions have been pointed out from time to time in the literature.

The problem of determining the conditions under which open conductors in general may give rise to further system troubles is of significance to the system operators, designers, and the manufacturers of electrical apparatus. Interpreting this reference to open conductors broadly, one may include also any type of switching device, such as single-pole disconnecting switches, oil circuit reclosers, horn gap switches, oil circuit breakers, etc. The interval of time between the closing or opening of the first and last phases may vary widely, depending on the type of switching device and also on the circuit being energized or de-energized. For example, in fuse operations, one phase or two phases may be open for a relatively long time. On the other hand, in an oil circuit breaker, the interval of time during which one contact would be closed and two opened, for example, would be very short. The time required for the transition to the condition of sustained voltage depends on the circuit constants and the characteristics of the interrupting device. It may vary from a few cycles to a relatively long period of several seconds. The phenomena under discussion in this chapter may be associated with any interrupting device that has characteristics such that the interval of time between the

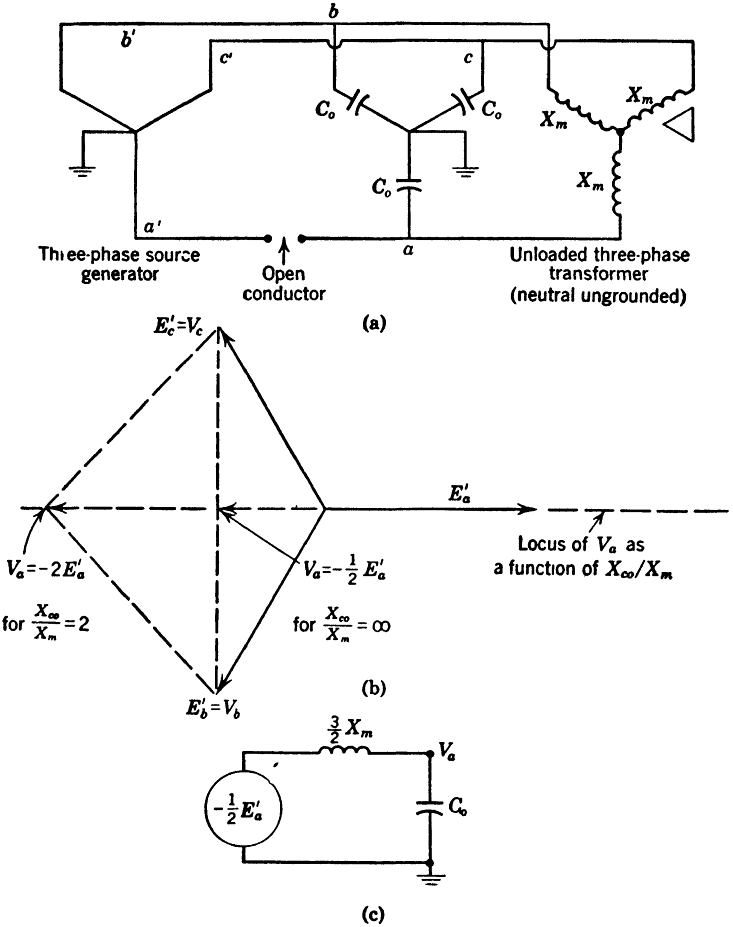
b
b'
c'
c
C_o
C_o
C_o
X_m
X_m
X_m
a'
a
Three-phase source generator
Open conductor
Unloaded three-phase transformer (neutral ungrounded)
(a)
$E_c'=V_c$
E_a'
$V_a=-2E_a'$ for $\frac{X_{co}}{X_m}=2$
$V_a=-\frac{1}{2}E_a'$ for $\frac{X_{co}}{X_m}=\infty$
Locus of V_a as a function of X_{co}/X_m
$E_b'=V_b$
(b)
$\frac{3}{2}X_m$
V_a
$-\frac{1}{2}E_a'$
C_o
(c)

FIG. 9·1. (*a*) Basic circuit which may produce overvoltages with one open conductor. Generator impedance assumed negligible. C_0 is zero sequence capacitance. X_m is magnetizing reactance of the unloaded transformer bank. (*b*) Vector diagram. (*c*) Equivalent circuit.

opening or closing of the first and last phases is of sufficient length to permit the ultimate steady-state voltage conditions to be attained.

9·2 ONE OPEN CONDUCTOR—BASIC CIRCUIT

The simplest form of circuit that may produce phase reversal and overvoltage as a consequence of opening one conductor is shown in Fig. 9·1(*a*). The phenomena may be described briefly as follows.

In this circuit there is a path for currents from the closed conductors *b* and *c* through the magnetizing impedance X_m of the ungrounded neutral unloaded transformer bank and the capacitance C_0 in the open phase *a* to ground. If C_0 is small so that X_{c0} is large as compared to X_m, then V_a, the voltage to ground at *a*, is small. If X_{c0} and X_m are of the same order of magnitude, then a series resonant condition is approached and overvoltages will exist.

The vector diagram of Fig. 9·1(*b*) shows clearly the relationship existing among the phase voltages-to-ground, V_a, V_b, and V_c. Before the conductor is opened, the voltages are balanced as indicated by the solid-line vectors. If C_0 is 0, so that X_{c0} is infinite, and the conductor is opened, the voltage on phase *a* moves along the indicated locus to the left to the position V_a. The terminus of V_a is on a straight line joining the termini of V_b and V_c, which remain fixed in position. It is to be noted that, whereas the original phase sequence was *abc*, it is now changed to *bac*. The magnitude of V_a is half of what it was before the conductor was opened.

By means of Thévenin's theorem it is possible to replace the entire system of Fig. 9·1(*a*) by an internal voltage $-\frac{1}{2}E_a'$ and the value of impedance looking in at terminal *a*. This simple equivalent circuit is shown in Fig. 9·1(*c*). The voltage across the capacitor C_0 is now the voltage V_a that is of interest in Figs. 9·1(*a*) and (*b*). From this

$$\begin{aligned} V_a &= -\frac{1}{2}E_a'\left(\frac{-jX_{c0}}{j\frac{3}{2}X_m - jX_{c0}}\right) \\ &= E_a'\frac{X_{c0}}{3X_m - 2X_{c0}} \\ &= E_a'\frac{X_{c0}/X_m}{3 - 2(X_{c0}/X_m)} \qquad [9\cdot 1] \end{aligned}$$

and it is evident that V_a moves further to the left along the locus indicated in Fig. 9·1(*b*) as X_{c0}/X_m decreases. Thus V_a increases negatively as X_{c0}/X_m decreases. For the particular value of $X_{c0}/X_m = 2$, $V_a =$

$-2E_a'$ and it is to be noted that *balanced* line-to-line voltages with reversed phase rotation are obtained. If X_{c0}/X_m is further decreased, then V_a increases to greater negative values and, at the particular value of $X_{c0}/X_m = 3/2$, V_a is infinite in magnitude. A further decrease in X_{c0}/X_m makes V_a positive and large in magnitude. As the ratio approaches unity, V_a approaches its original value of E_a'. It is only of academic interest to point out that, when $X_{c0}/X_m = 1.0$, V_a is equal to E_a', since any losses in the system would alter this situation. The effects of saturation of X_m have been completely ignored also in outlining these basic phenomena.

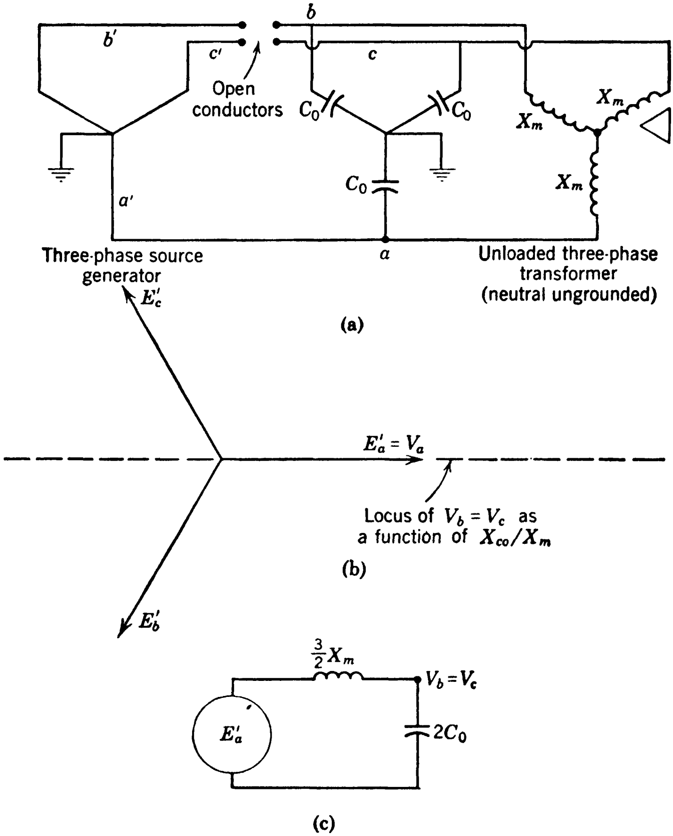
b
b'
c'
c
Open conductors
C_0
C_0
X_m
X_m
C_0
X_m
a'
a
Three-phase source generator
Unloaded three-phase transformer (neutral ungrounded)
(a)
E_c'
$E_a' = V_a$
Locus of $V_b = V_c$ as a function of X_{c0}/X_m
(b)
E_b'
$\frac{3}{2}X_m$
$V_b = V_c$
E_a'
$2C_0$
(c)

FIG. 9·2. (*a*) Basic circuit that may produce overvoltages with two open conductors. C_0 is zero sequence capacitance. X_m is magnetizing reactance of the unloaded transformer bank. (*b*) Vector diagram. (*c*) Equivalent circuit.

9·3 TWO OPEN CONDUCTORS—BASIC CIRCUIT

The simplest form of circuit that may produce overvoltages as a consequence of opening two conductors is shown in Fig. 9·2(*a*). This circuit will not produce phase reversal. The phenomena may be described briefly as follows. In this circuit there is a path for current to flow from the unopened phase *a* through the magnetizing reactances of the transformer and thence to ground through zero-sequence capacitance. The paths are identical for phases *b* and *c*; consequently V_b and V_c are equal. If *b* and *c* are opened and there is no zero-sequence capacitance C_0, then $V_b = V_c = V_a = E_a'$. Again from Thévenin's theorem, the equivalent circuit can be obtained as shown in Fig. 9·2(*c*). The magnitude of V_b and V_c as a function of X_{c0}/X_m is

$$\begin{aligned} V_b = V_c &= E_a' \frac{-jX_{c0}/2}{j\frac{3}{2}X_m - j(X_{c0}/2)} \\ &= E_a' \frac{X_{c0}}{X_{c0} - 3X_m} \\ &= E_a' \frac{X_{c0}/X_m}{X_{c0}/X_m - 3} \qquad [9\cdot2] \end{aligned}$$

From this equation, as X_{c0}/X_m is decreased from infinity, $V_b = V_c$ increases positively reaching a value of infinity when $X_{c0}/X_m = 3$. Further decrease of X_{c0}/X_m causes $V_b = V_c$ to acquire large negative values which decrease in magnitude as X_{c0}/X_m is further decreased.

It is to be noted that if $C_1 \neq C_0$, then the only change in either Fig. 9·1(*c*) or 9·2(*c*) is to add a capacitance of $\frac{2}{3}(C_1 - C_0)$ in parallel with $\frac{3}{2}X_m$.

9·4 TRANSIENT ANALYZER RESULTS

Although the foregoing analysis is very helpful in acquiring a clear physical picture of the basic phenomena involved, it is limited in its usefulness so far as the more general phenomena are concerned, including effects of losses, saturation, $C_1 \neq C_0$, faults, neutral impedance, source impedance, transformer connections, and other factors. In order to evaluate the effects of a number of these factors on the basic phenomena, the transient analyzer was used. The more general circuit studied is shown in Fig. 9·3.

Since the highest voltages are apt to occur with the unloaded or lightly loaded transformer bank, the study of this circuit was limited to the unloaded condition. Voltage conditions were obtained with transformer banks made up of three single-phase units and with three-phase transformers of the shell type and core type, the windings on the line side and also the secondaries being connected alternately in delta and wye for each case.

It was assumed that the three-phase system is large relative to the kilovolt-ampere rating of the transformer bank in Fig. 9·3, and therefore it can be represented by an equivalent generator with balanced line-to-line voltages and negligible positive- and negative-sequence impedances. If the system is grounded, the equivalent generator will also have balanced line-to-ground voltages and its zero-sequence impedance will be negligible.

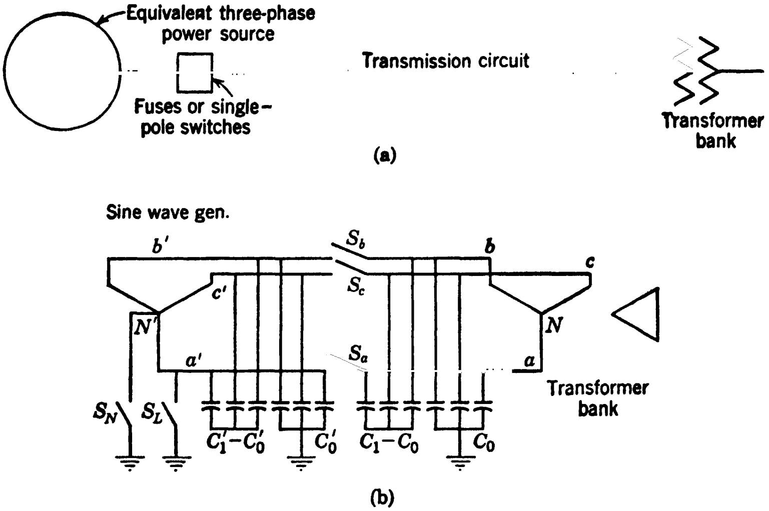

FIG. 9·3. System studied. (*a*) One-line diagram. (*b*) Miniature-system equivalent circuit.

In the three-phase diagram of Fig. 9·3(*b*) the impedance of the sine-wave generator is very low relative to the exciting reactance of the unloaded transformer bank and the line capacitive reactances used, so that balanced line-to-line voltages are applied to the circuit at *a'b'c'* under all conditions. The system becomes a grounded one with balanced line-to-ground voltages when switch S_N is closed. Resistance and reactance are neglected in the transmission circuit since they are insignificant relative to the capacitive reactances for the lengths of line under discussion. C_1' and C_0' represent the positive- and zero-sequence capacitances, respectively, on the system side of the fuses or single-pole switches; C_1 and C_0, the positive- and zero-sequence capacitances of the circuit supplying the ungrounded unloaded transformer bank on the transformer side, which in Fig. 9·3(*b*) is shown as wye connected on the line side and delta connected on the load side.

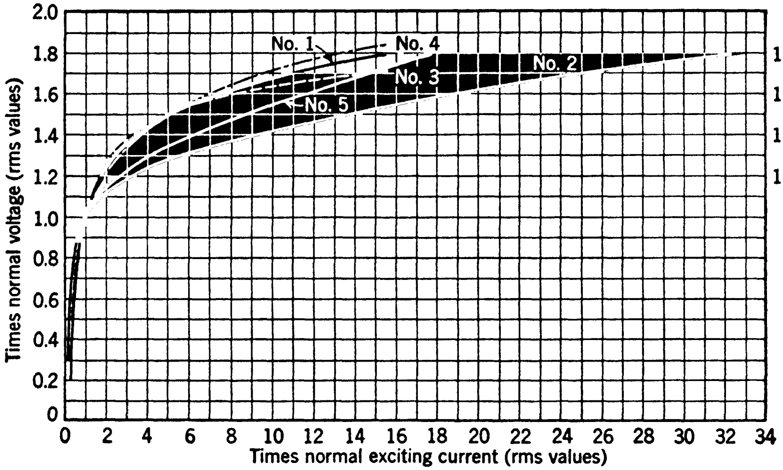

FIG. 9·4. Transformer saturation curves used in making calculations and in the miniature system. Curve 1, miniature-system single-phase transformers, 135 rms volts normal, 1,185 ohms normal magnetizing impedance; curve 2, assumed curve having more saturation than curve 1, used in making calculations to determine the effect of this factor; curve 3, miniature-system three-phase core-type transformer, 158 rms volts line-to-line normal wye, 840 ohms per phase normal magnetizing impedance; curve 4, miniature-system three-phase shell-type transformer, 144 rms volts line-to-line normal wye, 520 ohms per phase normal magnetizing impedance; curve 5, miniature-system special transformers 110 rms volts normal, 13,000 ohms normal magnetizing impedance, single-phase transformers.

In obtaining the results herein summarized, the miniature-system transformers were operated at normal flux densities corresponding to normal line-to-neutral voltage of unity on all the saturation curves of Fig. 9·4, except for curve 2 which is an assumed curve used in making calculations as noted on certain curves shown hereafter. All saturation curves give the values of the applied sinusoidal rms voltage and the exciting rms current in terms of normal voltage and normal exciting current, respectively.

A summary of cases studied follows.

- I. Power source grounded—load transformers ungrounded.
  - *A*. One phase open—no fault
    - 1. Single-phase load transformers.
      - *a*. Delta-connected on line side, wye- or delta-connected on load side.
      - *b*. Wye-connected on line side, delta-connected on load side.
      - *c*. Wye-connected on line side, wye-connected on load side.
    - 2. Three-phase load transformers
      - *a*. Delta-connected on line side, wye-connected on load side.
        - 1. Core type.
        - 2. Shell type.
      - *b*. Wye-connected on line side, delta-connected on load side.
        - 1 Core type
        - 2. Shell type
  - *B*. Two phases open—no fault
    Same as for *A*.
- II. Power source ungrounded—load transformers ungrounded
  - *A*. One phase open—no fault
    - 1. Single-phase load transformers.
      - *a*. Delta-connected on line side, wye- or delta-connected on load side
  - *B*. One conductor open—line-to-ground fault on system side of open conductor.
    - 1. Single-phase load transformers
      - *a*. Delta-connected on line side, wye- or delta-connected on load side
      - *b*. Wye-connected on line side, delta-connected on load side.
      - *c*. Wye-connected on line side, wye-connected on load side.
  - *C*. One conductor open—line-to-ground fault on load side of open conductor
    - 1. Single-phase transformers
      - *a*. Delta-connected on line side, wye- or delta-connected on load side.

The results of the miniature-system tests for these cases are shown in Figs. 9·5 to 9·13. For the solidly grounded system, data were obtained for $C_1 = C_0$ and $C_1 = 2C_0$. Throughout the following discussion, X_m is the magnetizing reactance to neutral of the transformer bank on the load side of the open conductor at normal voltage. In more general terms, it is the equivalent magnetizing reactance of the total transformer kilovolt-ampere capacity on the load side of the fuse or single-pole switch location. Thus the single-load transformer of Fig. 9·3(*b*) may be considered the equivalent of several smaller transformers all on the load side of the open conductor. Likewise the positive- and zero-sequence capacitive reactances, X_{c1} and X_{c0}, respectively, should be interpreted in a more general manner to correspond to total positive- and zero-sequence capacitances C_1 and C_0 on the load side of the open conductor. Thus the results assume much broader significance and are made much more widely applicable.

I Source grounded

A One phase open—no fault

1 Single-phase transformers

a Delta-connected on line side wye- or delta-connected on load side The curves of Fig 9 5(*a*) give the steady-state maximum peak voltage of the open conductor on the transformer side of the opening in terms of normal line-to-neutral peak voltage with the transformer bank delta-connected on the line side, and wye- or delta-connected on the load side as a function of X_{c1}/X_m for $C_0 = C_1$ Curves 1 and 2 are calculated by methods to be given later in this chapter Curve 1 gives calculated voltages using saturation curve 1 of Fig 9 4, which corresponds to the saturation of the miniature-system transformers used Points obtained by test on the miniature system are indicated The general shape of calculated and test curves is the same, although calculated voltages are higher except for large ratios of X_{c1}/X_m where both calculated and test voltages are less than normal

This case is the one discussed in detail in Section 9 2 and the dashed curve 3 of Fig 9 5(*a*) is obtained from equation [9 1]

It is seen that, when saturation is included, V_a is not a single-valued function of X_{c1}/X_m for the range of $X_{c1}/X_m < 1\ 5$ In this range three possible values of V_a exist two of which were obtained in the miniature system quite consistently the larger negative value and the smaller of the two positive values shown on the curves All three values were obtained for very low values of X_{c1}/X_m of about 0 2 Figure 9 6 shows oscillograms of the voltages and currents for the three possible stable conditions, any one of which may occur depending on the instant the switch is opened With the switch opened at random a number of times, the high negative voltage or the low positive voltage occurred more often than the medium positive voltage

Figure 9 5(*b*) is similar to Fig 9 5(*a*) except that $C_1 = 2C_0$

b Wye-connected on line side delta-connected on load side

c Wye-connected on line side wye-connected on load side

Figure 9 5(*c*) for $C_1 = C_0$ and Fig 9 5(*d*) for $C_1 = 2C_0$, with the transformers wye-connected on the line side and delta- or wye-connected on the load side are similar to Figs 9 5(*a*) and (*b*) Values obtained from the miniature system for $C_1 = C_0$ and a wye-delta connection do not differ materially from those for the transformer delta-connected on the line side except that only two of the stable voltage conditions indicated by calculation for values of X_{c1}/X_m in the neighborhood of 1 and below were obtained in test Figure 9 7 shows oscillograms of the voltages for the two stable conditions obtained at $X_{c1}/X_m = 0\ 15$ with $C_1 = C_0$ No miniature-system results were obtained for the case of $C_1 = 2C_0$

It should be pointed out that curves 3 on Figs 9 5(*a*) and (*c*) can be calculated from equation [9 1] Curves 3 on Figs 9·5(*b*) and (*c*) can be calculated by modifying equation [9 1] so as to include

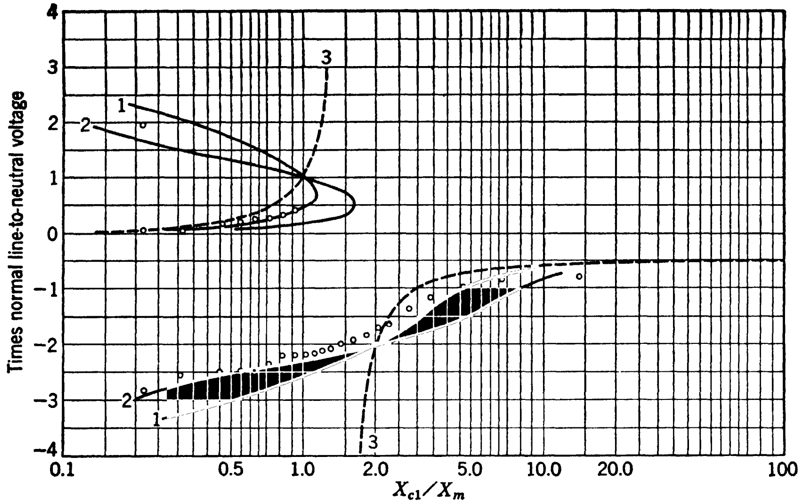

FIG. 9·5(*a*). Steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded. $C_1 = C_0$, transformers delta-connected on line side and delta- or wye-connected on load side; curve 1, saturation curve 1, Fig. 9·4; curve 2, saturation curve 2, Fig. 9·4; curve 3, no saturation. o, test points, miniature system with saturation curve 1, Fig. 9·4.

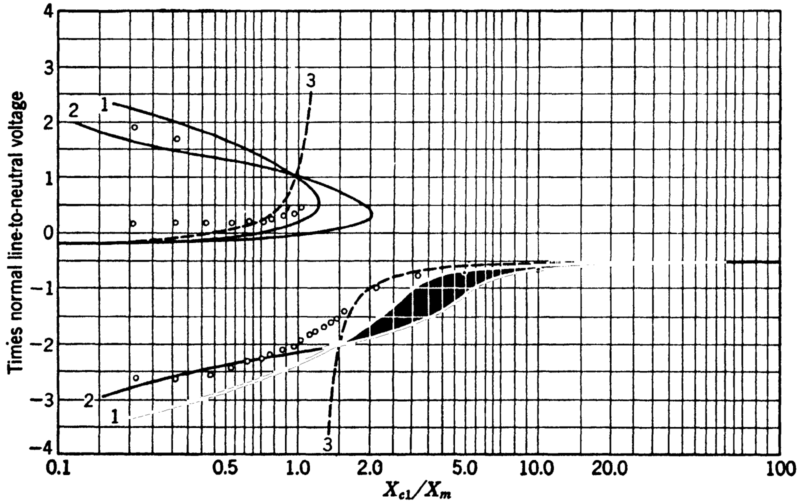

FIG. 9·5(*b*). Steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded. $C_1 = 2C_0$, transformers delta-connected on line side and delta- or wye-connected on load side; curve 1, saturation curve 1, Fig. 9·4; curve 2, saturation curve 2, Fig. 9·4; curve 3, no saturation. o, test points, miniature system with saturation curve 1, Fig. 9·4.

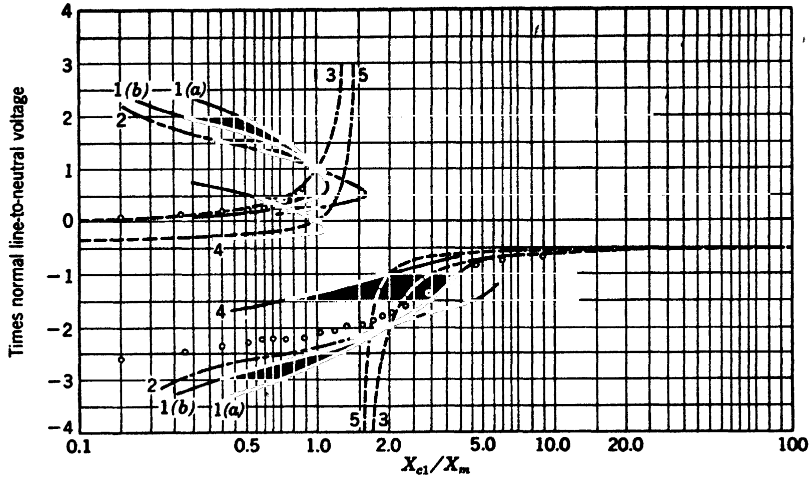

FIG. 9·5(c). Steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded. $C_1 = C_0$, transformers wye-connected on line side, wye- and delta-connected on load side. Curve 1, saturation curve 1, Fig. 9·4 (a) wye-wye, (b) wye-delta; curve 2, saturation curve 2, Fig. 9·4, wye-wye; curve 3, no saturation; curve 4, neutral displacement voltage, saturation curve 1, Fig. 2, connection wye-wye; curve 5, neutral displacement voltage, no saturation. o, test points, miniature system with saturation curve 1, Fig. 9·4. Connection wye-delta.

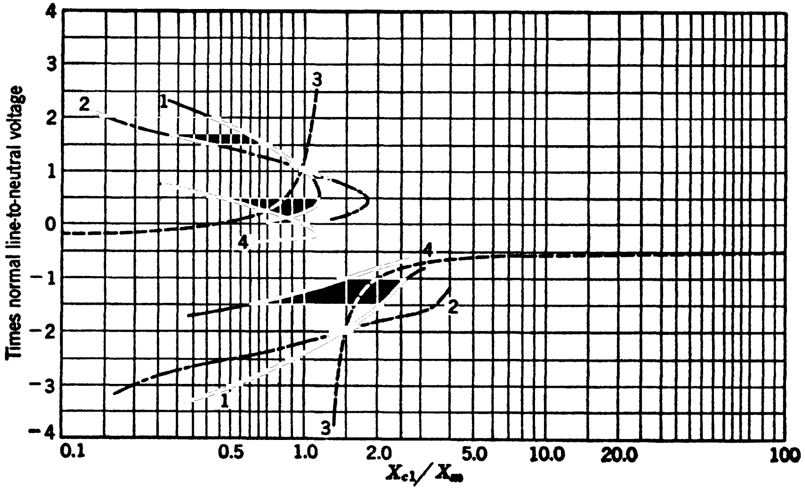

FIG. 9·5(d). Calculated steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded. $C_1 = 2C_0$, transformers wye-connected on line side and delta-connected on load side; curve 1, saturation curve 1, Fig. 9·4; curve 2, saturation curve 2, Fig. 9·4; curve 3, no saturation; curve 4, neutral displacement voltage, saturation curve 1, Fig. 9·4.

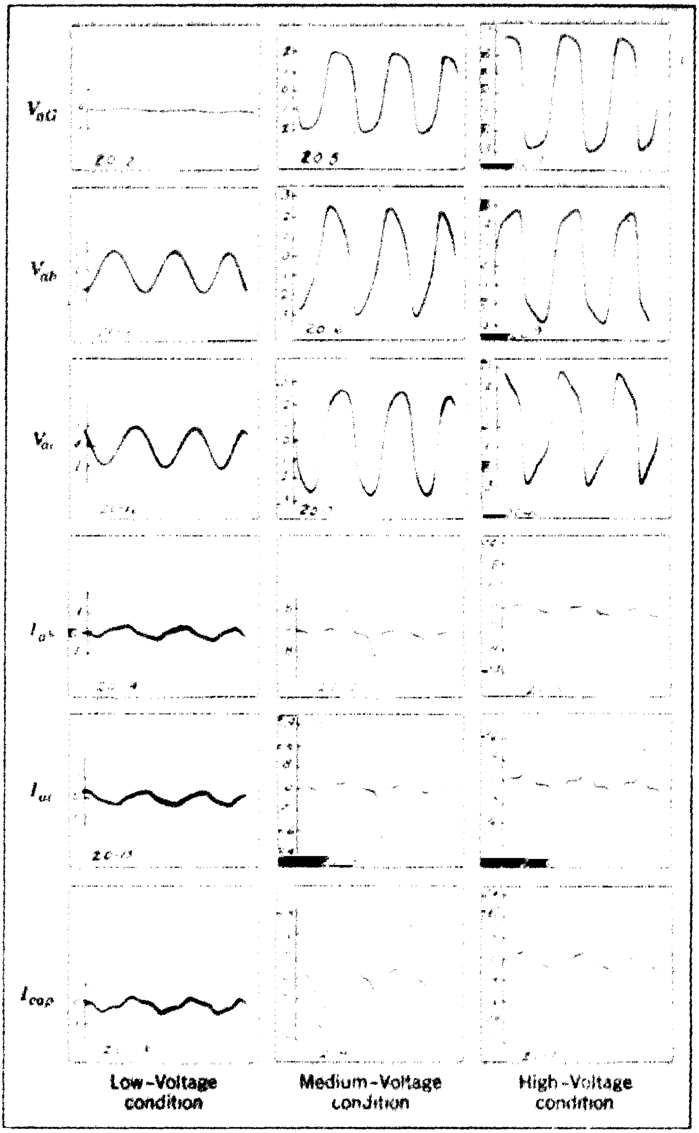

FIG. 9·6. Representative oscillograms, delta-connected load transformer; $C_0 = C_1$, 135 volts line-to-line; $X_{c1}/X_m = 0.212$. Calibrations: current, times normal crest transformer exciting current; voltage, times normal line-to-neutral crest voltage. Low-, medium-, and high-voltage conditions can be identified by referring to Fig. 9·5(a) at $X_{c1}/X_m = 0.212$.

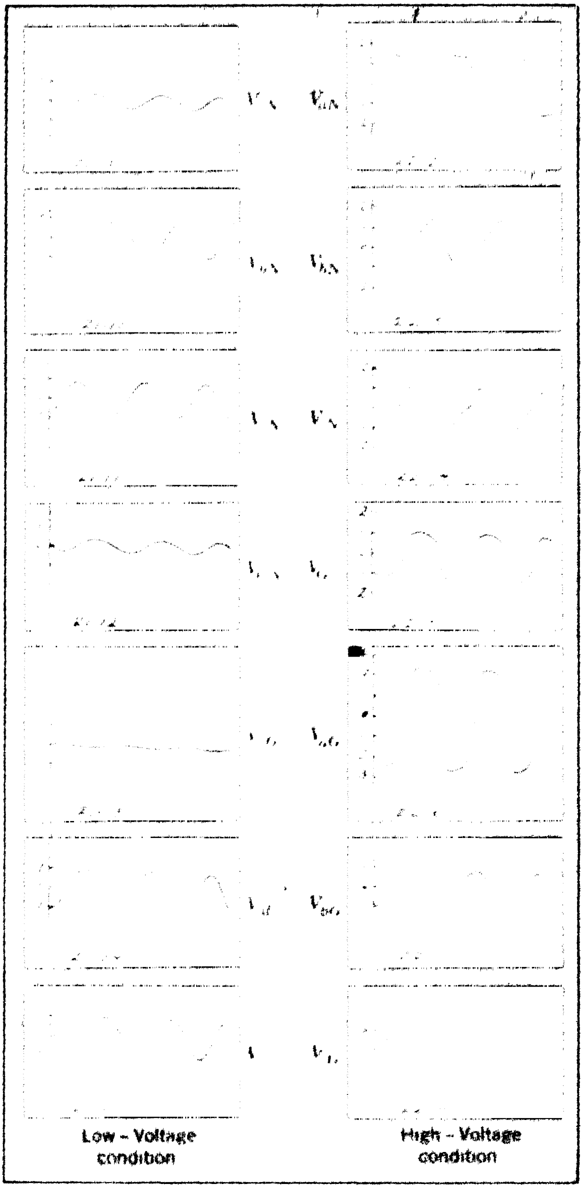

FIG. 9·7. Representative oscillograms, wye-connected load transformer; $C_0 = C_1$; 120 volts line-to-line; $X_{c1}/X_m = 0.15$. Calibration: times normal line-to-neutral crest voltage. Low- and high-voltage conditions can be identified by referring to Fig. 9·5(c) at $X_{c1}/X_m = 0.15$.

$\frac{2}{3}(C_1 - C_0)$ in parallel with $\frac{3}{2}X_m$ in the equivalent circuit of Fig. 9·1(*c*) as was pointed out in Section 9·3.

2. Three-phase transformers.

a. Delta-connected on the line side, wye-connected on the load side. Figure 9·8(*a*) gives miniature-system values of the voltage to ground of the open phase for both core-type and shell-type transformers, delta-connected on the line side and wye-connected on the load side with saturation curves 3 and 4 of Fig. 9·4 with $C_1 = C_0$. Calculated values of voltage for saturation curves 3 and 4 of Fig. 9·4 are not given, but they would not differ materially from those for saturation curve 1 of Fig. 9·4 given by curve 1 of Fig. 9·5(*a*). In general, the curves of Fig. 9·8(*a*) agree with those of Fig. 9·5(*a*), except that the medium voltage of the three calculated voltage conditions, indicated by calculations for values of X_{c1}/X_m in the neighborhood of 1 and below, does not appear. Instead, for X_{c1}/X_m lower than 0.7, higher voltages than indicated by calculations appear in some cases, giving three stable voltage conditions. These high voltages have pronounced subharmonics and therefore could not be predicted by the methods used. However, they are but little higher than the calculated voltages of curve 1, Fig. 9·5(*a*).

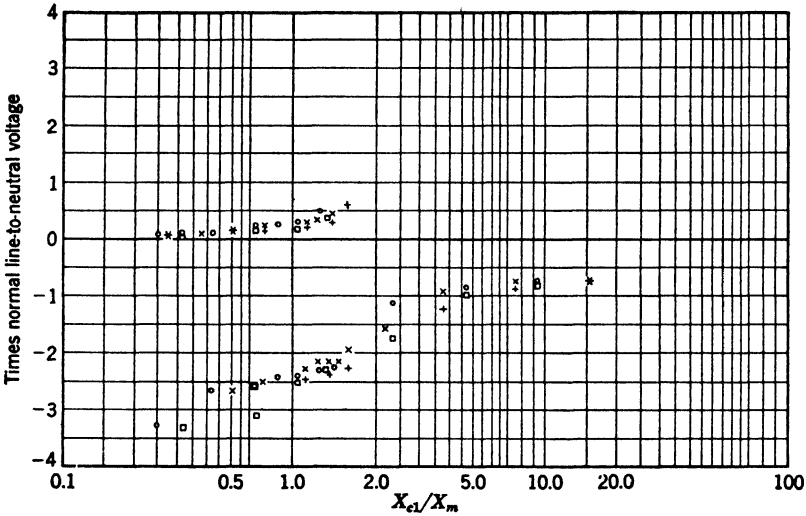

FIG. 9·8(*a*). Steady-state voltages on open phase with one conductor open; no fault on system; power source grounded; $C_1 = C_0$; three-phase load transformer connected line delta, load wye, ungrounded. Normal line-to-line voltage = 91 volts for core-type, 83 volts for shell-type. ×, shell-type; middle leg unopened. +, shell-type; end leg unopened. ○, core-type; middle leg unopened. □, core-type; end leg unopened.

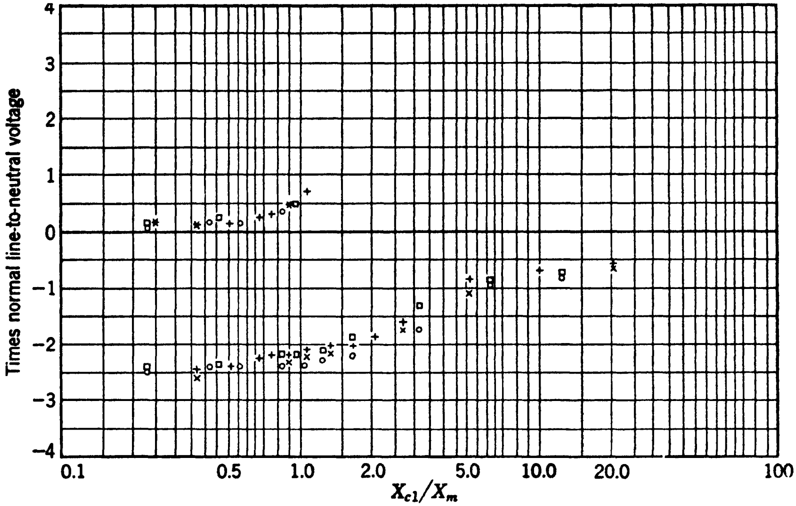

FIG. 9·8(*b*). Steady-state voltages on open phase with one conductor open; no fault on system; power source grounded; $C_1 = C_0$; three-phase load transformer connected line wye, load delta, ungrounded. Normal line-to-line voltage = 79 volts for core-type, 72 volts for shell-type. ○, core-type; middle leg opened. □, core-type; end leg opened. ×, shell-type; middle leg opened. +, shell-type; end leg opened.

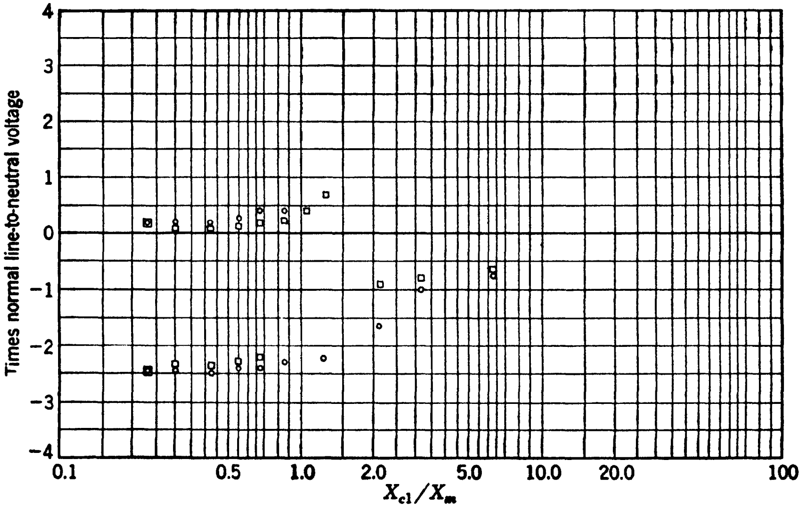

FIG. 9·8(*c*). Steady-state voltages on open phase with one conductor open; no fault on system; power source grounded; $C_1 = 2C_0$; three-phase load transformer connected line wye, load delta, ungrounded. Normal line-to-line voltage = 79 volts for core type. ○, core-type; middle leg opened. □, core-type; end leg opened.

b. Wye-connected on line side, delta-connected on load side. Figs. 9·8(*b*) for $C_1 = C_0$ and 9·8(*c*) for $C_1 = 2C_0$ are similar to Fig. 9·8(*a*) and serve to indicate that the transformer connections have little effect on the phenomena.

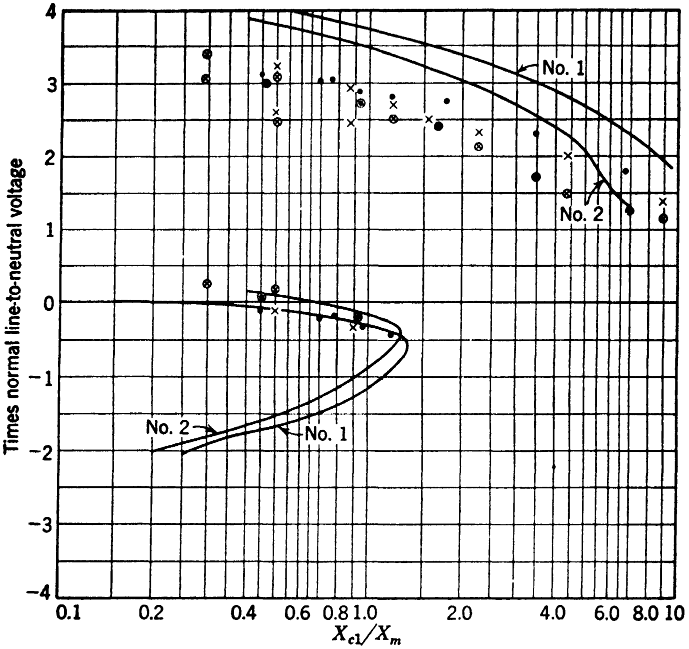

FIG. 9·9. Steady-state voltages on open phase with two conductors open. No fault on system. Power source solidly grounded. Curve 1, calculated using curve 1, Fig. 9·4 for $C_1 = C_0$ and transformers delta-connected on line side and wye-connected on load side. Curve 2, same but $C_1 = 2C_0$. Test points as follows, miniature system using saturation curve 1, Fig. 9·4, single-phase transformers: •, $C_1 = C_0$, transformers delta line, wye load; ⊙, $C_1 = 2C_0$, transformers delta line, wye load; ×, $C_1 = C_0$, transformers wye line, wye or delta load; ⊗, $C_1 = 2C_0$, transformers wye line, wye or delta load.

B. Two phases open—no fault.

Although numerous factors were investigated in connection with this condition as outlined in the summary of cases studied, detailed results will not be shown. Instead, a composite curve showing the range of values obtained for all the conditions studied using single-phase transformers is shown in Fig. 9·9. The narrow range of values obtained indicates that the transformer connection and its construction (core, shell, or single-phase) have little effect on the overvoltages obtained. Figure 9·10 shows some typical oscillograms for one case with single-phase transformers.

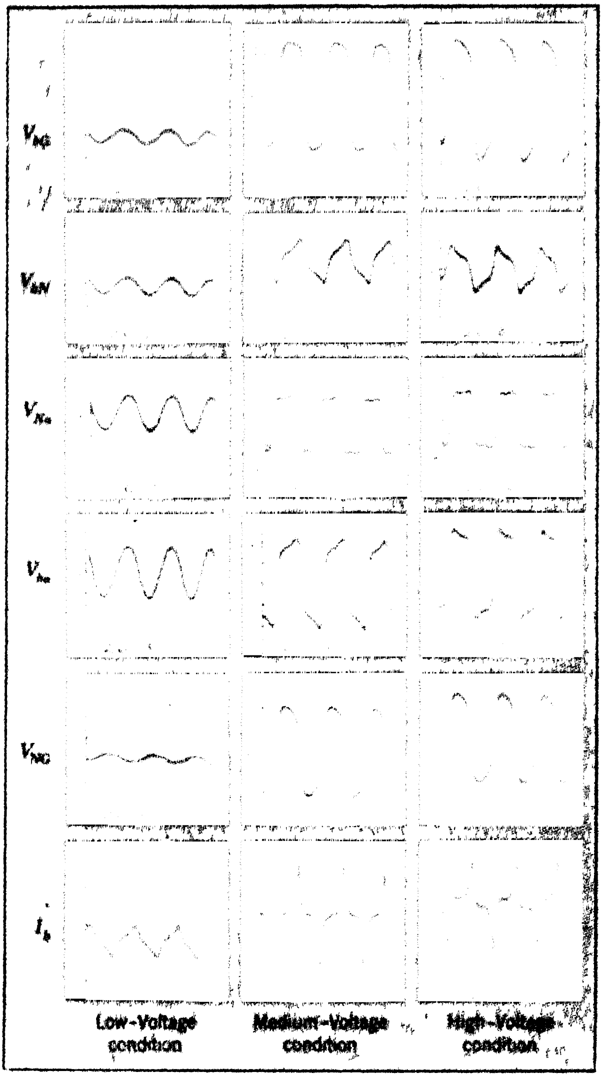

FIG. 9·10. Representative oscillograms, wye-connected load transformer; $C_0 = C_1$; $X_{c1}/X_m = 0.89$. Phases b and c open. Calibrations: current, times normal crest transformer exciting current; voltage, times normal line-to-neutral crest voltage. Low-, medium-, and high-voltage conditions can be identified by referring to Fig. 9·9 at $X_{c1}/X_m = 0.89$.

II. Power source ungrounded—load transformers ungrounded.

A. One phase open—no fault.

1. Single-phase transformers.

a. Delta-connected on line side, wye- or delta-connected on load side. Figure 9·11 gives the calculated voltages to ground of the open phase on the load side of the opening in per-unit of normal peak line-to-neutral voltage with X_{c1}/X_m as abscissa and X_{c0}'/X_{c1} as parameter for $C_1 = C_0$. Test points from the miniature system with saturation curve 1 of Fig. 9·4 for various values of X_{c0}'/X_{c1} are indicated. The voltages are independent of X_{c1}'.

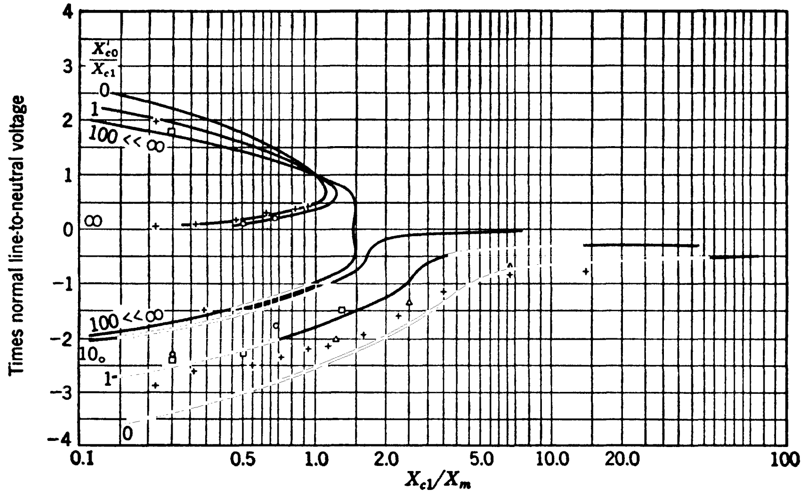

Fig. 9·11. Steady-state maximum voltages on open phase on load side with one conductor open. No fault on system. Power source ungrounded. $C_1 = C_0$, transformers delta-connected on line side and wye-connected on load side. Test points, miniature system with saturation curve 1, Fig. 9·4: +, $X_{c0}'/X_{c1} = 0$; △, $X_{c0}'/X_{c1} = 0.1$; □, $X_{c0}'/X_{c1} = 0.5$; ○, $X_{c0}'/X_{c1} = 1.0$; ×, $X_{c0}'/X_{c1} = 10$.

B. One conductor open—fault to ground on the system side of the open conductor—single-phase transformers.

Figure 9·12 shows the range of voltages obtained for the various transformer connections. Here again the transformer connections make little difference except for very low ratios of X_{c1}/X_m. The wye-delta connection tends to reduce the overvoltages somewhat in this region.

C. One conductor open—line-to-ground fault on the load side of the open conductor—single-phase transformers delta-connected on line side—wye- or delta-connected on load side.

Figure 9·13 shows the voltages obtained for this assumed condition. It should be noted that the severity of this system from the standpoint of overvoltages can be defined for all practical purposes by the ratio of total zero-sequence capacitive reactance to transformer magnetizing reactance. The ratio of X_{c0}'/X_{c0} has relatively little effect. This will be made clearer in a discussion later in this chapter of the equivalent circuits applying for this case.

If the total capacitive reactance of the system is large (capacitance on both sides of the opening), no high voltages are produced. As this reactance is decreased, voltages are increased. Only one voltage value is obtained until the ratio of total zero-sequence system capacitive reactance to magnetizing reactance, $X_{(c0+c0')}/X_m$, reaches a value of about 5.0. Below this value, three stable voltage conditions were obtained in some cases. As the capacitive reactance was decreased, increasingly higher maximum voltages were obtained. With a relatively low loss transformer (curve 1 of Fig. 9·4), the high-voltage condition could be obtained for values of $X_{(c0+c0')}/X_m$ down to 0.03, although the low-voltage condition occurred more often. In other words, the higher the

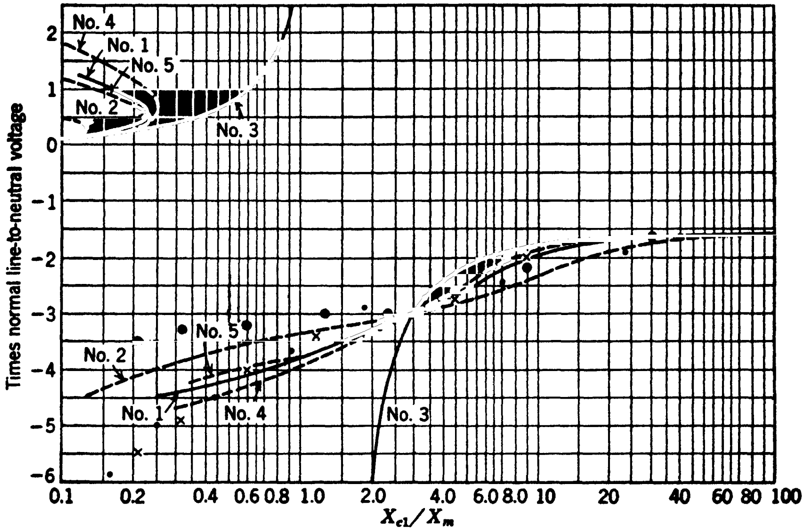

FIG. 9·12. Maximum steady-state voltages-to-ground with one conductor open. Line-to-ground fault on source side of open conductors. System neutral isolated. $C_1 = C_0$. These results are independent of C_1' and C_0'. No load on system. Single-phase transformers delta-connected on line side, delta- or wye-connected on load side: curve 1, calculated voltages based on saturation curve 1, Fig. 9·4; curve 2, calculated voltages based on saturation curve 2, Fig. 9·4; curve 3, calculated voltages based on no saturation (●, test points, miniature-system saturation curve 1, Fig. 9·4, single-phase transformers, wye-connected on line side); curve 4, calculated voltages based on saturation curve 1 of Fig. 9·4 with wye-connected load side; curve 5, same as curve 4, but delta-connected load side. (⊙, test points, miniature-system saturation curve 1, Fig. 9·4, delta-connected load side; ×, same but wye-connected load side.)

voltage, the less would be the probability of reaching that voltage. With the transformers (curve 5 of Fig. 9·4) which had slightly higher losses, the high-voltage condition did not occur for values of $X_{(c0+c0')}/X_m$ below 0.2, approximately. This elimination refers only to the steady-state condition, however. In many cases, long transient conditions were observed, which could be quite hazardous to insulation or connected apparatus. It is important to bear this in mind in connection with the protective criteria offered in the next section of this chapter.

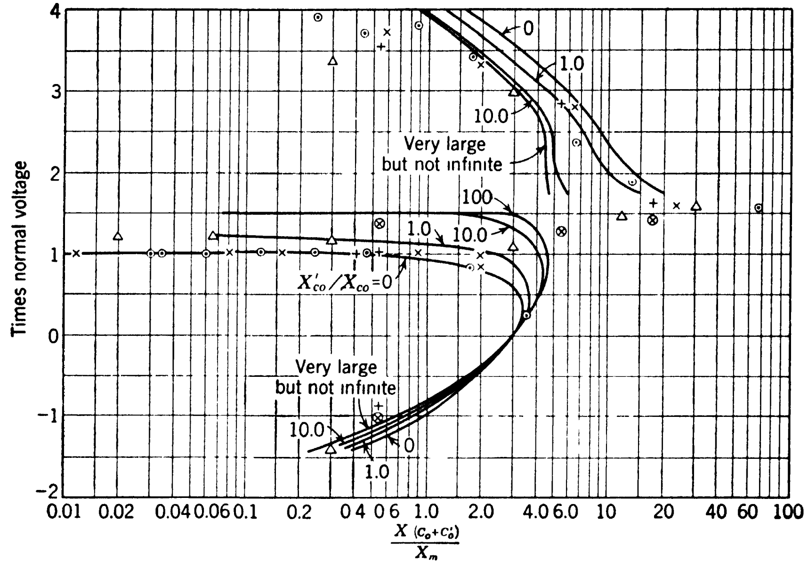

FIG. 9·13. Isolated-neutral system—steady-state voltages on open phase with one conductor open. Fault-to-ground on load side of open conductor. Curves calculated using saturation curve 1 of Fig. 9·4. ⊙, test points for $X_{c0}'/X_{c0} = 0$ and saturation curve 1, Fig. 9·4. Test points, saturation curve 5, Fig. 9·4: ×, $X_{c0}'/X_{c0} = 0$; +, $X_{c0}'/X_{c0} = 0.1$; △, $X_{c0}'/X_{c0} = 1.0$; ⊗, $X_{c0}'/X_{c0} = 10$.

Figure 9·13 indicates that phenomena such as these can take place in a relatively large percentage of ungrounded systems. It is likely that the occurrence of faults followed by the blowing of fuses may account for much of the heretofore obscure transient phenomena occurring in isolated-neutral systems.

9·5 CRITERIA FOR LIMITING OVERVOLTAGES

The manner of connecting the windings of an ungrounded transformer bank does not materially affect the open-conductor phenomena. Furthermore, the type of transformer, that is, either core-type or shell-type, has little effect.

The method of calculating sustained voltages-to-ground, when one or two conductors of the three-phase system supplying an unloaded, ungrounded transformer bank are open, gives values that approximately check tests made on the miniature system. Except where the calculated voltages are less than normal system voltage, the method gives values that with few exceptions in the very high-voltage region are too high. The method can therefore be considered to give conservative results.

Although tests points of maximum peak voltage agree quite well with calculated results, it may be well to point out that, for some of the conditions which gave rise to very high voltages, it was possible to obtain variations in the wave shape, depending on the initiating conditions. For instance, the high voltage might, for one opening of the switch, be essentially of fundamental frequency. For a second opening of the switch, it might be essentially of the same magnitude but contain pronounced subharmonics. Some of these subharmonics were of very low frequency.

For the neutral-grounded system or the isolated-neutral system effectively grounded through its zero-sequence capacitance, a fault-to-ground on the open conductor, or conductors, on the system side of the opening or openings does not affect the sustained voltages at the transformer bank terminals. A fault-to-ground on the transformer side of the opening reduces the voltage there to zero. The highest voltages are obtained when two conductors are open and there is no fault. Such a condition can exist when a line-to-line fault is cleared by the blowing of two fuses; and also on a circuit controlled by single-pole switches when one phase is closed before the other two.

If the peak value of the sustained voltage-to-ground of the open conductor on the load side of the opening in a solidly grounded system with ungrounded transformers bank is to be kept below $\sqrt{3}$ times normal line-to-neutral peak voltage, the approximate length of line, l, between the opening and the transformer bank should not exceed a value that can be approximately determined from the normal voltage of the system and the rated kilovolt-amperes and exciting current of the transformer.

The positive-sequence capacity susceptance of overhead transmission circuits, $2\pi f C_1$, in micromhos per mile at 60 cycles varies from about five to seven, depending on the diameter of conductors and the spacing between them. Since these, in turn, depend on voltage, $2\pi f C_1$, at a given voltage and frequency, can be estimated with fair accuracy. The following values will be taken as typical at 60 cycles.

Kv	$2\pi fC_1$ Micromhos per Mile
230–115	5.2
69–34.5	5.5
34.5–13.8	6.0
Below 13.8	6.5

The zero-sequence capacitance of overhead transmission circuits is less than the positive but usually greater than one-half of the positive.

From Fig. 9·9, it is seen that calculated voltages in the region of $\sqrt{3}$ times normal line-to-neutral voltage are greater than test voltages. Allowing for this, and with $C_1 > C_0 > C_1/2$, voltages above $\sqrt{3}$ times normal will probably not occur with transformers of the usual degree of saturation if the ratio of X_{c1}/X_m is 6.0 or greater. With a criterion of $X_{c1}/X_m = 6$,

$$\frac{X_{c1}}{X_m} = I_m X_{c1} = I_m \frac{10^6}{2\pi fC_1 l} \times \frac{\text{kva}}{\text{kv}^2\ 10^3} = 6$$

$$\therefore l = I_m \frac{\text{kva} \times 10^3}{\text{kv}^2 \times 6(2\pi fC_1)} \qquad [9\cdot3]$$

where l = allowable length of line in miles on the transformer side of the open conductor, kva = rated kilovolt-amperes total of the transformers on the transformer side of the open conductor, kv = normal rms line-to-line voltage of the circuit in kilovolts, I_m = average per-unit rms exciting current at normal voltage, $2\pi fC_1$ = positive-sequence capacity susceptance of the circuit in micromhos per mile.

The transformer exciting current I_m lies between 1 per cent and 7 per cent, with 3.5 per cent not an unusual value. Figure 9·14, drawn with I_m = 3.5 per cent, gives the approximate maximum allowable length of line at various voltages supplying a transformer bank of given kilovolt-ampere rating and 3.5 per cent average rms magnetizing current which can be operated by fuses or single-pole switches without the risk of voltages-to-ground exceeding line-to-line voltages when two conductors are open. This allowable length of line varies directly as the per-unit rms exciting current of the bank and inversely as the ratio X_{o1}/X_m selected. For an rms exciting current of 7 per cent, the allowable lengths of line would be doubled; for an exciting current of 1 per cent, they would be only about one-third those of Fig. 9·14.

Since for some cases the maximum allowable length of line is very short, it may be necessary under such conditions to take into account the internal capacitance of the transformer bank itself. Generally this will not have to be done. For cases requiring this refinement, representative values of internal capacitance of power transformers are given in a convenient form by L. V. Bewley.[18] In Fig. 8 of Bewley's paper the capacitance is given between windings per phase which may be considered as the capacitance to ground, since the capacitance of the low-voltage winding to ground is usually much larger, whereas the capacitance of the high-voltage winding to ground is small. The largest values of capacitance given are in the order of 0.01 microfarad which would be equivalent to one mile of line, or less under most conditions. For the higher voltages, the internal capacitance would be equivalent to considerably less than a mile of overhead line. Bushing capacitance, being very small relatively, can be neglected.

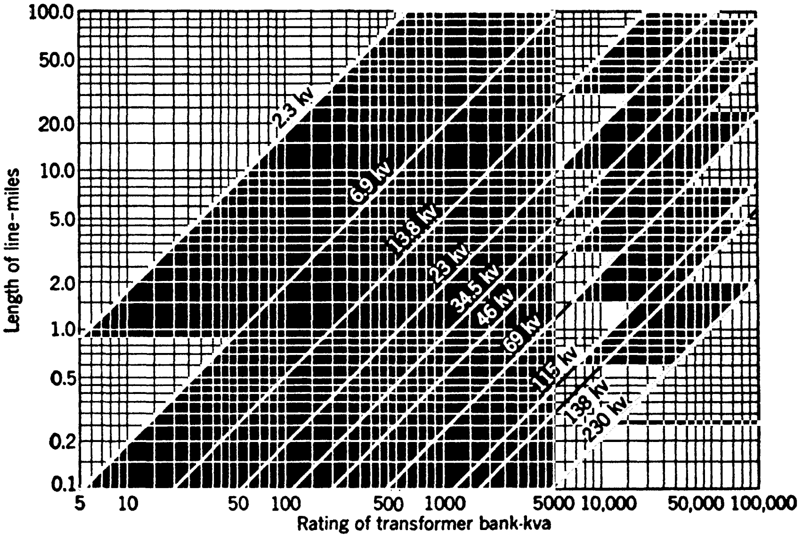

FIG. 9·14. Solidly grounded system—approximate maximum allowable length of line between fuses or single-pole switches and load transformer to limit line-to-ground voltage on open phase to $\sqrt{3}$ times normal, with two conductors open. $X_{c1}/X_m = 6$; $I_m = 3.5$ per cent rms average exciting current. Note: Allowable length of line varies directly with rms average exciting current.

When the transmission circuit contains some cable, its equivalent in terms of miles of overhead line may be used. For that purpose, Fig. 9·15 has been prepared which makes it possible to evaluate this equivalent readily.

For the ungrounded system, it is important to note that the criterion for the solidly grounded system is adequately conservative for conditions not involving faults. See Fig. 9·11. However, should a permanent fault occur on the system side, the criterion must be changed if voltages are to be limited to maximum values of $\sqrt{3}$ times normal. Figure 9·12 shows that a minimum ratio of $X_{c1}/X_m = 25.0$, approximately, is required. This criterion alone is not sufficient, however. If a permanent fault on the load side is assumed, then the total zero-sequence capacitance of the system must be very large or else very small in order to prevent overvoltages. See Fig. 9·13. Therefore, in addition to imposing the criterion of $X_{c1}/X_m \geqq 25.0$, restrictions must also be imposed on the ratio of X_{c0}'/X_m.

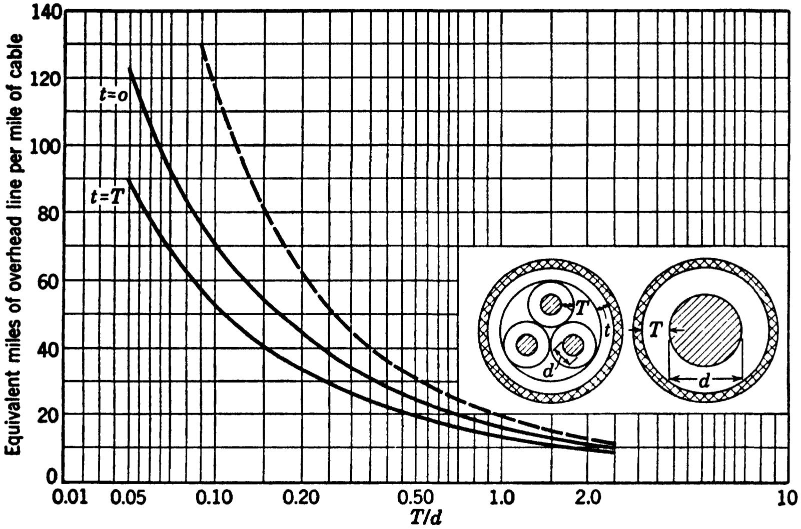

FIG. 9·15. Overhead line capacitance equivalent of cables. Dashed curve, single conductor. Solid curves, three conductor. For paper-insulated cables assumed dielectric constant 3.8. For varnished-cambric or rubber insulation multiply by 1.35. Overhead line equivalent based on positive-sequence capacity susceptance of 6.0 micromhos per mile.

From Fig. 9·13, the ratio of $X_{(c0+c0')}/X_m$ should be less than approximately 0.04 or greater than a value varying from 4.7 to 25.0 approximately, depending on the ratio X_{c0}'/X_{c0}. In the light of our first restriction on X_{c1}/X_m this can be conservatively interpreted to impose the restriction that X_{c0}'/X_m must be less than 0.04 or greater than 40.0, approximately.

Figure 9·14 can be used to interpret the first criterion in terms of total transformer kilovolt-amperes, system voltage, and maximum allowable lengths of line on the transformer side of the open conductor location. This figure is based on a ratio of $X_{c1}/X_m = 6.0$, and, since for the isolated system this must be increased to approximately 25.0, it is necessary to divide the miles of line shown by approximately 4.0 in order to conform to the required criterion.

In Fig. 9·16 the second criterion, imposing the restrictions on the lengths of line on the system side of the open conductor, is shown. This curve shows that overvoltages may be expected as a result of faults and subsequent fuse blowing for wide ranges of system constants.

In applying these criteria, it should be pointed out that these are approximate values. Occasionally it may be necessary to make a more detailed calculation. Exact line configuration, exact magnetizing reactance, and more detailed knowledge concerning the particular transformer saturation characteristics may be necessary. However, the curves serve to indicate in general the range of system constants that can give rise to high overvoltages in isolated-neutral systems. Under lightly loaded conditions, it would seem that a relatively high percentage of isolated-neutral systems are subject to overvoltages as a result of fuse blowing or single-pole switching under permanent fault conditions.

Obviously, there are other combinations of faults and open conductors possible in isolated systems which may tend to shift somewhat the range of constants causing high voltages. In distribution circuits involving single-phase feeders, a single blown fuse can sometimes cause overvoltages if the system constants are of certain values. Generally, such instances can be interpreted in terms of three-phase constants so that some one of the numerous curves included in this chapter can be used directly in determining the overvoltages possible.

The effect of load on the transformers is to reduce the voltages on the open phases obtained with the transformers unloaded. With the transformers loaded and phase a open, V_a, the voltage of the open phase on the transformer side of the opening referred to the voltage on the system side, will have a quadrature component in addition to the positive or negative in-phase component. If the in-phase component is negative and greater than -0.5, V_{a2} (negative-sequence voltage) will be greater than V_{a1} (positive-sequence voltage) and there will be phase reversal. Running motors, however, will not slow down if the difference between the positive- and negative-sequence torques is equal to that required to maintain rotation in the positive direction, but, if it is less, they will slow down and stop. They will then reverse

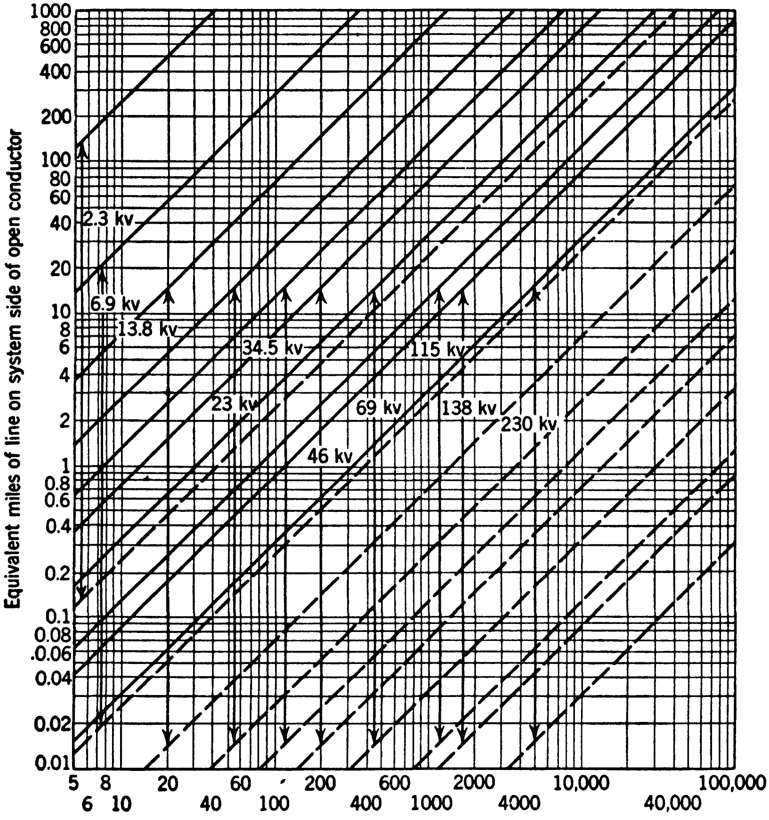

FIG. 9·16. Isolated-neutral system—approximate range of equivalent line lengths on system side of fuse or single-pole switch location which may cause voltages greater than $\sqrt{3}$ times normal with one conductor open and a single line-to-ground fault on the load side. X_{c0}'/X_m must be greater than 40.0 or less than 0.04; I_m = 3.5 per cent rms average exciting current. Note 1: Allowable length of line varies directly with rms average exciting current. Note 2: This criterion alone is not sufficient for elimination of overvoltages. The criterion of Fig. 9·14 must be met also. Maximum permissible miles of line in Fig. 9·14 should be divided by 4.0 for the isolated-neutral system.

their direction of rotation if the negative-sequence torque is great enough to overcome the positive-sequence torque and provide the torque required to start the motors from rest. With a motor running, the positive-sequence torque is greater than the negative for equal positive- and negative-sequence voltages. At standstill, these torques are equal for equal voltages.

The conditions under which loaded motors will reverse their direction of rotation will depend on the characteristics of the motors, their loads, and the voltages at their terminals, the latter being influenced by the presence of the motors and other loads on the transformers. Since reversal of the direction of rotation of motors has occurred in practice following the opening of a conductor, it is reasonable in view of the foregoing to assume that, the higher the negative voltage produced on the open phase with the transformer unloaded, the more apt is reversal of the direction of rotation of motors to occur.

9·6 CONCLUSIONS

1. In a three-phase power system, high sustained voltages may in some cases result from the opening of one or two conductors which separates ungrounded transformers from the rest of the system.
2. The transformer connections (wye or delta) and construction (single-phase, or three-phase, core- or shell-type) make little difference in the magnitudes of voltage or in the regions in which high voltages are encountered.
3. High overvoltages can be eliminated in grounded systems by grounding the neutrals of the load transformers.
4. The regions within which abnormal voltages are obtained are considerably extended by the effects of transformer saturation.
5. The regions wherein voltages in solidly grounded systems in excess of rated line-to-line voltages occur, and therefore of importance in the application of lightning arresters, are for the usual cases confined to ratios of X_{c1}/X_m less than three for one conductor open, and less than six for two conductors open.
6. To avoid the possibility of obtaining unduly high abnormal voltages in solidly grounded systems, single-pole switches or fuses should not be used where the length of line, or equivalent capacitance, between the break and ungrounded transformers supplying the load is greater than that given by equation [9·3]. Figure 9·14 can be used directly when the rms exciting current is 3.5 per cent.
7. The highest voltages appear in an ungrounded system with the simultaneous occurrence of an open conductor and line-to-ground fault, such as might correspond to the breaking and falling to ground of a line conductor.

8. From the standpoint of open conductors without faults, the criterion given in conclusion 6 is always conservative for ungrounded systems. However, if the criteria are to include the effects of permanent faults, then it is necessary to impose two approximate requirements, namely, X_{c1}/X_m must be greater than 25.0, and X_{c0}'/X_m must be either less than 0.04 or greater than 40.0. Figures 9·14 and 9·16 can be used when the rms exciting current is 3.5 per cent.

9. The voltages arising from open conductors can be calculated with reasonable accuracy by consideration only of the fundamental-frequency components of voltage and current as outlined in the following section.

9·7 EQUIVALENT CIRCUITS AND METHODS OF CALCULATION

Perhaps the most fruitful method of gaining insight into the phenomena associated with open conductors including effects of saturation is to construct the equivalent circuits for some typical cases. When such circuits have been constructed, it becomes relatively easy to visualize the behavior of circuits which has been discussed in the preceding sections. Furthermore, such an approach lends itself well to an extension of these studies to include numerous other similar nonlinear circuits. Although calculations could be made on a cut-and-try basis,[1] a graphical approach which will be outlined permits more rapid accumulation of data and has the additional advantage of affording a clearer insight concerning the fundamental phenomena.

In Fig. 9·3(*b*) let the following assumptions be made:

1. The sustained voltages to be determined are substantially sinusoidal voltages of fundamental frequency.

2. The effective exciting impedance of the transformers is a reactance that can be determined from the saturation curve of the transformer, determined by applying sinusoidal voltages. The saturation curves as in Fig. 9·4 therefore are interpreted to express the relationship between fundamental-frequency voltage and equivalent fundamental-frequency current. The actual impedance is then a ratio of rms voltage to rms current and is a *constant* for a *given rms voltage.* The actual impedance can be expressed as a constant X_m multiplied by a factor δ which is defined as:

$$\delta = \frac{\text{times normal rms voltage}}{\text{times normal rms exciting current}}$$

3. Resistance in the transformers and in the transmission circuit can be neglected.

Consider, then, the isolated-neutral system with one conductor open, and no fault, again referring to Fig. 9·3(*b*). If C_0 on phase *a* is thought of as a load impedance which is to be inserted between *a* and ground, then the entire system can readily be replaced (by Thévenin's theorem) by the impedance looking in at the terminals *a* and ground with the capacity C_0 in phase *a only* equal to zero. The voltage energizing this circuit is the voltage that exists on phase *a* with C_0 in phase *a only* equal to zero. Using the normal voltage on phase *a* as reference, this open-circuit voltage is $-\frac{1}{2}E_a'$. The impedance is ob-

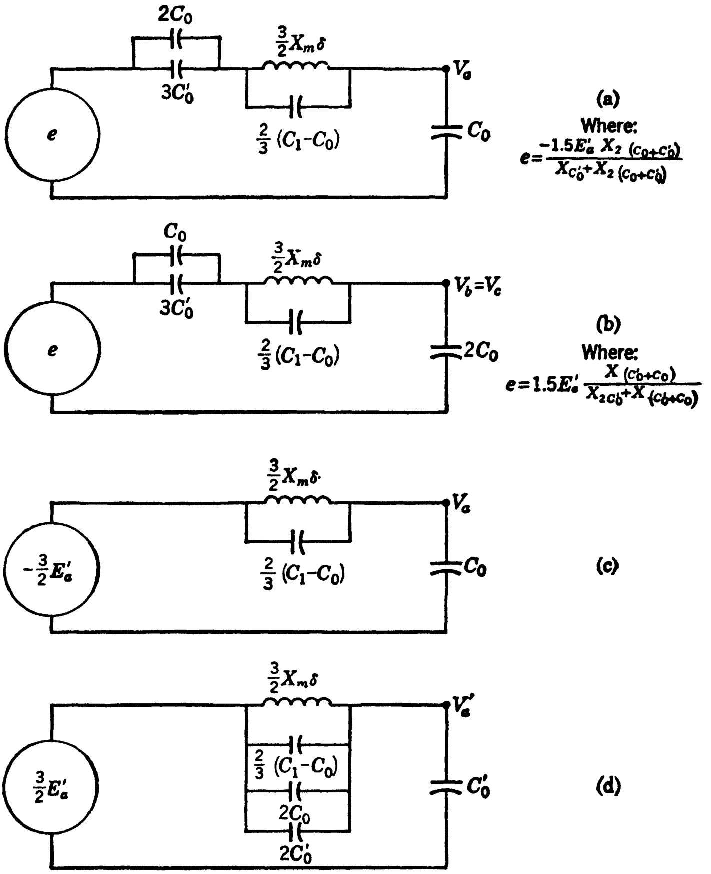

FIG. 9·17. Equivalent circuits for some isolated-neutral system open conductor cases. (*a*) One conductor open, no fault. (*b*) Two conductors open, no fault. (*c*) One conductor open, fault on source side. (*d*) One conductor open, fault on load side.

tained by looking back into the system at a, and the entire circuit is as shown in Fig. 9·17(a). The voltage across the capacitor C_0 in this circuit gives the voltage V_a on the open conductor on the load side of the switch in Fig. 9·3(b) for the assumed conditions.

It is interesting to point out that, if C_0' is sufficiently large so that it can be replaced by a short circuit and if $C_1 = C_0$, then Fig. 9·17(a) reverts to Fig. 9·1(c), as it should. The only difference is the factor δ. Similarly Fig. 9·17(b) reverts to Fig. 9·2(c). In other words, if C_0' is sufficiently large, then the source side may be thought of as being solidly grounded through capacitance.

This factor δ is a dimensionless saturation coefficient and is introduced so that the effects of saturation can be more clearly seen and taken into account. It is a function of (1) the transformer saturation characteristic, (2) the voltage V_a, (3) the transformer connections, and (4) the circuit under investigation. If there is no saturation, then $\delta = 1.0$ for all values of V_a and any transformer connection and conventional linear circuit theory applies. If there is saturation, then for a given saturation characteristic, given circuit under study, and given transformer connections δ is a function of V_a only.

As a detailed example, consider the simple case included in Fig. 9·17(a). The voltage V_a can be written directly.

$$V_a = e\left[\frac{-jX_{c0}}{-jX_{c0} - j\dfrac{(X_{c0}/2)(X_{c0'}/3)}{(X_{c0}/2) + (X_{c0'}/3)} - \dfrac{jX_{\frac{2}{3}(c_1-c_0)}(j\frac{3}{2}X_m\delta)}{-jX_{\frac{2}{3}(c_1-c_0)} + j\frac{3}{2}X_m\delta}}\right]$$

$$= e\left[\frac{X_{c0}/X_m}{(X_{c0}/X_m) + (A/X_m) + \dfrac{X_{\frac{2}{3}(c_1-c_0)}(\frac{3}{2}\delta/X_m)}{\frac{3}{2}\delta - (X_{\frac{2}{3}(c_1-c_0)}/X_m)}}\right] \qquad [9\cdot4]$$

where

$$A = \frac{(X_{c0}/2)(X_{c0}'/3)}{(X_{c0}/2) + (X_{c0}'/3)}$$

If $C_1 = C_0$ and C_0' becomes very large, $e = \frac{1}{2}E_a'$ and the solidly grounded case of Fig. 9·1(a) is obtained. In equation [9·4], A/X_m becomes zero and the last term in the denominator becomes $-\frac{3}{2}\delta$. Therefore,

$$V_a = -\frac{1}{2}E_a'\left(\frac{X_{c0}/X_m}{(X_{c0}/X_m) - \frac{3}{2}\delta}\right)$$

$$= E_a'\left(\frac{X_{c0}/X_m}{3\delta - 2(X_{c0}/X_m)}\right) \qquad [9\cdot5]$$

which checks equation [9·1] except for the saturation factor δ. In order to evaluate the saturation factor, the relation between V_a and the voltage across the transformer windings must be known. The vector diagram of Fig. 9·1(*b*) applies, and, if a delta-connected winding is assumed, the voltages $V_{ab} = V_{ac}$ must first be determined since they are the voltages impressed across the saturable elements. The following equation expresses this relationship.

$$|V_{ab}| = |V_{ac}| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(V_a + \frac{1}{2}\right)^2} = \sqrt{1 + V_a + V_a^2} \qquad [9\cdot6]$$

This follows from the vector diagram of Fig. 9·1(*b*). From equation [9·6] and Fig. 9·4 the following table can be constructed.

TABLE 9·1

V_a in P.U.	$V_{ab} = V_{ac}$ in P.U. Based on		δ (From Fig. 9·4)	
$= V_a/E_a'$	*L-N Voltage*	*L-L Voltage*	Curve 1	Curve 2
3.0	3.60	2.08	0.08	0.0345
2.5	3.12	1.805	0.112	0.0545
2.0	2.65	1.532	0.264	0.100
1.5	2.18	1.262	0.600	0.268
1.0	1.73	1.00	1.00	1.00
0.5	1.33	0.769	1.34	1.60
0	1.0	0.578	1.46	1.90
−0.5	0.866	0.500	1.5	2.00
−1.0	1.0	0.578	1.46	1.90
−1.5	1.33	0.769	1.34	1.60
−2.0	1.73	1.00	1.00	1.00
−2.5	2.18	1.262	0.600	0.268
−3.0	2.65	1.532	0.264	0.100
−3.5	3.12	1.805	0.112	0.0545
−4.0	3.60	2.08	0.08	0.0345

If, now, δ is plotted as in Fig. 9·18 for curves 1 and 2 as a function of V_a and on this same figure is plotted a family of curves of V_a from equation [9·5] with X_{c0}/X_m as a parameter, the *intersections* are the solutions for V_a that are sought. It is to be observed that as many as three intersections may occur for one given value of X_{c0}/X_m. These cases correspond to the triple values of voltage found in many cases as shown in the miniature-system results. Results taken from Fig. 9·18 will be found to check those plotted in Fig. 9·5(*a*) for saturation corresponding to curves 1 and 2 of Fig. 9·4.

Similarly other cases can be analyzed graphically. In all cases, the proper family of curves of V_a as a function of δ with appropriate

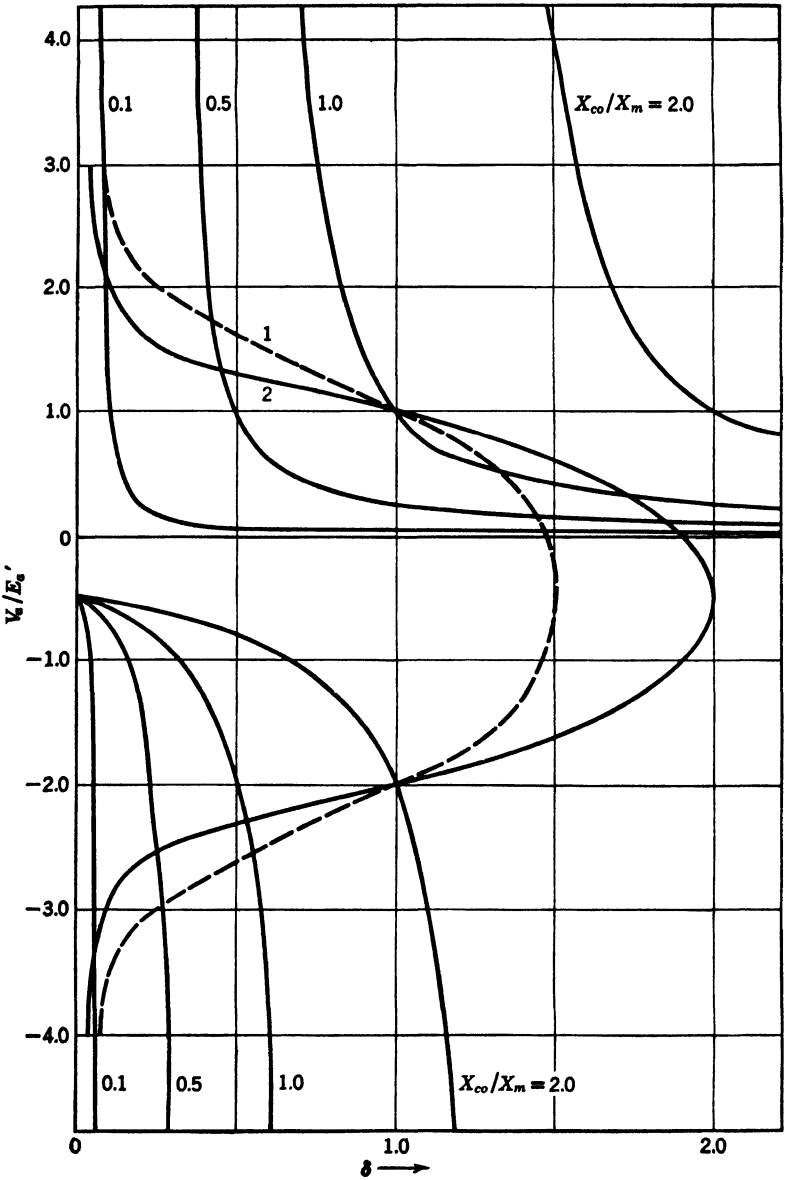

FIG. 9·18. Graphical solution for grounded neutral source, one conductor open, $C_1 = C_0$, and delta-connected transformer. The intersections are the solutions sought and are plotted in Fig. 9·5(*a*). Curves 1 and 2 from Table 9·1; other curves from equation [9·5].

variation in system constants as a parameter should be calculated. Then δ should be evaluated as a function of V_a plotted on the same figure. The solutions sought will be given by the intersections of one family of curves with the other.

The stable solutions can, in general, be recognized by applying the criterion

$$\frac{(\partial V_a/\partial \delta)_{\text{system}} - (\partial V_a/\partial \delta)_{\delta}}{V_a} > 0 \qquad [9\cdot 7]$$

to the family of curves such as in Fig. 9·18 for each point of intersection. Equation [9·7] defines the points of intersection that are stable solutions. The first term in the numerator is simply the slope of the V_a versus δ curve as plotted from equation [9·5] at the intersection under test. The second term in the numerator is similarly the slope of the V_a versus δ curve 1 or 2 of Fig. 9·18 at the same intersection under test. V_a is the voltage at the intersection under test. This criterion, defined by equation [9·7], has been found very useful in identifying stable and unstable solutions to a variety of non-linear circuit problems.

If this criterion is applied to Fig. 9·18, it will be found that all the high positive values of V_a will be unstable whereas the low positive values and high negative values will be stable, as was generally found to be true for the miniature-system results. There were exceptions, arising from the fact that harmonics are neglected in this entire analysis. In the miniature system the harmonics were present, and undoubtedly this fact accounts for discrepancies between miniature-system results and results obtained with equation [9·7] as a criterion. It is beyond the scope of this analysis to include the effects of these harmonics.

9·8 OTHER EXAMPLES

a. Open Conductors in Ground-Fault Neutralizer Systems.[15] If in Fig. 9·3(*b*), a reactance is inserted between neutral N' and ground, being properly tuned to $3(C_0' + C_0)$, the system is known as a ground-fault neutralizer system. Such systems have the inherent ability to clear automatically single-line faults to ground by virtue of the fact that, losses being neglected, the impedance viewed from any conductor is infinite, and therefore no fault current can flow when a fault occurs.

In such systems, while the *parallel* impedance (zero-sequence capacitive reactance and ground-fault neutralizer inductive reactance) is infinite, the *series* impedance is zero. Therefore any means which can generate a series voltage in this circuit may produce high voltage. It is well known that in unbalanced circuits, currents of one sequence produce voltage drops of another. Therefore it is important in Ground-Fault Neutralizer systems to keep the line impedances balanced among all three phases by appropriately transposing overhead conductors. With early air core Ground-Fault Neutralizers, this was a troublesome factor in connection with Ground-Fault Neutralizer application and operation. Present-day Ground-Fault Neutralizer applications are made, the system being carefully considered from the standpoint of transpositions and then generally detuned (GFN reactance slightly less than $X_{c0}/3$) so that the system is only slightly out of resonance. This is no way impairs the ability of the system to clear line-to-ground faults automatically.

Because of the resonant nature of the zero-sequence circuit it is generally undesirable to use any single-pole interrupting devices in such systems. The reason becomes clear when the system of Fig. 9·3(*b*) is equipped with a ground-fault neutralizer connected between N' and ground. From the technique used in the foregoing sections, the equivalent circuits for calculating neutral voltage V_N' with one and two conductors open are shown in Fig. 9·19. The circuits are obtained as before by replacing the two-terminal network between neutral and ground by the voltage existing at the neutral e_n (with the neutral open) in series with the impedance Z_N viewed from the neutral (with the neutral open). This gives a simple two-terminal network to replace the entire system as far as the neutral voltage V_N' is concerned for either one or two conductors open.

Although it will be impossible to investigate all the possibilities of these equations here, it should be pointed out that, in general, in Fig. 9·19 Z_N will be a capacitive impedance which is comparable in magnitude to X_N which is inductive. If, also, the open conductor is near the source so that X_{c0}' approaches infinity, then $e_n = \frac{1}{2}E_a'$ and this voltage acting in a series circuit tuned to fundamental frequency gives rise to overvoltages limited principally by saturation in the iron core of X_N.

Similarly, if two conductors are open, e_n will act in a series circuit tuned to the fundamental frequency, and saturation of the ground-fault neutralizer will be an important factor in limiting the overvoltages. It is principally for reasons disclosed by this analysis that single-pole switches and fuses are not generally used in ground-fault neutralizer systems, even though overvoltages may be limited by saturation as mentioned or by overvoltage arresters.

b. Single-Pole Switching of Grounding Banks. When a grounding bank is opened in one or two phases, overvoltages may be encountered also. The equivalent circuits for one and two conductors open are shown in Fig. 9·20. Again the circuit is fundamentally a capacitance in series with a saturable reactor so that trouble may be expected over a large range of system constants. For this reason fuses and single-

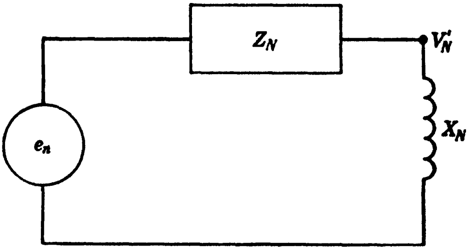

FIG. 9·19. Equivalent circuit for calculating V_N', the voltage at the neutral for one and two open conductors. Refer to Fig. 9·3(*b*) for identification of constants.

1. With one open conductor (phase *a*)

$$e_n = E_a'\left(\frac{\tfrac{1}{2}X_{c0}' - B}{X_{c0}' + B}\right)$$

$$Z_N = \frac{X_{c0}'B}{X_{c0}' + B}$$

$$V_N' = e_n\frac{X_N}{X_N + Z_N}$$

where

$$B = \frac{X_{2(C_0'+C_0)}(X_{c0} + Z)}{X_{2(C_0'+C_0)} + X_{c0} + Z}$$

$$Z = \frac{\dfrac{X_2}{3}(C_1 - C_0)\left(\dfrac{3}{2}X_m\right)}{\dfrac{3}{2}X_m - \dfrac{X_2}{3}(C_1 - C_0)}$$

2. With two conductors open (phases *b* and *c*)

$$e_n = E_a'\left(\frac{D - X_{2c0}'}{D + X_{2c0}'}\right)$$

$$Z_N = \frac{X_{2c0}'D}{X_{2c0}' + D}$$

where

$$D = \frac{X_{(c0+c0')}(X_{2c0} + Z)}{X_{(c0+c0')} + X_{2c0} + Z}$$

Z and V_N' are as under (1).

pole switches should not be used with grounding banks unless a careful study reveals that the zero-sequence capacitive reactance and the magnetizing reactance bear such a relation to each other that overvoltages cannot occur. In Fig. 9·20(*b*), it should be pointed out that to obtain actual voltages V_b and V_c, the existence of a quadrature component of $-j(\sqrt{3}/2)E_a'$ on phase *b* and $+j(\sqrt{3}/2)E_a'$ on phase *c* should be included in the analysis.

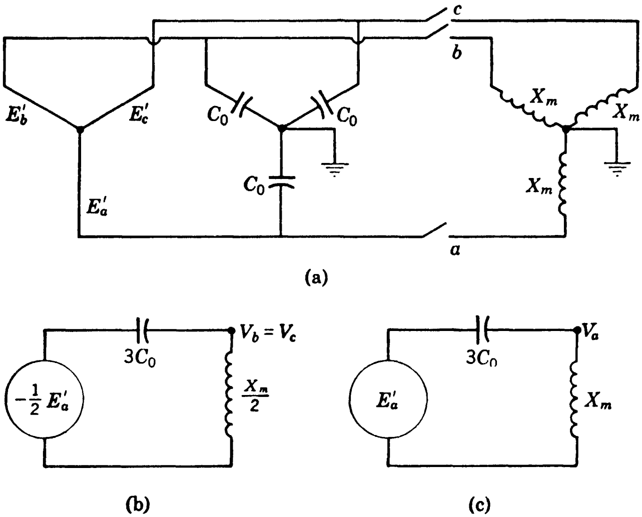

FIG. 9·20. Essential elements of isolated-neutral system with one and two phases of grounding bank open. (*a*) Three-phase circuit; (*b*) one phase open (phase *a*); (*c*) two phases open (phases *b* and *c*).

c. Unbalanced Load and Open Conductors. Unbalanced load on a three-phase system may cause overvoltages and phase reversal under certain conditions. Only the general equations will be derived here, and the equivalent circuits shown.

In Fig. 9·21 is shown the basic circuit under discussion. The impedances Z_{ab} and Z_{ac} are unequal. Any impedance between *b* and *c* will have no effect, and so it is not shown. The circuit represented by Z_1, Z_2 is any three-phase load in which positive- and negative-sequence impedances Z_1 and Z_2 are not necessarily equal.

Following the procedure of replacing a source by the open-circuit voltage in series with appropriate impedance viewed from the open-circuited terminals, consider everything to the left of Z_1, Z_2 a source that is to be replaced by sources of positive- and negative-sequence

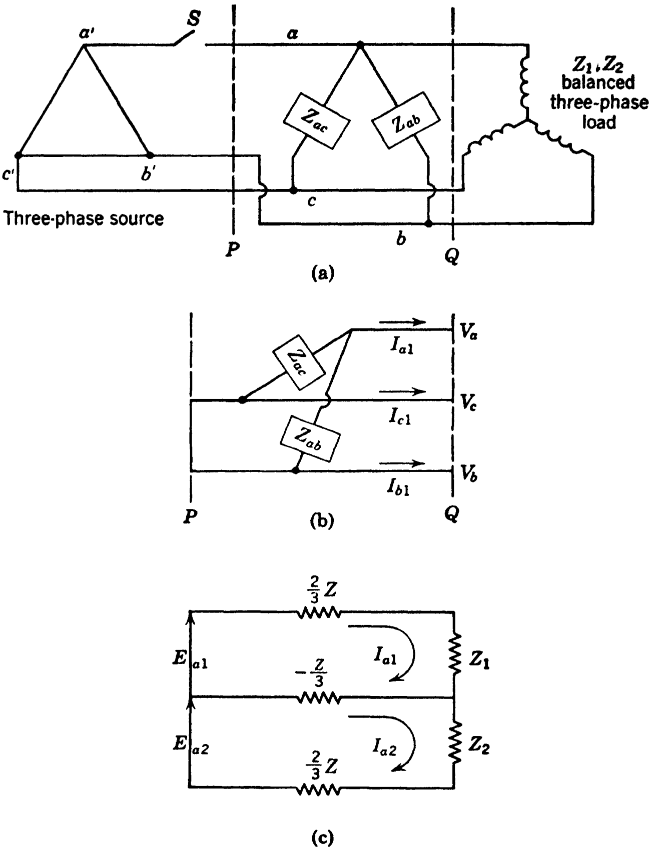

FIG. 9·21. (*a*) Essential elements of three-phase system supplying an unbalanced load with one open conductor. (*b*) Circuit for determining sequence impedances between P and Q. (*c*) Equivalent circuit for (*a*).

voltages in series with appropriate positive- and negative-sequence impedances. Let the voltage E_{cb} (the voltage from c to b) be the reference. The voltages on open circuit (balanced load removed) are

$$\begin{aligned} E_{cb} &= E_{cb} \\ E_{ac} &= E_{cb}\frac{-Z_{ac}}{Z_{ac}+Z_{ab}} \qquad [9{\cdot}8] \\ E_{ba} &= E_{cb}\frac{-Z_{ab}}{Z_{ac}+Z_{ab}} \end{aligned}$$

Resolving these three voltages in terms of their symmetrical components

$$\begin{aligned}
E_{cb1} &= \frac{E_{cb}}{3}\left(1 - \frac{aZ_{ac}}{Z_{ac}+Z_{ab}} - \frac{a^2Z_{ab}}{Z_{ac}+Z_{ab}}\right) \\
E_{cb2} &= \frac{E_{cb}}{3}\left(1 - \frac{a^2Z_{ac}}{Z_{ac}+Z_{ab}} - \frac{aZ_{ab}}{Z_{ac}+Z_{ab}}\right) \\
E_{cb1} &= \frac{E_{cb}}{3}\left[\frac{(1-a)Z_{ac} + (1-a^2)Z_{ab}}{Z_{ac}+Z_{ab}}\right] \\
E_{cb2} &= \frac{E_{cb}}{3}\left[\frac{(1-a^2)Z_{ac} + (1-a)Z_{ab}}{Z_{ac}+Z_{ab}}\right]
\end{aligned} \qquad [9\cdot 9]$$

The line-to-neutral components of these voltages are

$$\begin{aligned}
E_{a1} &= j\frac{E_{cb1}}{\sqrt{3}} \\
E_{a2} &= -j\frac{E_{cb2}}{\sqrt{3}}
\end{aligned} \qquad [9\cdot 10]$$

These are the line-to-neutral positive- and negative-sequence voltages which replace the source through sequence impedances yet to be determined. To determine these, consider Fig. 9·21(b). Here, with b and c shorted at P, it is only necessary to inquire what the voltages V_a, V_b, and V_c are with positive-sequence currents flowing.

$$\begin{aligned}
V_a &= -I_{a1}Z \qquad \text{where } Z = \frac{Z_{ab}Z_{ac}}{Z_{ab}+Z_{ac}} \\
V_b &= 0 \\
V_c &= 0
\end{aligned} \qquad [9\cdot 11]$$

From these,

$$\begin{aligned}
V_{a1} &= \frac{-I_{a1}Z}{3} \\
V_{a2} &= \frac{-I_{a1}Z}{3}
\end{aligned} \qquad [9\cdot 12]$$

and it is evident that, with only positive-sequence currents flowing, negative-sequence voltages occur also. Similarly, with negative-sequence currents only flowing,

$$V_{a1} = \frac{-I_{a2}Z}{3}$$
$$V_{a2} = \frac{-I_{a2}Z}{3} \qquad [9\cdot 13]$$

Therefore, from equations [9·10], [9·12], and [9·13], the general equations for replacing everything to the left of Q in Fig. 9·21(a) can be written. They are

$$E_{a1} = I_{a1}\frac{Z}{3} + I_{a2}\frac{Z}{3} + V_{a1Q}$$
$$E_{a2} = I_{a2}\frac{Z}{3} + I_{a1}\frac{Z}{3} + V_{a2Q} \qquad [9\cdot 14]$$

If V_{a1Q} and V_{a2Q} are replaced by their respective impedance drops in the balanced load to the right of Q, then

$$E_{a1} = I_{a1}\frac{Z}{3} + I_{a2}\frac{Z}{3} + I_{a1}Z_1$$
$$E_{a2} = I_{a2}\frac{Z}{3} + I_{a1}\frac{Z}{3} + I_{a2}Z_2 \qquad [9\cdot 15]$$

and the circuit of Fig. 9·21(a) can be represented by the relatively simple circuit of Fig. 9·21(c).

This circuit has been studied by a number of individuals.[16, 17] It has been shown clearly that the circuit for certain unbalanced load conditions may produce overvoltages on the open conductor, as well as phase reversal so that an induction motor load, for instance, could reverse its direction of rotation as a consequence of the opening of the conductor.

It is clear from the problems discussed in this chapter that open conductors in polyphase power systems, with or without faults, can give rise to overvoltages and, occasionally, phase reversal for wide ranges of system constants. Methods of calculation are generally tedious, but may yield satisfactory results. Frequently, physical insight, born of experience, is extremely helpful in determining whether or not open conductors can give rise to overvoltage troubles in a given system.

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10 SOME ADDITIONAL NON-LINEAR CIRCUITS

10·1 INTRODUCTION

There are numerous non-linear circuits which have been encountered and studied with varying degrees of thoroughness at different times. Some of these circuits cannot be analyzed at all by the fundamental-frequency approach used in the preceding chapter; others can be studied only to a limited extent by that approach. The miniature-system approach has proved extremely useful, therefore, as a tool in systematically studying several different non-linear circuits in detail. Results of several of these studies will be presented in this chapter. The results will be presented as far as practical in the form of generalized curves in such a way that they will be useful to many individuals who have occasion to be concerned with such phenomena but do not have the facilities for carrying out such detailed analyses.

10·2 WYE-GROUNDED POTENTIAL TRANSFORMERS

Grounded-neutral, wye-connected potential transformers with wye- or broken delta-connected secondaries have been applied to three-phase circuits, the neutrals of which are temporarily or permanently ungrounded. The broken delta secondary connection affords a means of ground-fault detection, the voltage across the break in the delta under such fault conditions being used to operate relays or otherwise indicate the presence of a ground fault.

Experience has shown that such a circuit is unstable. Although the arrangement gives correct indications of true faults, indications of faults may sometimes be obtained when no faults exist. These false indications may sometimes result in higher than normal fault-indicating voltages appearing across the break in the delta secondary. Sometimes higher than normal voltages appear on the system itself.

The phenomenon is caused by the inherent instability of a circuit of this type when the amount of capacitive reactance to ground and the magnetizing reactance of the transformers involved are in certain relative proportions.

The miniature system shown in Fig. 10·1 is the basic circuit studied. The reactance of the sine-wave generator was negligible compared with the external miniature-system impedances. Thus balanced sinusoidal line-to-line voltages were applied at the transformer terminals.

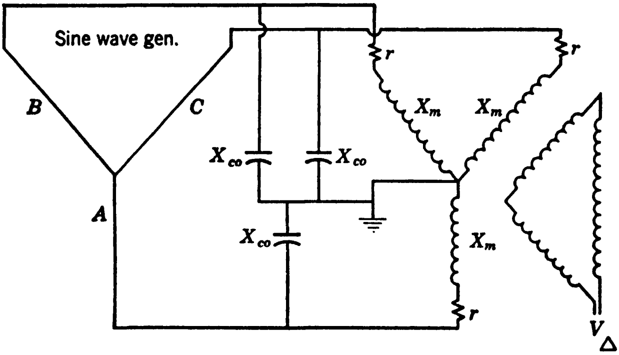


FIG. 10·1. Miniature system studied. X_{c0}, zero-sequence capacitive reactance; r, d-c resistance of transformer winding; X_m, ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E = 1.0$ in Fig. 10·2. Transformer turn ratio 1:1.

Consequently the phenomena were independent of positive-sequence capacitance so that only zero-sequence capacitive reactance X_{c0} was involved.

Transformers used were single phase and their saturation characteristics are shown in Fig. 10·2. Curve 1 is representative of most transformers and therefore was used throughout the study except for one case in which curve 2 was used to indicate the effect of more abrupt saturation. The value of X_m is, as in Chapter 9, the ratio of normal line-to-neutral rms sinusoidal exciting volts to normal-phase rms exciting current at normal frequency. The circuit, therefore, can be completely defined (neglecting losses) by the ratio X_{c0}/X_m when the saturation curve used and the normal operating voltage are specified.

Provision was made to control the excitation of the miniature-system sine-wave set so that system voltage could be varied. It was not deemed practical to consider operation at voltages higher than normal, as indicated on the saturation curves, since, in the system studied, a line-to-ground fault would result in extremely high magnetizing currents in the two unfaulted phases.

The study was made with and without load impedance in the break in the delta. This (neglecting saturation effects) is equivalent to putting load impedance across each phase in the case of wye-connected secondaries, the total ohmic resistance being the same in either case. Various impedances were used as burdens to determine the effect of impedance burden magnitudes and power factor, both lagging and leading.

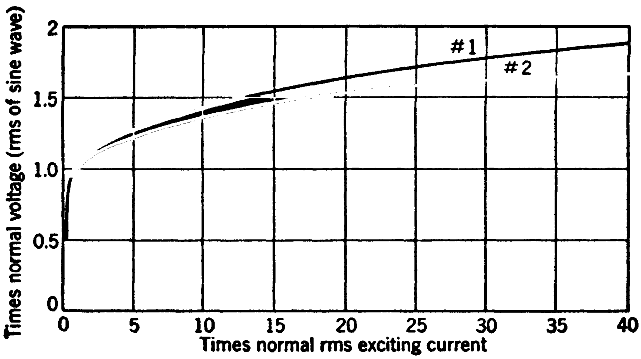


FIG. 10·2. Transformer saturation curves used in the miniature system. Curve 1, typical saturation curve; curve 2, curve having more saturation than curve 1.

In practically all cases studied, the abnormal conditions were obtained by suddenly energizing the miniature system or by applying and removing a fault-to-ground on one phase, although, in some instances, instability could be obtained merely by gradually raising the excitation of the sine-wave generator. However, the manner of obtaining the abnormal circuit conditions, as long as it is reasonable, is of secondary importance relative to the question whether abnormal behavior of the circuit is possible or not for given constants.

In the results as summarized here, it is sometimes true that regions of possible abnormality may overlap regions of possible normal circuit behavior. In other words, in regions of abnormal circuit behavior it is sometimes possible for the circuit to behave normally also, but it is never possible in a region designated as stable to have a sustained abnormal circuit condition of the type under discussion.

Regions of instability as defined by these results are regions wherein the abnormal conditions persist in the steady-state condition. It is possible for relatively long-time transients to exist near the borderline in the stable region. These will die out eventually, however, and normal conditions will always be restored ultimately.

For a particular set of transformer characteristics, the regions of neutral instability of the system shown in Fig. 10·1 are defined in terms of the ratio of system parameters X_{c0}/X_m and the system voltage E, where E is the per-unit system operating voltage. $E = 1.0$ corresponds to system operating voltage $E = 1.0$ on the saturation

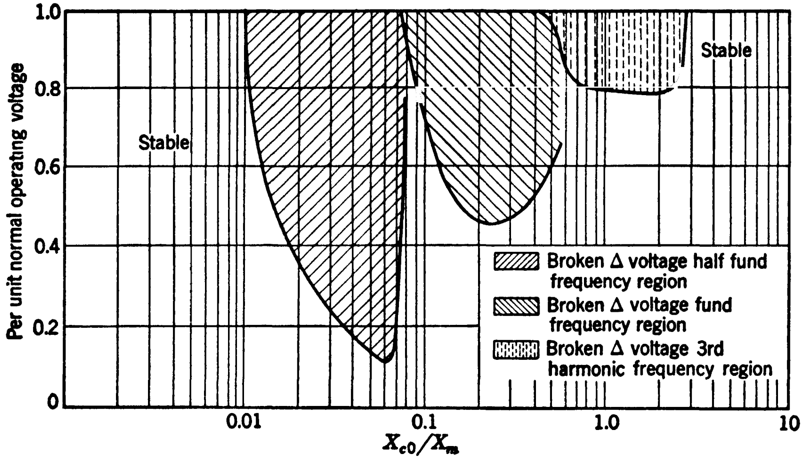


FIG. 10·3. Regions of possible instability of the circuit of Fig. 10·1 with transformer saturation corresponding to curve 1 of Fig. 10·2. $r/X_m = 0.0003$ for the transformer; r = d-c resistance of the winding; X_m is the ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E = 1.0$ in Fig. 10·2.

curves of Fig. 10·2. In the application of potential transformers as a means of ground-fault detection, it is customary to select a transformer rated at line-to-line system voltage, and operate it at system leg voltage; that is, $E = 0.58$. In some cases, however, circuits for which $E = 1.0$ may be encountered if in a normally grounded system the wye-grounded transformer bank is temporarily connected to an otherwise ungrounded but energized circuit, or if single-phase transformers are connected line-to-neutral in a three-phase system which has an open circuit in the neutral at the system grounding point.

The data of Fig. 10·3 were obtained using a low-loss transformer with typical saturation characteristics, curve 1, of Fig. 10·2. The regions outlined are those in which instability may occur as the result of a shock, such as a line-to-ground fault applied and removed.

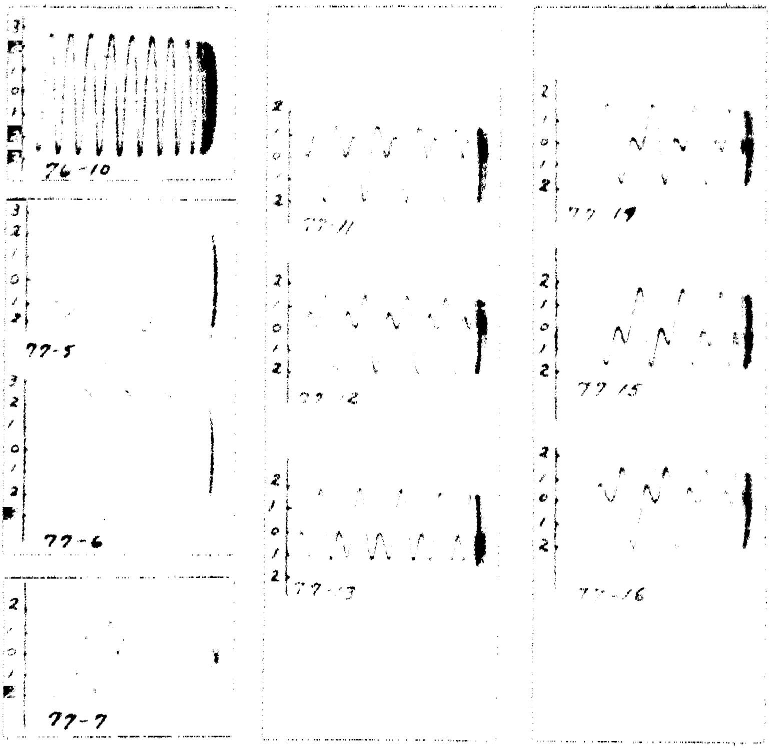
Stable operation may be obtained within portions of the unstable regions if system voltage is gradually applied or if the delta secondary is closed until normal steady-state conditions exist.

At each test value of X_{c0}/X_m the system voltage was raised to a value corresponding to $E = 1.0$. If the application and removal of a line-to-ground fault created an unstable condition as indicated by the presence of an abnormal voltage across the break in the delta, the system voltage was then reduced to the boundary value at which the unstable delta voltage collapsed.

As indicated on Fig. 10·3, neutral instability for this circuit may occur in one of three distinct modes, depending on the values of X_{c0}/X_m and E. For values of X_{c0}/X_m less than 0.01, no instability can occur for values of E less than 1.0. For values of X_{c0}/X_m between 0.01 and 0.07, corresponding to relatively large values of zero-sequence capacitance, the unstable voltage appearing across the broken delta is approximately of one-half normal frequency. Intermediate values of X_{c0}/X_m (between 0.07 and 0.55) give rise to a fundamental-frequency delta voltage, and large values of X_{c0}/X_m (between 0.55 and 2.8) give rise to a third-harmonic delta voltage.

The first of these three abnormal conditions is illustrated by the oscillograms in Fig. 10·4(a). For $X_{c0}/X_m = 0.026$ and $E = 0.58$, oscillogram 76-10 shows the broken delta voltage V_Δ with a single line-to-ground fault on phase a. This, of course, is of fundamental frequency and approximately three times normal leg voltage. Oscillogram 77-5 shows the transient V_Δ to the same time and amplitude scale following the removal of the fault on phase a at a normal current zero without any subsequent restriking in the fault. It will be observed that this transient voltage is approximately one-half fundamental frequency. Oscillogram 77-6 shows, with the same calibration, the ultimate steady-state voltage appearing across the break in the delta. It can be seen that this also is approximately one-half fundamental frequency, of very good wave form, and essentially of the same amplitude as though a line-to-ground fault were on the system.

This phenomenon is referred to as being essentially of one-half frequency. Actually it differs from one-half fundamental frequency by a small amount so that line-to-neutral voltages vary with time at a very low beat frequency. Oscillogram 77-7 shows the transient recovery voltage on phase a. Oscillograms 77-11, 77-12, and 77-13 show the steady-state line-to-neutral voltages on phases a, b, and c, respectively. Oscillograms 77-14, 77-15, and 77-16 show the same three respective voltages at a later time to illustrate how slowly the voltages-to-ground are shifting, owing to the "slip" frequency involved giving rise to a beat frequency component of voltage. Phase voltages reach a maximum of two times normal line-to-neutral crest for this case. Transformer currents may be very high for this condition because of the presence of this low-frequency component of voltage as will be shown later.



(a)

Fig. 10·4(a). Oscillograms illustrating the abnormal conditions giving rise to one-half normal frequency broken delta voltages and balanced phase voltages-to-ground. $X_{c0}/X_m = 0.026$, Fig. 10·3. $E = 0.58$. See text for identification of oscillograms. Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer.

The fundamental-frequency phenomenon is illustrated by the oscillograms in Fig. 10·4(b) for $X_{c0}/X_m = 0.123$ and $E = 0.58$. Oscillograms 32-1, 32-2, and 32-3 show steady-state voltages to ground on

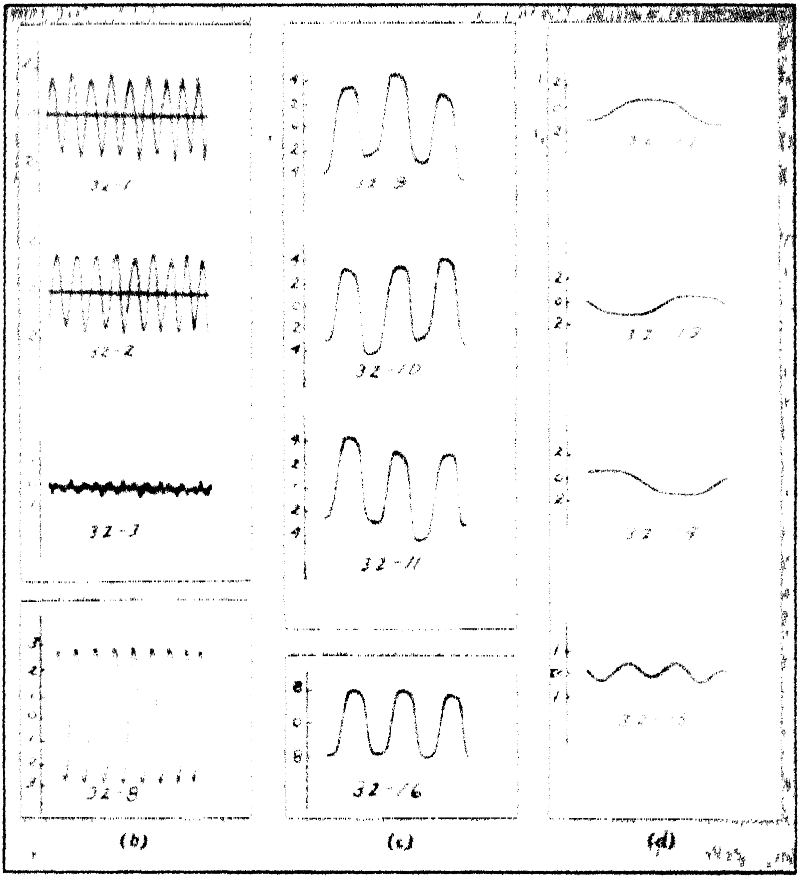


FIG. 10·4(*b*). Oscillograms illustrating the abnormal condition giving rise to fundamental frequency broken delta voltages and unbalanced phase voltages-to-ground. $X_{c0}/X_m = 0.123$, Fig 10·3; $E = 0.58$ See text for identification of oscillograms. Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer.

FIG. 10·4(*c*). Oscillograms illustrating the abnormal condition giving rise to third-harmonic broken delta voltages and balanced phase voltages-to-ground. $X_{c0}/X_m = 0.594$, Fig 10·3; $E = 1.0$. See text for identification of oscillograms. Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer.

FIG. 10·4(*d*). Oscillograms illustrating the normal condition of some third-harmonic voltage across the delta break with balanced voltages-to-ground. $X_{c0}/X_m = 0.594$, Fig. 10·3; $E = 1.0$. See text for identification of oscillograms. Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer.

phases *a*, *b*, and *c*, respectively, whereas oscillogram 32-8 shows the sustained voltage appearing across the break in the delta. This condition is very nearly the same as if a fault existed on phase *c* to ground. The voltage on phase *c* is very low—in fact, the fundamental-frequency component appears to be practically zero. Voltages-to-ground on phases *a* and *b* are essentially line-to-line voltages of fundamental frequency, and the broken delta voltage gives every indication of a fault although there is no fault on the system. The presence of subharmonics of one-half and one-ninth frequency are evident in the oscillograms.

The third-harmonic phenomenon is illustrated in Fig. 10·4(*c*) for $X_{c0}/X_m = 0.594$ and $E = 1.0$. Oscillograms 32-9, 32-10, and 32-11 show voltages-to-ground (one cycle of fundamental frequency) on phases *a*, *b*, and *c*, respectively, whereas oscillogram 32-16 shows the voltage appearing across the break in the delta. The broken delta voltage is of third-harmonic frequency with good wave form and of higher magnitude than that obtained for a fault on the system. Voltages-to-ground on all three phases are the same in magnitude (about four times normal line-to-neutral crest). There are no subharmonics present in this case.

The normal condition for the same system ($X_{c0}/X_m = 0.594$) and operating voltage ($E = 1.0$) is illustrated by the oscillograms in Fig. 10·4(*d*). Although the system constants are identical with those for the case discussed in the preceding paragraph, the voltage conditions are quite different, corresponding to "normal" behavior of the circuit. As shown previously, circuits involving saturable impedance elements may give rise to voltage and current conditions that are not single-valued functions of the circuit parameters. For this case ($E = 1.0$ and $X_{c0}/X_m = 0.94$), as in many others that could be used as illustrations, either of these conditions may be obtained, the one "abnormal" and the other "normal." Oscillograms 32-12 to 32-14, inclusive, show the balanced voltages-to-ground on phases *a*, *b*, and *c*, respectively, whereas oscillogram 32-15 shows the broken delta voltage. A third-harmonic component is in evidence in this case also, but it is only a very small fraction of that present for the abnormal condition illustrated in oscillograms 32-9 to 39-11, inclusive, and oscillogram 32-16. It is also much lower in magnitude than that obtained for the condition of a fault on the system.

Dividing lines between these three distinct modes of instability are not always very clear-cut. For example, in the vicinity of $X_{c0}/X_m = 0.01$ but in the unstable region, one-quarter frequency components of voltage were in evidence, although the one-half frequency components predominated. Other subharmonics were also present, sometimes of very low frequency, in this portion of the unstable region.

In the vicinity of $X_{c0}/X_m = 0.08$, the unstable phenomenon was mainly a combination of fundamental frequency and one-half fundamental frequency. The dividing line might be considered as an indication of where for increasing values of X_{c0}/X_m the phenomenon changes gradually from something predominantly of one-half fundamental frequency to something of predominantly fundamental frequency.

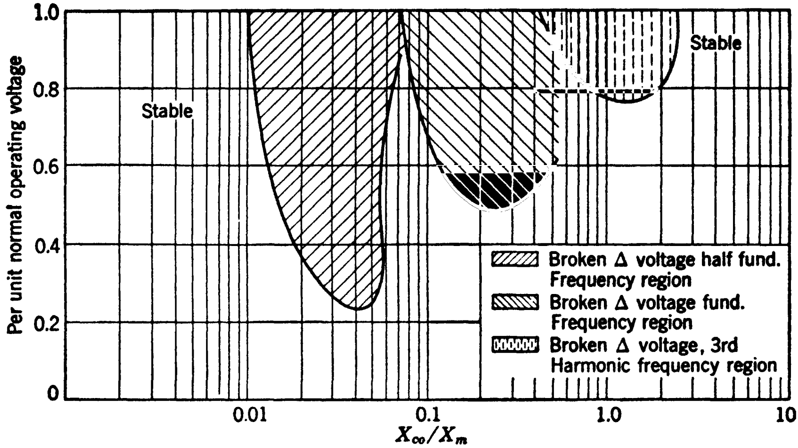


Fig. 10·5. Regions of possible instability of the circuit of Fig. 10·1 with transformer saturation corresponding to curve 1 of Fig. 10·2. $r/X_m = 0.0007$ for the transformer; r is the d-c resistance of the winding; X_m is the ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E = 1.0$ in Fig. 10·2.

In the vicinity of $X_{c0}/X_m = 0.5$, the phenomena included a component of approximately twice normal frequency. This was observed only for a very limited range of values of X_{c0}/X_m, and the other abnormal conditions, occurring over greater ranges of this ratio, were considered of far greater importance.

These facts are pointed out merely to indicate the physical consistency of the information plotted in Fig. 10·3. The ratio X_{c0}/X_m is proportional to the square of the natural frequency of the circuit studied. For very small values of this ratio, the circuit is always stable. As the ratio is increased beyond a certain value, instability is encountered and the unstable phenomena are characterized in succession by voltage components of one-quarter, one-half, one, two, and three times normal frequency. Thus the predominant frequency of the phenomena is seen to increase consistently with the natural frequency of the circuit.

The effect of increased transformer resistance is shown in Fig. 10·5, i.e., greater than that shown in Fig. 10·3. It is of importance to note that the regions of instability are only very slightly affected by resistance over a practical range. This fact makes the results of the investigation much more widely applicable.

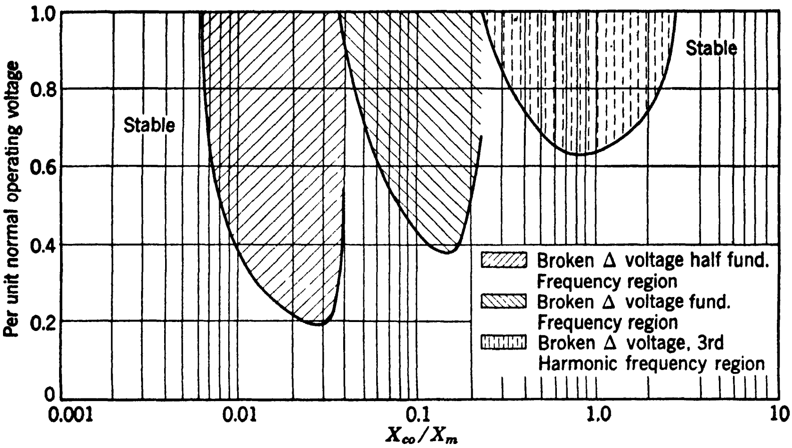


FIG. 10·6. Regions of possible instability of the circuit of Fig. 10·1 with transformer saturation corresponding to curve 2 of Fig. 10·2. $r/X_m = 0.0003$ for the transformer; r is the d-c resistance of the winding; X_m is the ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E = 1.0$ in Fig. 10·2.

The results shown in Fig. 10·6 were obtained for the saturation curve 2 of Fig. 10·2. This curve is somewhat steeper than that likely to be encountered generally, but the results are of interest in that they show how the unstable regions are broadened and shifted so as to extend the range of instability to lower values of the ratio of X_{c0}/X_m. This effect is consistent with other phenomena observed in connection with circuit analysis involving saturable impedance elements. The effect of saturation is to broaden the range of abnormality, the broadening effect increasing with increasing abruptness of saturation.

For line-to-line voltage rated potential transformers connected line-to-neutral $E = 0.58$ and the regions of instability given in Fig. 10·3 are $0.013 < X_{c0}/X_m < 0.075$ and $0.125 < X_{c0}/X_m < 0.45$. Crest values of broken delta voltage, line-to-ground voltage, and transformer current that may be obtained within these regions are given in Fig. 10·7. In the region corresponding to the fundamental-frequency broken delta voltage the voltage-to-ground of one phase is of small magnitude. The voltages-to-ground on the other two phases are approximately equal. Thus line voltages-to-ground as well as the broken delta voltage are essentially the same as if a fault were on the system.

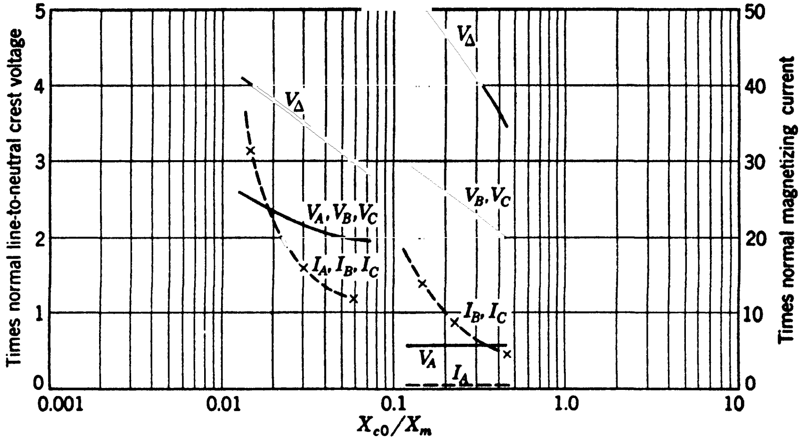


FIG. 10·7. Abnormal voltage and current conditions obtained for the circuit of Fig. 10·1. Transformer saturation corresponding to curve 1 of Fig. 10·2. $E = 1/\sqrt{3} = 0.58$; $r/X_m = 0.0003$. ———, voltage; ------, transformer current.

In the region corresponding to the one-half frequency broken delta voltage condition, the voltages-to-ground are the same in magnitude on all three phases. However, an interval of several seconds must be considered for this to be true because of the slow "slip" frequency involved as previously described.

The transformer current values shown in Fig. 10·7 are of interest also. In the region of one-half frequency phenomena these currents may reach values of 35 times normal magnetizing current in the vicinity of $X_{c0}/X_m = 0.015$. These currents are the same in magnitude in all three phases. In this range it might be possible to blow potential transformer fuses under such abnormal conditions. In the region of fundamental-frequency phenomena the currents are smaller in general and, furthermore, the current in one phase is lower than normal.

It is a significant fact that, while the voltage conditions in this range of X_{c0}/X_m approximate those obtained during a single line-to-ground fault, the currents flowing in the wye winding of the potential transformer may be 10 or 15 times as high as those obtained during a single line-to-ground fault condition, depending on the ratio of X_{c0}/X_m.

Figure 10·8 shows the results of inserting resistance R in the break in the delta secondary. The curves show that the heaviest burden (lowest resistance) to ensure stable operation is required when X_{c0}/X_m is about 0.02 or slightly less, depending on the normal system operating voltage. At the lowest point, $R/X_m = 0.15$ for operation at $E = 1.0$ and $R/X_m = 0.34$ for operation at $E = 0.58$. These values are on a basis of resistance per leg in per-unit of magnetizing impedance per leg for $E = 1.0$ on the saturation curves of Fig. 10·2. Therefore the curve gives the resistance required per phase for wye-connected secondaries. The value of impedance required across the break in the case of broken delta secondaries as in Fig. 10·1 is three times the quantity R/X_m of Fig. 10·8.

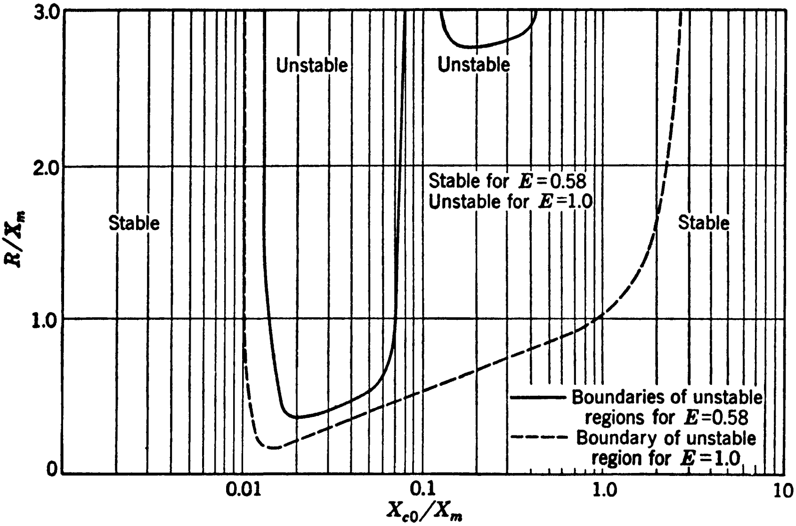


FIG. 10·8. Effect of resistance burden R in the broken delta on regions of instability. Saturation curve 1 of Fig. 10·1. $R/X_m = 0.0003$. Note: The total amount of resistance in the delta is three times the amount of R indicated by the curves since R/X_m is on a per-leg basis.

The effect of normal operating voltage E on the R/X_m ratio required for stabilization is shown in Fig. 10·9. It is of importance to note that, as E decreases, the ratio of R/X_m required for stability increases. The values of X_{c0}/X_m used in preparing these curves were purposely selected in the range requiring the lowest ratios of R/X_m as indicated in Fig. 10·8.

The effect of an inductive reactance burden X_L (power factor = 0.2) is indicated in Fig. 10·10. A lower ratio X_L/X_m is required for stabilization than for R/X_m as discussed in the preceding paragraph. The ratio X_L/X_m at its lowest point is 0.04 for operation at $E = 1.0$ and 0.28 for operation at $E = 0.58$.

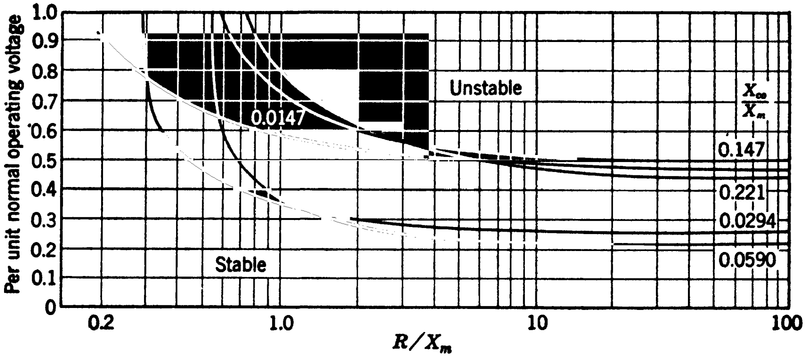


FIG. 10·9. Illustration of the relationship between operating voltage and resistance burden to obtain stability for various values of capacitive reactance.

The effect of a leading power factor (capacitive) burden in the break in the delta was investigated, and the results indicated that pure capacitance was not satisfactory. The same conclusion holds for wave traps which were made up of a reactance and capacitance tuned to the third harmonic. These were not effective because other complicating phenomena were encountered which were difficult to account for. Consequently a resistance burden appears to be the simplest and most effective means of ensuring stability by means of a secondary burden. Reactance was effective, although a reactance ohmic value would have to be smaller than that of a resistance to ensure stability.

Conclusions. 1. In using wye-grounded primary broken delta- or wye-conected secondary potential transformers for metering or relaying, it is necessary to consider the amount of capacitance to ground that may be permanently or temporarily connected at the terminals of the transformer in order to avoid the possibility of abnormal voltage conditions brought about by circuit instability which might cause incorrect indications of faults.

For line-to-line voltage rated potential transformers connected line-to-neutral, corresponding to general practice in the application of such devices ($E = 0.58$), the range of possible abnormality is defined approximately by $0.012 < X_{c0}/X_m < 0.6$. If a given circuit is found to fall within this range, normal behavior can be ensured by: (*a*) adding shunt capacitance to ground on the system side of the transformer until the resultant ratio X_{c0}/X_m becomes equal to or less than 0.012, or (*b*) connecting impedance across the break in the broken delta secondary or in each phase in the case of a wye-connected secondary. If resistance is used, it should have a value per phase of 33 per cent of the magnetizing reactance X_m per phase or less; if reactance is used, its ohmic value per phase should not be greater than 28 per cent of X_m per phase. In either case, X_m is referred to the secondary side in accordance with the transformer turn ratio. The impedances required for stabilization are high enough so as not to interfere with the normal functioning of the device as a fault detector. For these criteria, X_{c0} is the zero-sequence capacitive reactance to ground on the wye-grounded side of the transformer and X_m is the

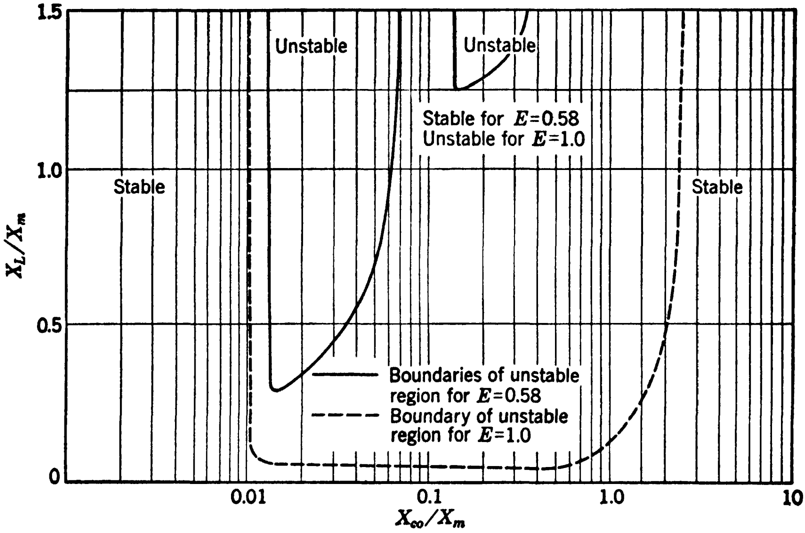


FIG. 10·10. Effect of inductive reactance burden X_L (0.2 power factor) in the broken delta on regions of instability. Saturation curve 1 of Fig. 10·1. Note: The total amount of reactance in the delta is three times the amount of X_L indicated by the curves since X_L/X_m is on a per-leg basis.

magnetizing reactance per leg corresponding to $E = 1.0$ on the saturation curve 1 of Fig. 10·2.

2. The regions of abnormality as well as the abnormal phenomena observed are practically unaffected by resistance in the transformer windings or by losses in transformer iron over the range investigated.

3. Increasing the steepness of the saturation curve tends to increase the range of possible abnormal circuit behavior. In particular, the region of abnormality is extended to lower values of X_{c0}/X_m as the steepness of saturation is increased.

4. For conditions of operation above transformer flux densities corresponding to the normal practice of using line-to-neutral connected potential transformer with line-to-line voltage rating ($E = 0.58$), abnormal circuit behavior is extended to include a range of $0.01 < X_{c0}/X_m < 3.0$ approximately for $E = 1.0$. Furthermore, the impedance in the open delta must be made smaller if stability is to be ensured. For $E = 1.0$, a resistance in the break in the delta not greater than 45 per cent of X_m (15 per cent of X_m per phase) or a reactance not greater than 12 per cent of X_m (4 per cent of X_m per phase) would be required to avoid abnormal circuit behavior independent of X_{c0}/X_m.

10·3 THIRD-HARMONIC POWER FROM SINGLE-PHASE FUNDAMENTAL-FREQUENCY SOURCE

In a circuit as shown in Fig. 10·11, it is possible to generate third-harmonic voltages of good wave shape from a single-phase fundamental frequency source. Furthermore, it is possible to deliver third-harmonic power to a load (represented by the resistor R in Fig. 10·11) with relatively good wave form and with good efficiency. For relatively small capacity requirements, the circuit offers a ready means of frequency conversion from fundamental frequency to third harmonic. It follows that a second unit appropriately designed could convert the

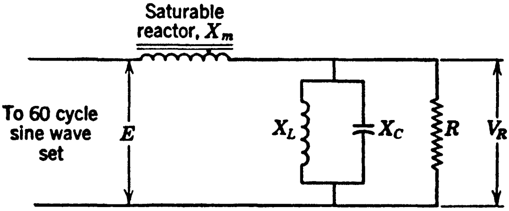


Fig. 10·11. Circuit studied to determine the feasibility of generating harmonic power, using static impedance elements and a single-phase fundamental-frequency output of the first unit to its third harmonic, thus converting from fundamental to ninth in two stages. Three stages would permit conversion from fundamental to twenty-seventh harmonic, and so forth.

The material included here will be that obtained by means of miniature-system tests converting from a 60-cycle fundamental frequency to a third harmonic of 180 cycles per second. The saturable reactor was one having the same per-unit saturation characteristic as curve 2 in Fig. 10·2 and is denoted by X_m in Fig. 10·11.

The operation of the circuit is as follows: First consider the reactor X_m alone connected across a 60-cycle sine-wave source of voltage. Because of its non-linear volt-ampere (or flux linkage current) characteristic, harmonic currents must flow from the source through X_m. The harmonic of greatest magnitude is the third, although the fifth, seventh, ninth, etc. (all odd frequencies), are present. Now, if a small reactor X_L which is linear is inserted in series with the reactor X_m, there will be a voltage of fundamental, third harmonic, and higher odd-harmonic frequencies across it. If we put a capacitive reactance X_C in parallel with X_L and adjust it in magnitude so that the parallel value of X_C and X_L prevents, say, the third-harmonic current from flowing, then there should be a high third-harmonic voltage appearing across this parallel "trap." Other frequencies would still pass freely through the trap, and the voltage across the trap would be predominantly of third-harmonic frequency. If a load, such as the resistor R in Fig. 10·11, is supplied from this source, it would be supplied at predominantly third-harmonic frequency.

It will be the purpose of this section to summarize results of investigating the effects of varying the parameters E, X_C, X_L, and R. X_m is taken as 1.0 when $E = 1.0$ on the saturation curve 2 of Fig. 10·2.

Figure 10·12 shows the effect of varying the 60-cycle voltage E for various values of X_C/X_L and no load. It is apparent from these curves that, in order to get any third-harmonic voltage of useful amplitude, the value of E must be above 1.2. For values of E above 1.2, the third-harmonic voltage depends on the ratio of X_C/X_L. The maximum voltages are not obtained when the trap is tuned to the third harmonic ($X_C/X_L = 9$), but when the *entire circuit* is tuned for the third harmonic. Therefore, since the saturable reactor is in parallel with X_L, it is to be expected that the value of X_L will always be greater than that required to tune X_C and X_L alone to the third harmonic. For this particular value of $X_C = 0.011$, the maximum third-harmonic voltage in terms of the circuit-exciting voltage E is obtained for $X_C/X_L = 5.5$.

In Fig. 10·13 the effect of loading the circuit by adding resistance R in Fig. 10·11 is shown for $X_C/X_L = 5.5$ and $X_C = 0.011$. Voltage regulation of the circuit is not good, although, for relatively constant load conditions, this need be of only small concern. Figures 10·14 and 10·15 show corresponding results for $X_C/X_L = 4.89$ and 4.4 with $X_C = 0.011$ for both cases.

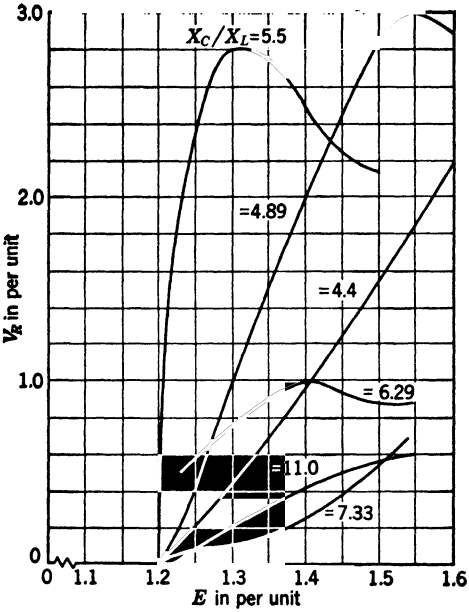


FIG. 10·12. Effect of E on V_R for various values of X_C/X_L. $X_C = 0.011$; no load; $R = \infty$.

In Fig. 10·16 the effect of loading is also shown. It is apparent from these curves, too, that the voltage regulation is poor. It is also apparent from these curves that, for very light loads, an increase in E results in a decrease in V_R. This is not always true, as shown in Figs. 10·17 and 10·18. These curves are for values of $X_C = 0.0055$ and 0.00376, respectively. In each case X_L was selected to give the highest third-harmonic voltage at no load. The ratio X_C/X_L required for this varies with X_C. As X_C decreases, the ratio X_C/X_L for maximum third-harmonic voltages increases and in the limit (for X_C, very small) this ratio will approach 9.0.

There is an advantage in making X_C small. If X_C is very small, then the impedance of the trap to fundamental frequency (inductive)

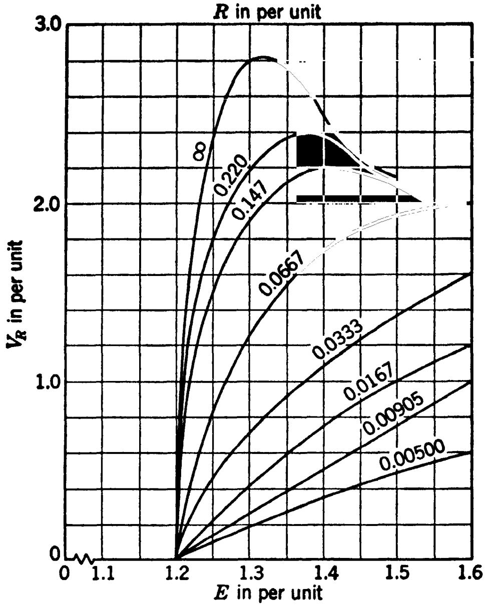


FIG. 10·13. Effect of load on V_R. $X_C = 0.011$; $X_C/X_L = 5.5$.

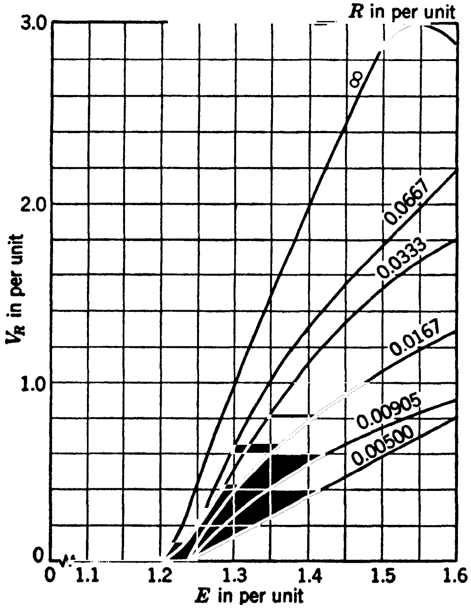


FIG. 10·14. Effect of load on V_R. $X_C = 0.011$; $X_C/X_L = 4.89$.

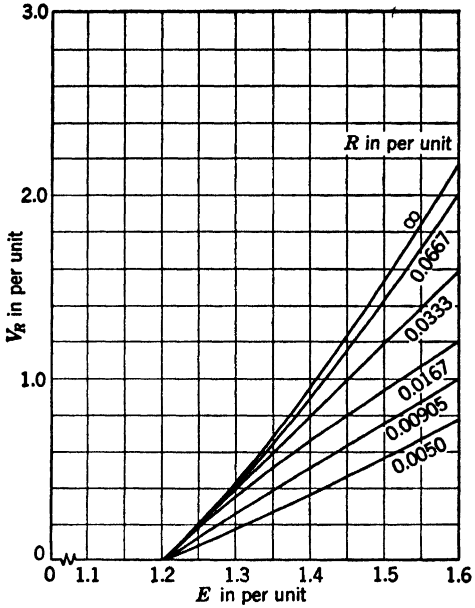


FIG. 10·15. Effect of load on V_R. $X_C = 0.011$; $X_C/X_L = 4.4$.

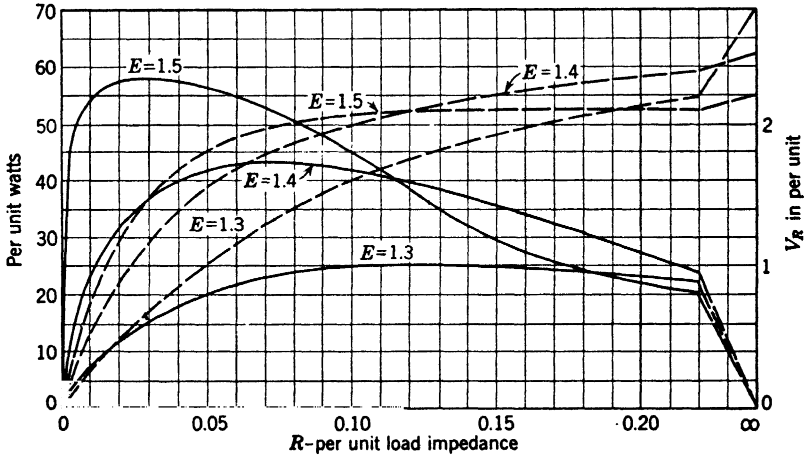


FIG. 10·16. Effect of E and R on third-harmonic voltage and power output. $X_C = 0.011$; $X_C/X_L = 5.5$; ———, watts; – – – – – –, V_R.

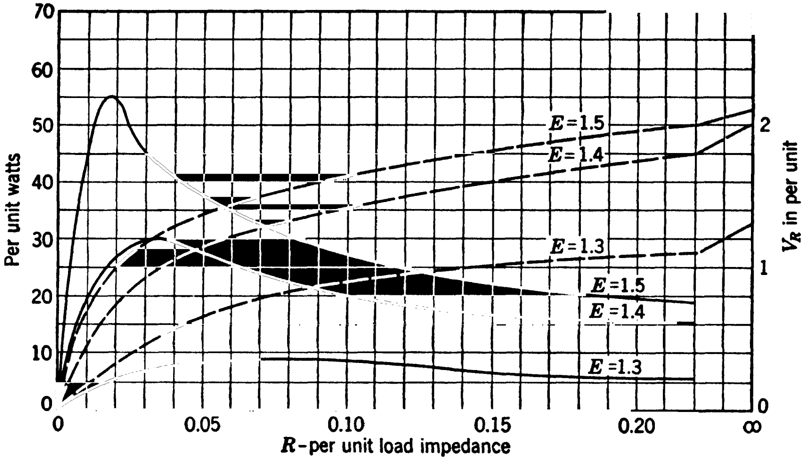


FIG. 10·17. Effect of E and R on third-harmonic voltage and power output. $X_C = 0.0055$; $X_C/X_L = 7.33$; ———, watts; ------, V_R.

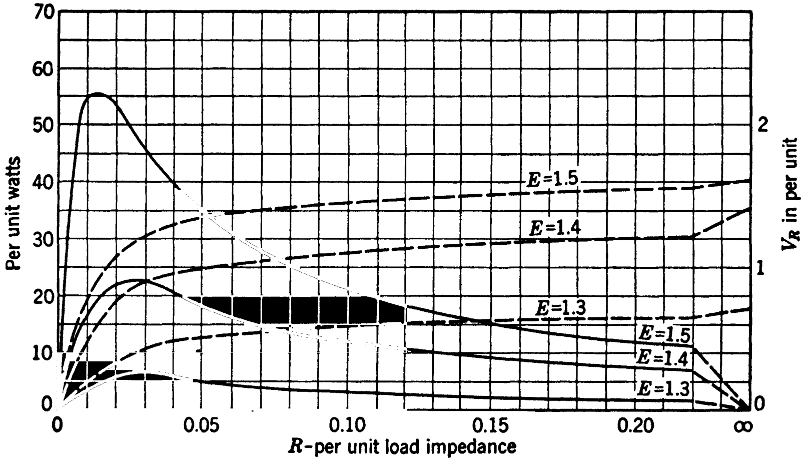


FIG. 10·18. Effect of E and R on third-harmonic voltage and power output. $X_C = 0.00376$; $X_C/X_L = 7.47$; ———, watts; ------, V_R.

is also small. It follows that the fundamental-frequency voltage across the trap is less, and consequently there is less fundamental-frequency voltage at the load.

Similarly there is an advantage in not going to an exceedingly high value of E for normal operation. As E is raised, the fundamental-frequency current increases more rapidly than the first power of E because of saturation. Consequently the fundamental-frequency voltage appearing across the load may become excessive if E is made greater than 1.5. This becomes apparent by inspecting carefully the oscillograms in Fig. 10·20 together with information in Table 10·1. Figure 10·19 is helpful also in this regard. This figure shows that more power for a given E can be obtained for the higher values of X_C. However, these higher values of X_C result in somewhat increased fundamental-frequency voltage. Although no detailed harmonic analysis

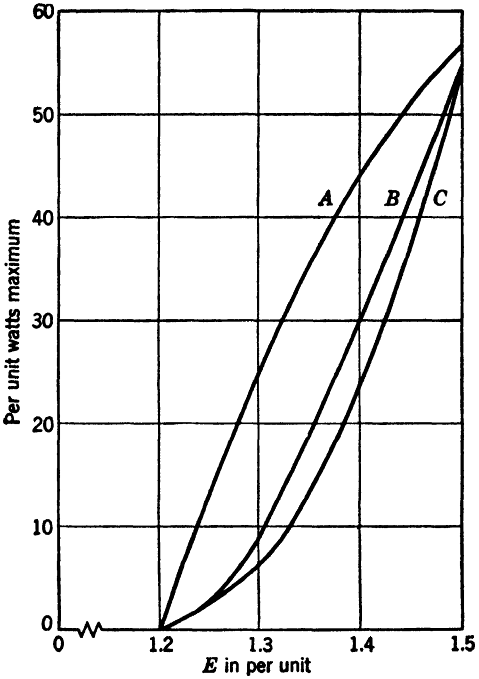


FIG. 10·19. Effect of circuit constants on third-harmonic maximum power output. From Figs. 10·16, 17, and 18. Curve A, $X_C = 0.011$, $X_C/X_L = 5.5$; curve B, $X_C = 0.0055$, $X_C/X_L = 7.33$; curve C, $X_C = 0.00376$, $X_C/X_L = 7.47$.

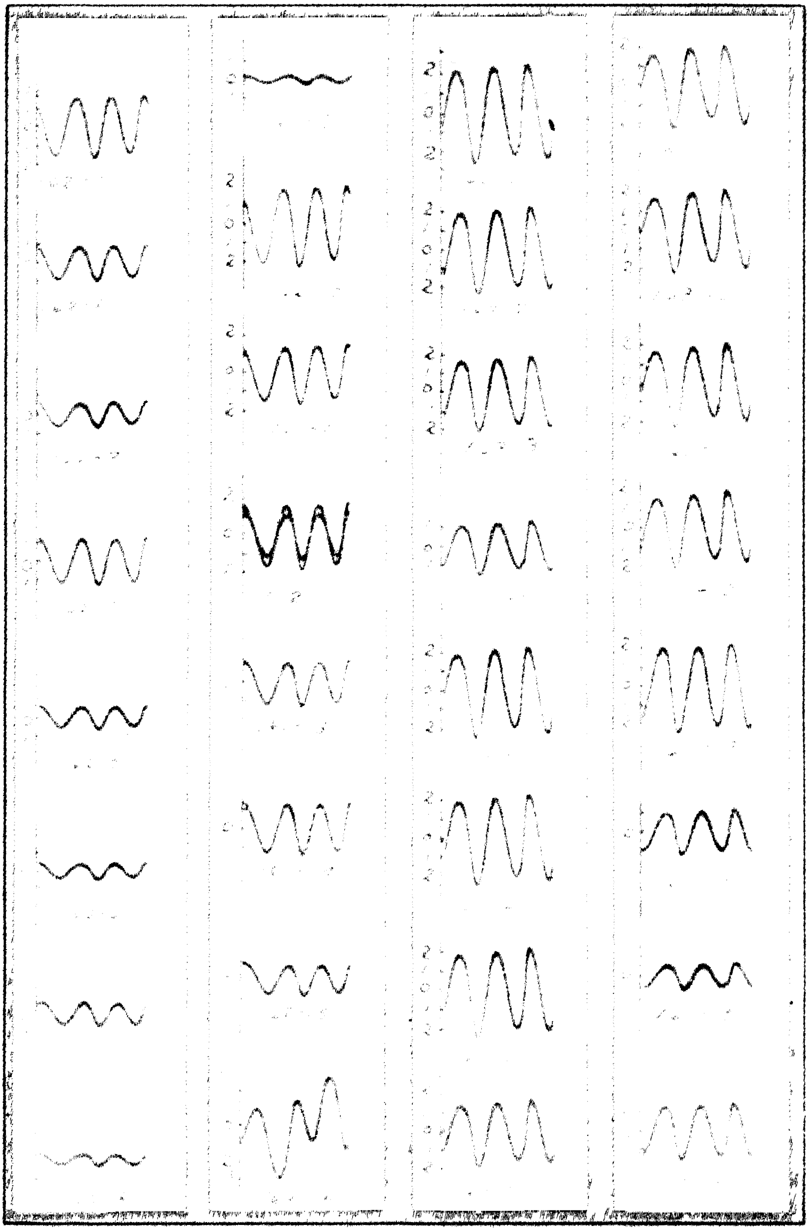


FIG. 10·20. Typical oscillograms for harmonic power generator shown in Fig. 10·11. Refer to Table 10·1 for identification.

TABLE 10·1

IDENTIFICATION OF OSCILLOGRAMS

Oscillogram	Voltage	X_C	X_C/X_L	R	E
162–1	V_R	0.00376	7.47	∞	1.5
–2	V_R	0.00376	7.47	0.0182	1.5
–3	V_R	0.00376	7.47	0.0091	1.5
–4	V_R	0.00376	7.47	∞	1.4
–5	V_R	0.00376	7.47	0.0182	1.4
–6	V_R	0.00376	7.47	0.0091	1.4
–7	V_R	0.00376	7.47	∞	1.3
–8	V_R	0.00376	7.47	0.0182	1.3
–9	V_R	0.00376	7.47	0.0091	1.3
–10	V_R	0.0055	7.33	∞	1.5
–11	V_R	0.0055	7.33	0.0733	1.5
–12	V_R	0.0055	7.33	∞	1.4
–13	V_R	0.0055	7.33	0.0733	1.4
–14	V_R	0.0055	7.33	∞	1.3
–15	V_R	0.0055	7.33	0.0733	1.3
–16	. . .*	0.0055	7.33	∞	1.5
163–1	V_R	0.011	5.5	∞	1.34
–2	V_R	0.011	5.5	0.22	1.34
–3	V_R	0.011	5.5	0.147	1.34
–4	V_R	0.011	5.5	0.073	1.34
–5	V_R	0.011	5.5	∞	1.41
–6	V_R	0.011	5.5	0.22	1.41
–7	V_R	0.011	5.5	0.147	1.41
–8	V_R	0.011	5.5	0.073	1.41
–9	V_R	0.011	5.5	∞	1.49
–10	V_R	0.011	5.5	0.22	1.49
–11	V_R	0.011	5.5	0.147	1.49
–12	V_R	0.011	5.5	0.0733	1.49
–13	V_R	0.011	5.5	∞	1.24
–14	V_R	0.011	5.5	0.22	1.24
–15	V_R	0.011	5.5	0.147	1.24
–16	V_R	0.011	5.5	0.0733	1.24

* Note: This shows the voltage across the saturable reactor.

of wave shapes has been made, it is believed that the evidence indicated by the oscillograms clearly points to the desirability of keeping X_C as small as possible consistent with size and cost limitations. In addition, E should not appreciably exceed 1.5 if freedom from fundamental-frequency voltage at the load is of importance.

Conclusions. A single-phase non-linear circuit of the type studied can be used as a third-harmonic power source. The output voltage is sinusoidal with a small amount of fundamental frequency and higher harmonics superimposed. For some applications this fundamental-frequency component may be of importance. In order to keep the fundamental-frequency component at a minimum the following suggestions are helpful.

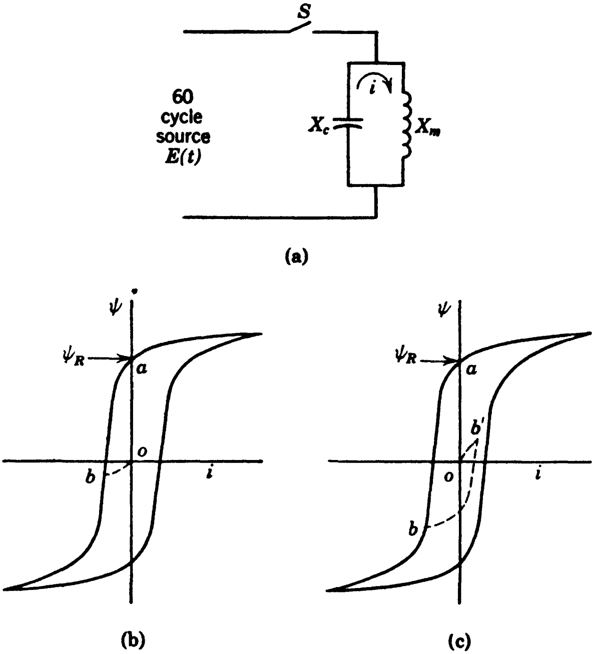
1. Make X_C small.
2. Select X_L to give maximum third-harmonic voltage at the lowest value of E. This might be termed the optimum combination of E and X_L for a given X_C.
3. Design the reactor so that, for the normal circuit voltage applied, its flux density will not exceed about 1.5 times the value corresponding to $E = 1.0$ in Fig. 10·2.
4. A closed cycle controller properly designed to control E in order to regulate V_R and/or, if efficiency is unimportant, to control R as parallel load is varied in order to regulate V_R conceivably would add greatly to the practical usefulness of such a circuit as a source of harmonic power.

10·4 REDUCTION OF RESIDUAL FLUX IN A TRANSFORMER

The problem of transformer inrush current has been the subject of much written and oral discussion at various times. With the advent of higher silicon steels and cold rolling, residual flux in transformers was increased. This increase in residual flux increases the possible maximum peak inrush current in accordance with well-known transformer theory. With high residual fluxes it is possible for the maximum peak inrush current during energizing a transformer to approach (but not equal or exceed) the peak current that could be obtained for a fully offset short-circuit current on the transformer secondary.

Now a capacitor connected permanently in parallel with a transformer can be used to reduce this residual flux to zero and thus avoid the extreme possible inrush current condition just mentioned. A complete investigation of all types of transformer steels and lamination thicknesses would entail a study far beyond the scope of this book. However, the fundamental problem will be outlined, and some representative results will be shown which will serve to indicate the fact that the values of capacitance that are completely effective in reducing residual flux to zero are quite critical for any given case.

Consider a simple circuit arrangement as in Fig. 10·21(*a*). With the switch closed, the transformer flux-current relationship might be that represented by solid lines in Fig. 10·21(*b*). If the switch S is opened at a normal current zero, say, following a positive half-cycle of current, then the residual flux is represented by Ψ_R. If there were no capacitor present, this flux Ψ_R would remain in the core until some future closing of the switch S, at which time there would be a new set of closing conditions. If there is a capacitor present, however, then, at the instant of opening switch S at a current zero as specified, this capacitor is left with a charge on it corresponding to the crest value of $E(t)$. It will discharge, after the switch is opened, through the transformer winding. If the circuit could oscillate freely, finally dying down to zero voltage, then the residual magnetism would be reduced to zero. However, for practical transformer steels the losses (hysteresis and eddy current) are so high that the circuit is not very freely oscillatory.



(a)

(b) (c)

FIG. 10·21. Effects of a parallel capacitor in reducing residual flux in a transformer. (*a*) Equivalent circuit; X_m, transformer magnetizing reactance; X_C, parallel capacitor reactance. (*b*) ψ–i relationship for reducing residual flux with smallest possible capacitance. (*c*) ψ–i relationship for reducing residual flux with next-larger critical value of capacitance.

An inspection of Fig. 10·21(*b*) suggests that it is not necessary for the circuit to be oscillatory to attain the desired objective. All that is necessary is that the current caused by the capacitor discharge through the transformer winding immediately after the opening of switch *S* shall have a single loop of peak value corresponding to point *b* as shown in Fig. 10·21(*b*). This peak value of current is rather critical. However, it might be expected that even a relatively small capacitor could accomplish the desired objective. If the capacitor is made greater, then the current flow will be too great on the first discharge and reverse residual flux will be left in the core. If the capacitance is further increased a reversal of current will also take place, but a critical value can be found which will lead to complete elimination of the residual as illustrated in Fig. 10·21(*c*). This critical value is that which will give a first loop designated by *b* and a second by *b′*. It follows that further increases in capacitance will lead to the discovery of other critical values of capacitance which will bring about the desired objective of reducing the residual flux to zero.

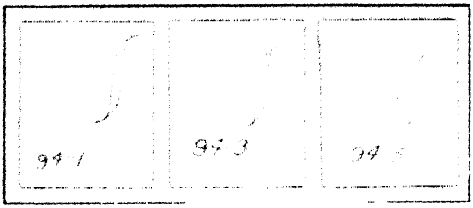


FIG. 10·22. Hysteresis loops for single-phase miniature-system transformers.

Oscillogram Number	60-Cycle rms Voltage
94-1	110
94-3	102
94-5	90

A typical case was worked out on the transient analyzer using the saturation curve 2 of Fig. 10·2. Hysteresis loops of this transformer are shown in Fig. 10·22. Results of varying the capacitance in parallel with the transformer are shown in Fig. 10·23. From these results it is clear that there are several distinctly different but critical values of capacitance that may be used to reduce residual transformer flux to zero. In this particular case, a value of capacitor kva equal to about 40 per cent of the transformer normal voltage magnetizing kva was the smallest capacitance that would accomplish the desired objective.

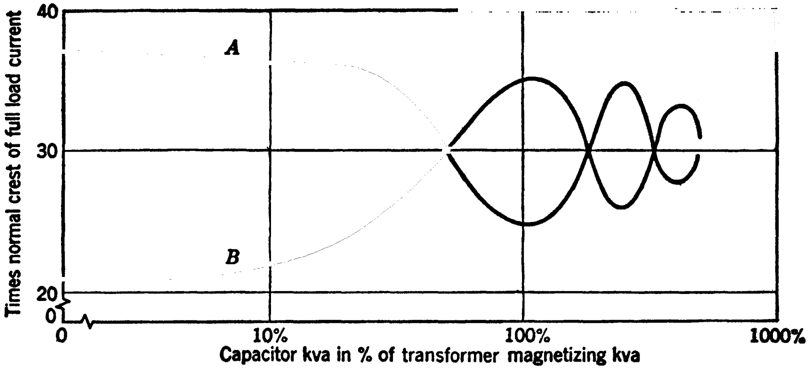


FIG. 10·23. Effect of capacitor on maximum transformer magnetizing inrush currents. Transformer de-energized under normal operating flux density conditions leaving the capacitor at normal voltage to discharge through the transformer each time before the transformer was again energized at an instant controlled to give maximum inrush. Curve A, last half-cycle of current in transformer before switch was opened of same polarity as first half-cycle of inrush current; curve B, last half-cycle of current in transformer before switch was opened of opposite polarity to the first half-cycle of inrush current.

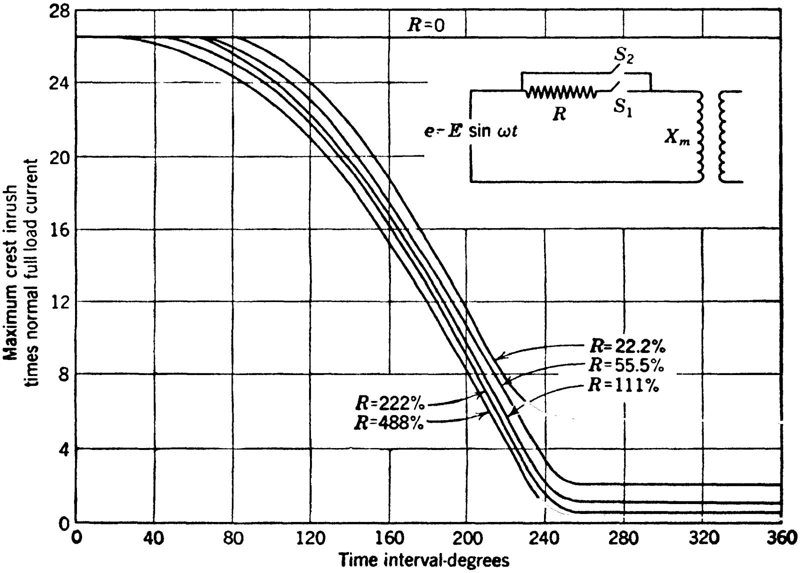


FIG. 10·24. Effect of resistor switching on inrush currents. Single-phase transformer. $E = 1.0$ on curve 2 of Fig. 10·2. Residual of polarity to give maximum inrush. Normal exciting current, 3 per cent. Time lag is interval between closing S_1 and S_2. For any given time lag, the inrush given by these curves is the maximum possible for the conditions specified.

Whereas this curve is only for a particular value of residual flux Ψ_R' corresponding to $E = 110$ volts, it was found in the miniature-system tests that the effectiveness of the given transformer and capacitor combination was relatively insensitive to the value of residual flux at the opening of the switch S. It is also not noticeably sensitive to the amount of transformer winding resistance for representative power and distribution transformers.

Another method of reducing the effects of residual flux in a transformer consists of an arrangement as shown in Fig. 10·24. The transformer and the operating conditions were the same as in the preceding case. If S_1 is closed first and S_2 is closed last, then the effects of the residual flux are eliminated. In fact, inrush currents can be reduced quite completely by proper proportioning of the resistor R relative to the value of X_m. The time interval between the closing of S_1 and S_2 is also important as shown by the curves. The values of R shown are in per cent of X_m, with the time interval in electrical degrees based upon 60-cycle frequency.

10·5 TRANSFORMER INRUSH CURRENTS—EFFECTS OF PARALLEL OPERATION

Although inrush current phenomena associated with the energizing of one transformer are well understood, there are certain elements of uniqueness sometimes encountered when one transformer is suddenly energized in parallel with another which was already in operation. In one instance, such tests on an actual system showed that, when one transformer was carrying light load and a second transformer (unloaded) was switched on in parallel, the line currents exhibited characteristics indicative of high saturation of the transformer cores which sometimes persisted for several seconds. One oscillogram showed a current equal to full load current still flowing but decaying very gradually after slightly over one second.

As an explanation, it was suggested by the late L. F. Blume that possibly external resistance (R in Fig. 10·25) in the system was causing a sufficient d-c voltage drop to result in a building up of current in the transformer already energized simultaneously with the decay of inrush current in the unit being energized. The result of such an occurrence would be a circulating d-c current in the windings of the transformers so that they would both be saturated, but *each* with a *different* polarity. This would cause a large symmetrical current in the line, indicative of saturation of both polarities, which would be in evidence as long as the d-c circulating current persisted in flowing in the windings. Since the time constant of this d-c current would be very large, it might take a long time for evidence of its presence to disappear from the line currents even though there might be quite high resistance in the system.

Such a phenomenon, it has since been shown, is characteristic of even a single-phase arrangement as shown in Fig. 10·25. It is also characteristic of any three-phase circuit where transformers are being energized in parallel. It is not characteristic of any three-phase circuit when a single transformer bank is being energized regardless of the connection of the windings, i.e., whether wye-wye, wye-delta, delta-delta, delta-wye, or open delta with or without the neutral points grounded. This is of particular importance from the standpoint of relay application because it indicates that, as far as correct relay operation is concerned, the only requirement is that current transformers be placed in the lines going to each transformer. It is not necessary to install current transformers inside a delta winding.

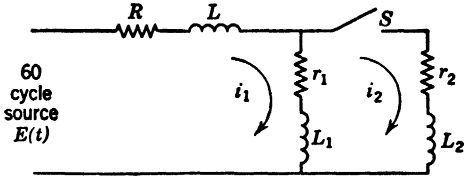


FIG. 10·25. Equivalent circuit for illustrating the effects of magnetizing inrush currents when transformers are operated in parallel.

For the purpose of analysis, consider the circuit arrangement shown in Fig. 10·25. The circuit might be thought of as a substation or source supplying a load through a line represented by R and L. One transformer represented by L_1 is already energized. The second one represented by L_2 will be energized by closing switch S. Now, in transformers, L_1 and L_2 are not constants, as has been pointed out and is well known. However, insight is gained by considering both L_1 and L_2 constant for preliminary purposes.

The differential equations for the loop currents flowing are:

$$e(p) = i_1(p)[R + r_1 + (L + L_1)p] - i_2(p)(r_1 + L_1p) \quad [10\cdot1]$$

$$0 = -i_1(p)(r_1 + L_1p) + i_2(p)[r_1 + r_2 + (L_1 + L_2)p] \quad [10\cdot2]$$

These equations, when solved, yield for the current in L_1 already connected to the bus

$$i_1 - i_2 = A_1 \sin(\omega t + \theta - \phi_1) + A_2 \sin(\theta - \alpha_1)\epsilon^{-b_1 t} + A_3 \sin(\theta - \alpha_2)\epsilon^{-b_2 t} \quad [10\cdot 3]$$

and for the transformer being connected

$$i_2 = B_1 \sin(\omega t + \theta - \phi_2) + B_2 \sin(\theta - \alpha_1)\epsilon^{-b_1 t} + B_3 \sin(\theta - \alpha_2)\epsilon^{-b_2 t} \quad [10\cdot 4]$$

The current in the line is the loop current i_1 which is the sum of equations [10·3] and [10·4].

Equations [10·3] and [10·4] give the currents which flow after the switch S is closed, assuming that the voltage *on the bus* is $E \sin(\omega t + \theta)$. In these equations

$$b_1 = \frac{B}{2A} - \sqrt{\left(\frac{B}{2A}\right)^2 - \frac{C}{A}}$$

$$b_2 = \frac{B}{2A} + \sqrt{\left(\frac{B}{2A}\right)^2 - \frac{C}{A}}$$

$$A = LL_1 + L_1L_2 + L_2L$$

$$B = RL_1 + RL_2 + r_1L_2 + r_1L + r_2L + r_2L_1$$

$$C = Rr_1 + r_1r_2 + r_2R$$

$$A_2 = \frac{R - Lb_1}{Bb_1 - 2C} \cos \alpha_1$$

$$A_3 = -\frac{R - Lb_2}{Bb_2 - 2C} \cos \alpha_2$$

$$\alpha_1 = \tan^{-1} \frac{\omega}{b_1}$$

$$\alpha_2 = \tan^{-1} \frac{\omega}{b_2}$$

$$B_2 = \frac{(R + r_1) - (L + L_1)b_1}{Bb_1 - 2C} \cos \alpha_1$$

$$B_3 = \frac{(R + r_1) - (L + L_1)b_2}{Bb_2 - 2C} \cos \alpha_2$$

$$A_1 = E \left| \frac{Z_2}{Z_1} \frac{(Z + Z_1)}{(ZZ_1 + ZZ_2 + Z_1Z_2)} \right|$$

$$B_1 = E\left|\frac{Z + Z_1}{Z_1Z_2 + ZZ_1 + ZZ_2}\right|$$

$$\phi_1 = \text{impedance angle of } \frac{Z_2}{Z_1}\frac{(Z + Z_1)}{(ZZ_1 + ZZ_2 + Z_1Z_2)}$$

$$\phi_2 = \text{impedance angle of } \frac{Z + Z_1}{Z_1Z_2 + ZZ_1 + ZZ_2}$$

The steady-state portion of the foregoing is of little interest. The transient terms are of special significance, however. First, consider b_1 and b_2, the two time constants involved. If it is assumed that $L_1 = mL_2$ and $r_1 = mr_2$, which is not unreasonable for practical cases involving transformers of the same voltage rating and comparable kva ratings, then it can be shown that

$$b_1 = \frac{r_1}{L_1} = \frac{r_2}{L_2}$$

and

$$b_2 = \frac{R + [mr_2/(1 + m)]}{L + [mL_2/(1 + m)]}$$

Thus the decay of the d-c transient component in the external series circuit is partly controlled by the system resistance R and the parallel value of L_1 and L_2, whereas the parallel d-c transient is largely controlled by the time constant of the loop resistance $r_1 + r_2$ and the loop inductance $L_1 + L_2$ or, as shown above, r_1 and L_1. It follows that in a representative case, b_1 is small and b_2 is large. While the series external circuit causes the initial inrush current to decay rapidly in accordance with the time constant $1/b_2$, at the same time a d-c current is being *built up* in L_1 so that, when the external circuit d-c transient has disappeared, there is a trapped circulating d-c current which does not depend on R for its rate of decay, but only on the resistance and inductance of the loop. This transient has a long time constant $1/b_1$ and may be on the order of seconds for the practical transformer constants.

Although this analysis affords insight into the mechanism of the phenomenon, it is only qualitative at best. It should be emphasized that only a small amount of d-c need be trapped in the loop to cause transformer saturation. Therefore the effects of a small amount of trapped d-c are greatly amplified in the external series circuit in practical transformer circuits.

It can be shown that the amount of circulating or trapped d-c current depends essentially on the ratio L_1/L_2. If $r_1 = r_2 = 0$, then the trapped current is (with L negligible)

$$\frac{E_{\max} \cos \theta}{\omega(L_2 + L_1)} = \frac{E_{\max} L_2 \cos \theta}{\omega L_2(L_2 + L_1)}$$

This is, of course, a maximum when $\cos \theta = 1$. The maximum d-c inrush is recognized as being $E_{\max}/\omega L_2$. Therefore, the part of this that is left to circulate is $L_2/(L_1 + L_2)$ and, as L_2 increases, more of the available inrush current is trapped in the loop. In the extreme case, as L_2/L_1 becomes very large, practically all the available d-c inrush will circulate as a trapped d-c current. Thus, if one transformer bank is being energized in parallel with ten others already energized, most of the d-c inrush to this one transformer will continue to circulate between the one transformer in series with the other ten units as a group in parallel.

10·6 EFFECT ON HYSTERESIS LOOPS OF SIMULTANEOUS EXCITATION AT TWO FREQUENCIES

It has been known for some years that, when two fluxes varying sinusoidally at different frequencies are superimposed, the hysteresis loss is largely supplied by the higher-frequency component. If the second frequency is made very high in comparison with the first, then essentially all the hysteresis loss is associated with this higher frequency, whereas there is essentially none associated with the lower frequency. This fact has special significance in connection with certain designs where losses are to be minimized for reasons of accuracy of a device, for example, a current transformer. It is desirable to minimize ratio and phase-angle errors in such devices, but hysteresis losses prevent maintaining good accuracy over the lower end of the current range. If hysteresis losses could be supplied from another source, an obvious extension of the range of current values over which accuracy could be maintained would result.

It is the purpose of this section merely to show how the effects of two simultaneous sources of excitation can be studied in detail, a cathode-ray oscillograph being used for the purpose of disclosing the resultant modified hysteresis loops.

Consider the circuit arrangement of Fig. 10·26. With the winding on the middle leg unexcited, and with the capacitor and resistor circuits as shown, one can observe the normal hysteresis loop of the transformer on the screen of a cathode-ray oscillograph. If the two windings on the outside legs are wound with equal turns, then there is no flux in the middle leg, provided that the core is symmetrical about the center line of the middle leg. Therefore, there is no voltage induced in the coil on the center leg due to excitation on the outer leg windings as shown.

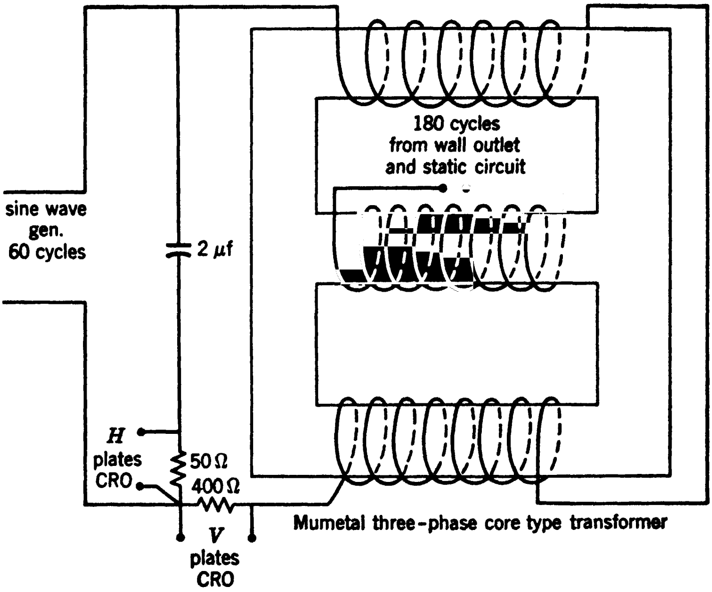


FIG. 10·26. Arrangement for obtaining hysteresis loops for transformers with simultaneous excitation at two frequencies.

If now the outside winding is unexcited and the inside winding is excited, then there would be no voltage induced across the outside winding input terminals because the windings are wound in such a way that the voltages induced in each outer leg are in opposition to each other. Yet there is flux in both outside legs as well as in the center leg due to this excitation.

If both circuits are excited simultaneously, then, although there is no mutual coupling between the two circuits, there is a superposition of fluxes in the two outside legs and the hysteresis loop now traced out on the cathode-ray oscillograph screen would portray the effect of the excitation of the middle leg. If this middle leg is excited at a higher frequency (say, 180 cycles as shown in Fig. 10·26), then the area of the resultant hysteresis loop should be noticeably reduced. The amount of this reduction in area is a measure of the extent to which hysteresis loss of the transformer core has been reduced by the superposition of the higher-frequency flux.

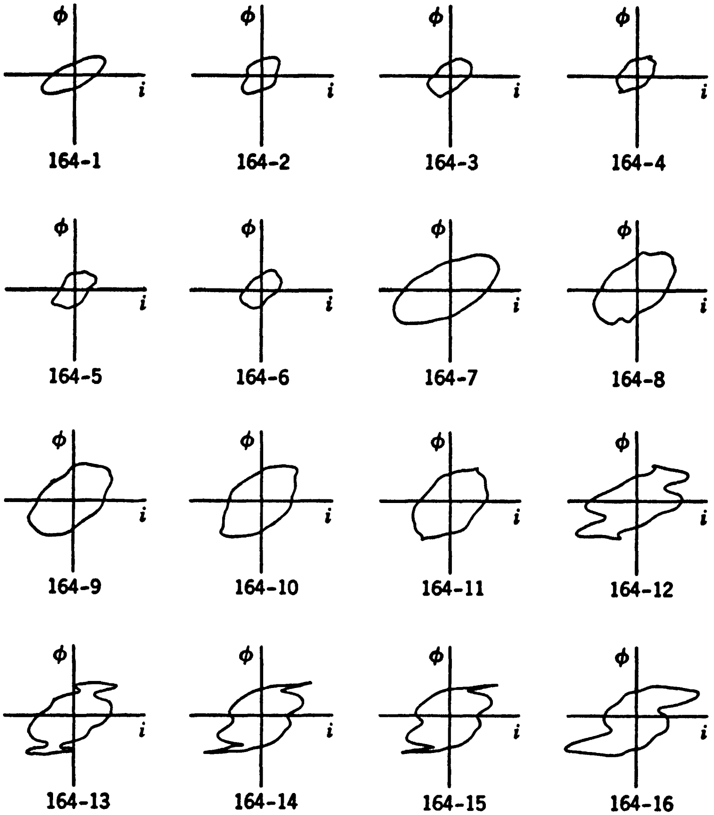


FIG. 10·27. Oscillograms showing hysteresis loops for transformers with simultaneous excitations at two frequencies. See Table 10·2 for identification.

No detailed results can be included here, but it is desirable to include some illustrations. In Fig. 10·27 are some oscillograms that show the effects outlined. Table 10·2 identifies these oscillograms and gives the flux densities for each excitation in each case. A miniature-system transformer having a saturation curve similar to that of Fig. 10·2 was used. The salient feature of these oscillographic hysteresis loops is the fact that the area of the loop is reduced by impressing an additional high-frequency excitation on the core. This is clearly apparent from an inspection of the oscillograms.

TABLE 10·2

IDENTIFICATION OF OSCILLOGRAMS

Oscillogram	E_{60}	60-Cycle Flux Density	E_{180}	Equivalent 180-Cycle Flux Density
164–1	5.8	3.2 kilolines/sq in.	0	0
164–2, 3, 4, 5, 6	5.8	3.2 kilolines/sq in.	75	26 kilolines/sq in.
164–7	11.0	6.1 kilolines/sq in.	0	0
164–8, 9, 10, 11	11.0	6.1 kilolines/sq in.	75	26 kilolines/sq in.
164–12, 13, 14, 15, 16	11.0	6.1 kilolines/sq in.	150	52 kilolines/sq in.

The exact shape of the loop depends on the phase relationship existing between the fundamental and third-harmonic frequency. The records shown are photographs taken at various instants of time to show what the range of loop shapes might be. Each record corresponds to a definite phase relationship, but the group of records for each set of parameters shows the range of loop shapes encountered if "slip" is present, i.e., if the high-frequency excitation is not an integral multiple of the fundamental frequency. Slip was present for this study, and hence pictures taken at various instants gave a variety of loop patterns. However, it was not feasible to determine the phase relationship existing between the two excitations corresponding to each record. Enough records were taken so that the deviations are well illustrated.

It is significant that, whereas the shapes of the resultant hysteresis loops change with phase relationship between the two sources of excitation, the *area* of these loops is not altered appreciably. The area seems to be relatively independent of this phase relationship. This is in agreement with the unpublished findings of B. M. Smith. His detailed results of a large number of tests showed that excitation losses on the low-frequency side were essentially independent of phase relationship between fundamental- and higher-frequency harmonic voltage sources.

The reduction in area of the hysteresis loops when high-frequency excitation is impressed is not so great for this miniature transformer as is obtained in a large power transformer. The reason for this lies apparently in the fact that, in the miniature transformer, eddy current losses are a higher percentage of total losses than would be true for an actual full-scale transformer.

10·7 BLOWING POTENTIAL TRANSFORMER FUSES DURING CLEARING OF GROUND FAULTS

In the operation of power systems, fuses in potential transformer circuits have been known to blow without any immediately apparent reason. In systems that are grounded through high neutral impedance it seems that such fuses could blow on an unfaulted phase *following* the occurrence of a line-to-ground fault on the system. There are distribution systems in the lower voltage classes that are operated with high neutral grounding impedance and that have potential transformers connected from line-to-ground for metering or other purposes.

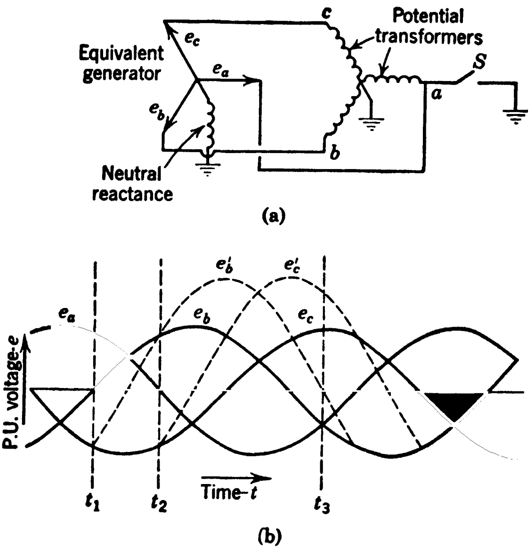


FIG. 10·28. Effect of line-to-ground fault on line-to-ground potential transformer magnetizing currents in high-impedance neutral grounded system. (*a*) Simplified equivalent circuit. (*b*) Voltage-time relationships: ———, before fault; - - - - - -, during fault.

For the purpose of illustration, consider a system grounded through high neutral impedance. For simplicity, let the system consist of an equivalent generator with neutral reactance as shown in Fig. 10·28. For further simplicity of illustration, let the reactance of this equivalent generator be negligible. Let there be potential transformers connected from the terminals of this generator to ground as shown; there might be a large number of these at appropriate buses in an extensive system.

Let the three per-unit voltages to neutral be

$$e_a = \sin t$$

$$e_b = \sin\left(t - \frac{2\pi}{3}\right) \qquad [10\cdot5]$$

$$e_c = \sin\left(t - \frac{4\pi}{3}\right)$$

When a fault to ground occurs on phase a, the voltage on that phase drops to zero. The voltages on phases b and c become

$$e_b' = e_b - e_a = \sqrt{3}\sin\left(t - \frac{5\pi}{6}\right)$$

$$[10\cdot6]$$

$$e_c' = e_c - e_a = \sqrt{3}\sin\left(t - \frac{7\pi}{6}\right)$$

The relationship between the voltage and flux linkages in any one of the potential transformers is

$$e_{a,b,c} = \frac{d\psi}{dt}\, a,\, b,\, c \qquad [10\cdot7]$$

or

$$\psi_{a,b,c} = \int_{t_1}^{t_2} e_{a,b,c}\, dt$$

Before the fault, taking phase b as an example,

$$\psi_b = \int_{2\pi/3}^{\pi+2\pi/3} \sin\left(t - \frac{2\pi}{3}\right) dt = 2 \qquad [10\cdot8]$$

gives the *change* in flux linkage from the time of a voltage zero to the time of the next subsequent voltage zero in phase b. This corresponds to the maximum change in flux linkages occurring under steady-state conditions and simply means that the flux has changed from its normal maximum negative value 1.0 to its normal maximum positive value 1.0.

Now from Fig. 10·28 it is apparent that, if a fault occurs on phase a at such an instant that the transformer on phase b is subjected to a flux change corresponding to the area under the e_b curve from t_1 to t_2

and the area under the e_b' curve from t_2 to t_3, then the flux change from the instant of voltage zero on phase b at $t = t_1$ until the next subsequent voltage zero on phase b is

$$\Psi_b' = \int_{t_1}^{t_2} \sin t\, dt + \sqrt{3}\int_{t_2}^{t_3} \sin\left(t - \frac{5\pi}{6}\right) dt = 3.73 \qquad [10\cdot 9]$$

This value of *change* of flux linkages in the potential transformer in phase b means that the flux changed from a normal maximum negative value -1 to a positive maximum value 2.73. Therefore at $t = t_3$, the potential transformer has 2.73 times normal flux in it and it may be oversaturated, thus requiring a high magnetizing current which may be sufficient to cause the transformer series fuse to blow. Of course, if the transformer is designed to operate at such a low flux density normally that even 2.73 times normal flux does not appreciably saturate the core, then there is no excessive current required.

An inspection of the preceding equations and Fig. 10·28 reveals that, although phase c is also subjected to an excess of flux linkages, the excess in phase b as analyzed is greater.

There are systems that can cause further amplification of this effect. For instance, a system with high neutral grounding resistance and a great deal of cable and overhead line (giving a large zero-sequence capacitance) may have a zero-sequence impedance viewed from the fault location which has a negative reactive component. For such cases, the unfaulted phase voltages-to-ground may be greater than 1.73 times normal leg voltage as the curves of Chapter 1 show. To the extent that these unfaulted phase voltages exceed the value 1.73, there is a further amplification of the flux linkages in the unfaulted phases when a single line-to-ground fault occurs.

It is impossible to summarize completely in this chapter, and in this book, detailed results for all non-linear circuits that have been studied. It is hoped that this chapter will serve merely to point out problems that require further investigation, from both a laboratory or miniature-system and analytical approach. No book is ever complete. It seems more appropriate to consider the material in this chapter as evidence that the broad subject of Transients in Power Systems is an active one and that operating experiences and new system design practices will require a continued awareness and increased understanding of such phenomena.

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APPENDIX I

Effect of Saliency on Overvoltages During Faults—Details of Analysis

A1·1 NOMENCLATURE

x_l = armature leakage reactance.

x_{ad} = mutual reactance of armature direct-axis winding with field or amortisseur winding.

x_f = field leakage reactance.

x_{aq} = mutual reactance of armature and amortisseur quadrature-axis windings.

x_d' = direct-axis transient reactance.

x_q = quadrature-axis synchronous reactance.

r_{kd} = direct-axis amortisseur resistance.

r_{kq} = quadrature-axis amortisseur resistance.

x_{kd} = direct-axis amortisseur leakage reactance.

x_{kq} = quadrature-axis amortisseur leakage reactance.

i_d = direct-axis armature current.

i_q = quadrature-axis armature current.

x_d'' = direct-axis subtransient reactance.

x_q'' = quadrature-axis subtransient reactance.

x_e = external reactance.

$$y = \frac{x_d'' - x_q''}{2}.$$

$$x = \frac{x_d'' + x_q''}{2}.$$

$$x_{ad}' = \frac{x_{ad}x_f}{x_{ffd}} = x_d' - x_l.$$

$x_{ffd} = x_{ad} + x_f$ = field self-reactance.

x_c = capacitive reactance.

$\alpha_d = \dfrac{r_{kd}}{x_{kd} + x_{ad}'}$ = direct-axis amortisseur decrement factor.

$\alpha_q = \dfrac{r_{kq}}{x_{kkq}}$ = quadrature-axis amortisseur decrement factor.

$x_{kkq} = x_{aq} + x_{kq}$ = quadrature-axis amortisseur self-reactance.
e_a = line-to-neutral armature voltage of the unfaulted phase.
$\theta = \theta_0 + t$ = angle between direct axis and phase-a axis.
ψ_d', ψ_{d2}' = components of direct-axis armature flux linkage.
ψ_q, ψ_{q2} = components of quadrature-axis flux linkage.

Subscripts 1, 2, 0, α, β refer to positive, negative, and zero sequence, and to α and β components.

Subscript e refers to external circuit.

e = voltage.
i = current.
Z = impedance.

A1·2 EQUATIONS FOR DIFFERENTIAL ANALYZER

The equations as set up on the differential analyzer were:

$$\psi_d' = -x_d' i_d + \psi_{d2}' \tag{A[1·1]}$$

$$\psi_{d2}' = (x_d' - x_d'')i_d - \alpha_d \int \psi_{d2}'\,dt \tag{A[1·2]}$$

where

$$\alpha_d = \frac{r_{kd}}{x_{kd} + x_{ad}'}$$

$$x_{ad}' = \frac{x_{ad}x_{fd}}{x_{ffd}}$$

$$\psi_{d20}' = -\left(\frac{x_d' - x_d''}{x_c}\right) - \left(1 - \frac{x_d'}{x_c}\right)\left(\frac{x_{ad}'}{x_{kd} + x_{ad}'}\right)$$

$$= \text{initial value of } \psi_{d2}'$$

$$\psi_q = -x_q i_q + \psi_{q2} \tag{A[1·3]}$$

$$\psi_{q2} = (x_q - x_q'')i_q - \alpha_q \int \psi_{q2}\,dt \tag{A[1·4]}$$

where

$$\alpha_q = \frac{r_{kq}}{x_{kkq}}$$

$$\psi_{q20} = 0$$

$$\psi_d' = \int (r_a i_d + \psi_q)\,dt + \int e_a d(\sin\theta) \tag{A[1·5]}$$

$$\psi_q = \int (r_a i_q - \psi_d')\,dt + \int e_a d(\cos\theta) \tag{A[1·6]}$$

where

$$
\begin{aligned}
i_{d0} &= -\frac{1}{x_c} \\
\psi_{q0} &= 0 \\
e_{a0} &= -\sin\theta_0 \\
i_{q0} &= 0 \\
\psi_{d0}' &= \frac{x_d''}{x_c} + \left(1 - \frac{x_d'}{x_c}\right)\left(\frac{x_{kd}}{x_{kd} + x_{ad}'}\right) \\
e_a &= \int x_c i_d d(\sin\theta) + \int x_c i_q d(\cos\theta) \qquad \text{A[1·7]}
\end{aligned}
$$

It will be noted that these equations are in an integral form similar to those of reference 9, Chapter 2. Moreover, the initial conditions are in a simplified form valid only for reasonably small r_a, since for the range of line resistance covered in this investigation the load angle δ of the machine was usually negligible and the terminal (i.e., at the terminals of the capacitance) voltage e_a before the short circuit occurred could consequently be expressed as: $e_{q0} = 1$, $e_{d0} = 0$.

In a few cases it was necessary to take into account the initial value of i_q, which could be found by the formula

$$i_{q0} = \frac{\sin\delta}{x_c} \qquad \text{A[1·8]}$$

where from reference 10 of Chapter 2

$$\tan\delta = \frac{r_a}{x_c - x_q}$$

Equations A[1·1] to A[1·7] may be derived from Park's synchronous-machine equations (reference 11, Chapter 2), in exactly the same manner as were the equations of reference 9, Chapter 2, and so they need not be derived here. It should only be pointed out that in equation A[1·5] the expression $e_a d(\sin\theta) = e_a \cos\theta\, dt = e_d\, dt$, and similarly for the similar expressions of equations A[1·6] and A[1·7]. This substitution was made in order to obtain the most desirable analyzer setup.

With the equations expressed as above, eight integrators are seen to be required. In addition, the two remaining integrators were used

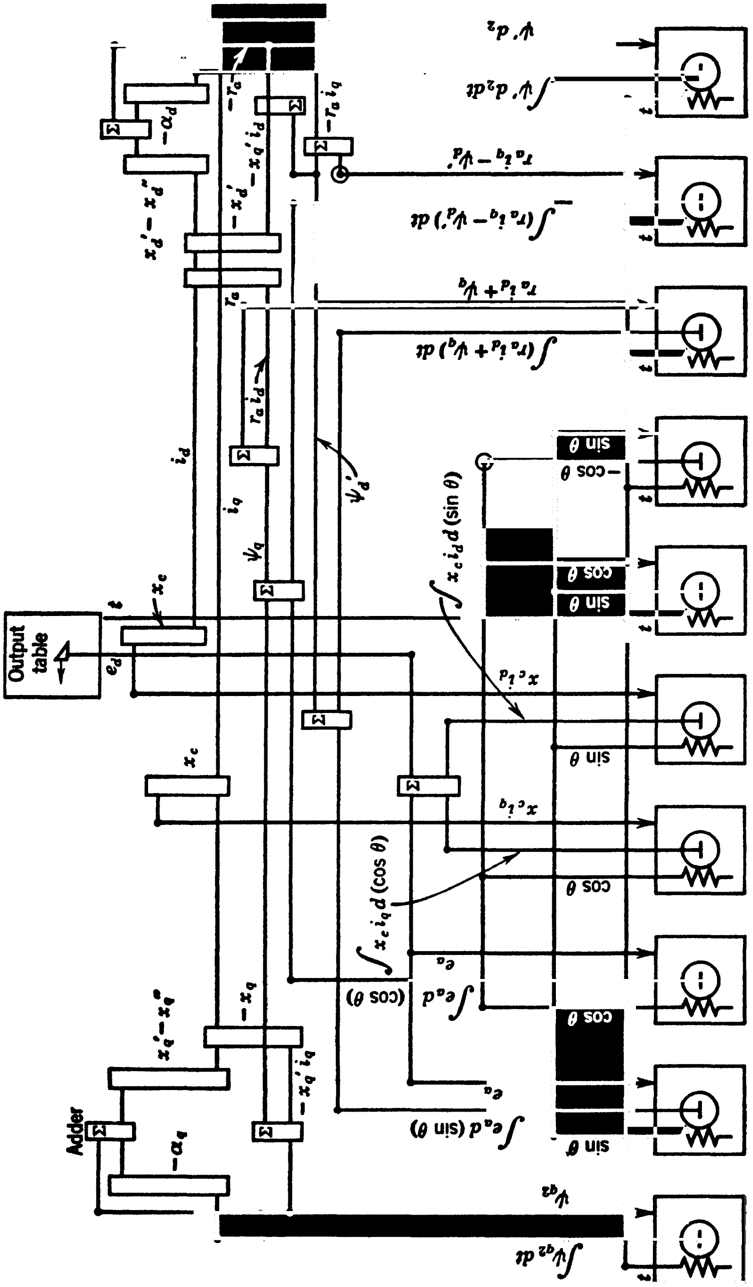

FIG. A1·1. Differential analyzer connections.

to generate the required sinusoidal functions sin θ, cos θ, and so the setup was entirely self contained and required no operators. Figure A1·1 shows a schematic diagram of the analyzer connections. Figure A1·2 shows the General Electric Differential Analyzer.

FIG. A1·2. General Electric Differential Analyzer.

A1·3 EQUATIONS FOR CALCULATION OF OVERVOLTAGES

The equations of an ideal synchronous machine with any static terminal impedance under the conditions that (1) all rotor resistances are negligibly small, (2) the external circuit parameters are constants, (3) the external circuit is symmetric about phase a of the synchronous machine, and (4) the rotor speed is constant are most simply and conveniently expressed for the present purpose as:

$$\begin{aligned} e_\alpha &= -[p(x + y\cos 2\theta) + r]i_\alpha - py\sin 2\theta i_\beta = Z_\alpha(p)i_{\alpha e} \\ e_\beta &= -py\sin 2\theta i_\alpha - [p(x - y\cos 2\theta) + r]i_\beta = Z_\beta(p)i_{\beta e} \qquad \text{A[1·9]} \\ e_0 &= -px_0 + r]i_0 = Z_0(p)i_{0e} \end{aligned}$$

where

$$\begin{aligned} x &= \tfrac{1}{2}(x_d'' + x_q'') \\ y &= \tfrac{1}{2}(x_d'' - x_q'') \end{aligned} \qquad \text{A[1·10]}$$

r = armature circuit resistance

$Z_\alpha(p)$, $Z_\beta(p)$, $Z_0(p)$ = operational impedances of the external connected circuit to the α, β, and 0 components of current, respectively

i_α, i_β, i_0 are components of the stator currents defined in terms of the phase currents by the equations:

$$\begin{aligned} i_\alpha &= \tfrac{2}{3}i_a - \tfrac{1}{3}i_b - \tfrac{1}{3}i_c \\ i_\beta &= \frac{1}{\sqrt{3}}i_b - \frac{1}{\sqrt{3}}i_c \\ i_0 &= \tfrac{1}{3}i_a + \tfrac{1}{3}i_b + \tfrac{1}{3}i_c \end{aligned} \qquad \text{A[1·11]}$$

$i_{\alpha e}$, $i_{\beta e}$, i_{0e} are the similar currents flowing in the external circuit. The currents i_α, i_β may also be expressed in terms of the direct- and quadrature-axis components of current by the equations:

$$\begin{aligned} i_\alpha &= i_d \cos\theta - i_q \sin\theta \\ i_\beta &= i_d \sin\theta + i_q \cos\theta \end{aligned} \qquad \text{A[1·12]}$$

i_α is the component of armature current which produces a field in the axis of phase a of the synchronous machine and may be regarded as a current that flows through phase a and splits into two equal parts which flow through phases b and c in parallel. i_β is the component of armature current which produces a field in the common axis of phases b and c and may be regarded as a current flowing only through phases b and c in series. e_α, e_β, e_0 are components of armature voltage defined by voltage equations similar to equations A[1·11] and A[1·12]. In the special case of a machine without an amortisseur winding the subtransient reactances x_d'', x_q'' are replaced by x_d', x_q', respectively. Equations A[1·9] are most easily derived from the equations of a synchronous machine (with no external impedance) in terms of direct- and quadrature-axis quantities (reference 11, Chapter 2). These are, for zero rotor resistances,

$$\begin{aligned} e_d &= -(px_d'' + r)i_d + x_q''i_q \\ e_q &= -x_d''i_d - (px_q'' + r)i_q \end{aligned} \qquad \text{A[1·13]}$$

Now, if equation A[1·12] and the similar voltage equations are applied, there are equations A[1·9] except for the $Z_\alpha(p)$, $Z_\beta(p)$ terms. It is now evident from a consideration of the circuit and the physical interpretations of i_α, i_β, $i_{\alpha e}$, $i_{\beta e}$, e_α, e_β that, if a static network is connected to the machine terminals, the complete equations A[1·9] are obtained. Moreover, if the network is completely symmetric, there is:

$$Z_\alpha(p) = Z_\beta(p) = Z_1(p) = Z_2(p) \qquad \text{A[1·14]}$$

where $Z_1(p)$, $Z_2(p)$ are the positive- and negative-sequence operational impedances of the external circuit.

For a line-to-line fault, as shown in Fig. 2·1, there is

$$Z_\alpha(p) = \frac{x_c}{p} \qquad \text{A[1·15]}$$

and there are no zero-sequence currents or voltages.

Before the fault let

$$e_a = -\sin(\theta_0 - \phi + t)$$

where ϕ is the load angle. Then, from equation A[1·11]

$$e_\alpha = -\sin(\theta_0 - \phi + t)$$

$$e_\beta = \cos(\theta_0 - \phi + t)$$

The fault is equivalent to the application of a voltage at the terminals of the machine equal and opposite to e_β. Thus when the voltage $e_\beta = -\cos(\theta_0 + t)$ is applied to equations A[1·9], we obtain the components of current caused by the fault. Further the terminal voltage e_α is given by

$$e_\alpha = \frac{x_c}{p} i_\alpha \qquad \text{A[1·16]}$$

and the total resultant is obtained by adding e_α to the voltage existing before short circuit. If the speed is constant the current may be obtained as follows:

Equations A[1·9] may be written

$$\begin{aligned} &[p(1 + y' \cos 2\theta) + r' + Z_\alpha'(p)]xi_\alpha + (py' \sin 2\theta)xi_\beta = 0 \\ &(py' \sin 2\theta)xi_\alpha + [p(1 - y' \cos 2\theta) + r']xi_\beta = \cos(\theta_0 + t) \end{aligned} \qquad \text{A[1·17]}$$

where

$$y' = \frac{y}{x} \quad r' = \frac{r}{x} \quad Z' = \frac{Z}{x}$$

It is difficult to obtain a simple finite operational solution to equations A[1·17] because of the presence of the periodic time functions in the current coefficients. However a solution may easily be found by expanding the currents in powers of the troublesome coefficient y' about $y' = 0$. We then have

$$
\begin{aligned}
i_\alpha &= i_{\alpha 0} + i_{\alpha 0}'y' + \frac{i_{\alpha 0}''}{2!}y'^2 + \cdots \\
i_\beta &= i_{\beta 0} + i_{\beta 0}'y' + \frac{i_{\beta 0}''}{2!}y'^2 + \cdots
\end{aligned}
\qquad \text{A[1·18]}
$$

where the primes on the currents indicate differentiation with respect to y' and the subscripts zero indicate evaluation at $y' = 0$.

To obtain $i_{\alpha 0}$, $i_{\beta 0}$ we have merely to let $y' = 0$ in equations A[1·17] and then to solve the resulting two independent equations operationally. Next, to obtain $i_{\alpha 0}'$, $i_{\beta 0}'$ we must (1) differentiate equations A[1·17] with respect to y', (2) let $y' = 0$, (3) substitute for $i_{\alpha 0}$, $i_{\beta 0}$ their values as previously found, (4) obtain the operational solution to the resulting equation. To obtain $i_{\alpha 0}''$, $i_{\beta 0}''$ we proceed in exactly the same way by differentiating again and utilizing the previously found values of $i_{\alpha 0}'$, $i_{\beta 0}'$. For the general coefficients $i_{\alpha 0}^{n'}$, $i_{\beta 0}^{n'}$ of equations A[1·18] there are the equations,

$$
\begin{aligned}
[p + r' + Z_\alpha'(p)]i_{\alpha 0}^{(n-1)'} &= -np[(\cos 2\theta)i_{\alpha 0}^{(n-1)'} + (\sin 2\theta)i_{\beta 0}^{(n-1)'}] \\
[p + r']i_{\beta 0}^{n'} &= -np[(\sin 2\theta)i_{\alpha 0}^{(n-1)'} - (\cos 2\theta)i_{\beta 0}^{(n-1)'}]
\end{aligned}
\qquad \text{A[1·19]}
$$

The currents, and consequently the terminal voltage by equation A[1·16], are now determined as infinite series of transient and steady-state terms which, however, converge rapidly because in the usual case y' is much smaller than unity. Figure 2·10 shows for a particular case a comparison of voltage calculated by the formulas developed above with a curve taken on the differential analyzer and with a step-by-step solution of equations A[1·9]. Only the first two terms of equations A[1·18] were considered in calculating the points of Fig. 2·10.

If only the steady-state (as far as armature transients are concerned) voltage is desired the equations are very much simplified since then the currents and voltages appear only as a series of odd-harmonic terms and the operational impedances of equations A[1·9] and A[1·19] may be replaced by steady-state impedances at the proper frequency. Figure 2·9 shows a comparison of maximum overvoltages calculated by this simplified method with the overvoltages obtained by the differential analyzer.

A second method of procedure in solving equations A[1·9] is to assume that currents and voltages of fundamental frequency can be determined with sufficient accuracy by the method of symmetrical components. It can be applied to systems of any degree of complexity. Since symmetrical components are in general use, the procedure in determining instantaneous currents and voltages following faults will be given in terms of symmetrical components, the α- and β-components being used only to pass from direct and quadrature components to symmetrical components. An advantage of using α- and β-components over positive- and negative-sequence components is that the relations among the α and β currents and voltages may be expressed by real coefficients.

The following equations give the relations between the positive- and negative-sequence components of currents, voltages, and impedances and the α- and β-components.

$$\begin{aligned}
i_\alpha &= i_{a1} + i_{a2} \\
i_\beta &= -j(i_{a1} - i_{a2}) \\
e_\alpha &= e_{a1} + e_{a2} \\
e_\beta &= -j(e_{a1} - e_{a2}) \\
i_{a1} &= \tfrac{1}{2}(i_\alpha + ji_\beta) \\
i_{a2} &= \tfrac{1}{2}(i_\alpha - ji_\beta) \\
e_{a1} &= \tfrac{1}{2}(e_\alpha + je_\beta) \\
e_{a2} &= \tfrac{1}{2}(e_\alpha - je_\beta) \\
Z_\beta &= \tfrac{1}{2}(Z_1 + Z_2) \\
Z_\alpha &= 2\frac{Z_1 Z_2}{Z_1 + Z_2} \cong \frac{1}{2}(Z_1 + Z_2)
\end{aligned} \tag{A[1·20]}$$

Reference Vector. In the method of symmetrical components, sinusoidal voltages and currents of fundamental frequency are represented by vectors referred to some particular vector as reference. Thus, if the voltage e_a of phase a at some specified point leads the reference vector by an angle α, it is written

$$e_a = |\,e_a\,|\underline{/\alpha} = E_a\underline{/\alpha}$$

To express e_a as a function of time, let the fundamental-frequency component of e_a be written

$$e_{1a} = -E_{1a}\sin(\theta + \alpha_1) = E_{1a}\underline{/\theta + \alpha_1}$$

and the *n*th harmonic component of e_a

$$e_{na} = -E_{na}\sin(n\theta + \alpha_n) = E_{na}\underline{/n\theta + \alpha_n}$$

where θ is the position of the axis of the rotor of a specified machine in electrical degrees measured from the axis of phase a.

The following expressions for instantaneous values of currents and voltages of any frequency will be used interchangeably,

$$\begin{aligned}
-V\sin(n\theta \pm \alpha) &= V\underline{/n\theta \pm \alpha} \\
V\cos(n\theta \pm \alpha) &= -jV\underline{/n\theta \pm \alpha} = V\underline{/n\theta \pm \alpha - 90^\circ} \\
\pm jV\sin(n\theta + \alpha) &= V\sin(n\theta + \alpha \pm 90^\circ) = \pm V\cos(n\theta + \alpha) \\
\pm jV\cos(n\theta + \alpha) &= V\cos(n\theta + \alpha \pm 90^\circ) = \mp V\sin(n\theta + \alpha)
\end{aligned}$$

Let

$$\left.\begin{aligned}
i_{na1} &= \text{positive-sequence } n\text{th harmonic current} \\
&= -jI_{na1}\underline{/n\theta + \alpha_{n1}} = I_{na1}\cos(n\theta + \alpha_{n1}) \\
i_{na2} &= \text{negative-sequence } n\text{th harmonic current} \\
&= -jI_{na2}\underline{/n\theta + \alpha_{n2}} = I_{na2}\cos(n\theta + \alpha_{n2})
\end{aligned}\right\} \qquad \text{A[1}\cdot\text{21]}$$

where α_{n1} and α_{n2} are determined by the system impedances, type of fault, and load angle.

Generated Voltages. When $x_q'' = x_d''$, currents and voltages following faults will in the general case have a fundamental-frequency term, one or more natural-frequency terms, and a d-c term. These can be determined by applying a voltage at the fault equal and opposite to that which existed there before the fault and adding resulting changes in currents and voltages to initial values. When $x_q'' \neq x_d''$, additional voltages are generated. From equations A[1·9] the voltages generated by sinusoidal currents i_α and i_β of any frequency are

$$\left.\begin{aligned}
e_\alpha &= -py(\cos 2\theta i_\alpha + \sin 2\theta i_\beta) \\
e_\beta &= -py(\sin 2\theta i_\alpha - \cos 2\theta i_\beta)
\end{aligned}\right\} \qquad \text{A[1}\cdot\text{22]}$$

Replacing i_α and i_β in equations A[1·22] by their positive- and negative-sequence values from equations A[1·20] and substituting for them the expressions in equations A[1·21], positive- and negative-sequence voltages generated in a given machine can be determined as follows:

$$\begin{aligned}
e_\alpha &= -py[I_{na1}\{\cos 2\theta \cos (n\theta + \alpha_{n1}) + \sin 2\theta \sin (n\theta + \alpha_{n1})\} \\
&\qquad + I_{na2}\{\cos 2\theta \cos (n\theta + \alpha_{n2}) - \sin 2\theta \sin (n\theta + \alpha_{n2})\}] \\
&= -py[I_{na1} \cos \{(n-2)\theta + \alpha_{n1}\} \\
&\qquad + I_{na2} \cos \{(n+2)\theta + \alpha_{n2}\}] \\
&= (n-2)yI_{na1} \sin \{(n-2)\theta + \alpha_{n1}\} \\
&\qquad + (n+2)yI_{na2} \sin \{(n+2)\theta + \alpha_{n2}\} \\
e_\beta &= -py[I_{na1}\{\sin 2\theta \cos (n\theta + \alpha_{n1}) - \cos 2\theta \sin (n\theta + \alpha_{n1})\} \\
&\qquad + I_{na2}\{\sin 2\theta \cos (n\theta + \alpha_{n2}) + \cos 2\theta \sin (n\theta + \alpha_{n2})\}] \\
&= -py[-I_{na1} \sin \{(n-2)\theta + \alpha_{n1}\} \\
&\qquad + I_{na2} \sin \{(n+2)\theta + \alpha_{n2}\}] \\
&= (n-2)yI_{na1} \cos \{(n-2)\theta + \alpha_{n1}\} \\
&\qquad - (n+2)yI_{na2} \cos \{(n+2)\theta + \alpha_{n2}\}
\end{aligned} \qquad \text{A[1·23]}$$

If $n \geqq 2$

$$\begin{aligned}
je_\beta = -(n-2)yI_{na1} &\sin [(n-2)\theta + \alpha_{n1}] \\
&+ (n+2)yI_{na2} \sin [(n+2)\theta + \alpha_{n2}]
\end{aligned} \qquad \text{A[1·24]}$$

$$\begin{aligned}
e_{a1} &= (n+2)yI_{na2} \sin [(n+2)\theta + \alpha_{n2}] \\
&= -(n+2)yI_{na2}\underline{/(n+2)\theta + \alpha_{n2}} \\
&= (n+2)y\underline{/2\theta - 90^\circ}\, i_{na2}
\end{aligned} \qquad \text{A[1·25]}$$

$$\begin{aligned}
e_{a2} &= (n-2)yI_{na1} \sin [(n-2)\theta + \alpha_{n1}] \\
&= -(n-2)yI_{na1}\underline{/(n-2)\theta + \alpha_{n1}} \\
&= (n-2)y\underline{/-2\theta - 90^\circ}\, i_{na1}
\end{aligned} \qquad \text{A[1·26]}$$

If $n = 1$

$$\begin{aligned}
e_{a1} &= yI_{na1} \sin (\theta - \alpha_{n1}) = -yI_{1a1}\underline{/\theta - \alpha_{11}} = y\underline{/-2\alpha_{11} - 90^\circ}\, i_{1a1} \\
e_{a2} &= 0
\end{aligned} \qquad \text{A[1·27]}$$

From equations A[1·25], A[1·26], and A[1·27], positive- and negative-sequence voltages generated by harmonic current without decrements can be determined.

Steady State. Since zero rotor resistance has been assumed in the direct and quadrature axes, there will be no field decrements and fundamental-frequency and odd-harmonic terms of currents and voltages will have no decrements. Natural-frequency and even-harmonic terms will disappear in the steady state.

Table A1·1 gives the positive- and negative-sequence odd-harmonic voltages generated by positive- and negative-sequence odd-harmonic currents.

TABLE A1·1

POSITIVE- AND NEGATIVE-SEQUENCE * ODD-HARMONIC GENERATED VOLTAGES

Positive-Sequence Odd-Harmonic Voltages Generated in Machines by Negative-Sequence Currents

$$e_{3a1} = 3yI_{1a2}\sin(3\theta + \alpha_{12}) = -3yI_{1a2}\underline{/3\theta + \alpha_{2}} = 3y\underline{/2\theta - 90^\circ}\, i_{1a2}$$

$$e_{5a1} = 5yI_{3a2}\sin(5\theta + \alpha_{32}) = -5yI_{3a2}\underline{/5\theta + \alpha_{23}} = 5y\underline{/2\theta - 90^\circ}\, i_{3a2}$$

$$e_{7a1} = 7yI_{5a2}\sin(7\theta + \alpha_{52}) = -7yI_{5a2}\underline{/7\theta + \alpha_{25}} = 7y\underline{/2\theta - 90^\circ}\, i_{5a2}$$

$$e_{9a1} = 9yI_{7a2}\sin(9\theta + \alpha_{72}) = -9yI_{7a2}\underline{/9\theta + \alpha_{27}} = 9y\underline{/2\theta - 90^\circ}\, i_{7a2}$$

Negative-Sequence Odd-Harmonic Voltages Generated in Machines by Positive-Sequence Currents

$$e_{1a2} = yI_{3a1}\sin(\theta + \alpha_{31}) = -yI_{3a1}\underline{/\theta + \alpha_{31}} = y\underline{/-2\theta - 90^\circ}\, i_{3a1}$$

$$e_{3a2} = 3yI_{5a1}\sin(3\theta + \alpha_{51}) = -3yI_{5a1}\underline{/3\theta + \alpha_{51}} = 3y\underline{/-2\theta - 90^\circ}\, i_{5a1}$$

$$e_{5a2} = 5yI_{7a1}\sin(5\theta + \alpha_{71}) = -5yI_{7a1}\underline{/5\theta + \alpha_{71}} = 5y\underline{/-2\theta - 90^\circ}\, i_{7a1}$$

$$e_{7a2} = 7yI_{9a1}\sin(7\theta + \alpha_{91}) = -7yI_{9a1}\underline{/7\theta + \alpha_{91}} = 7y\underline{/-2\theta - 90^\circ}\, i_{9a1}$$

Procedure in Determining Steady-State Values of Currents and Voltages Following Faults. Assuming zero resistance in the field for machines without amortisseur windings and also in the amortisseur for those with amortisseur windings, the procedure for determining currents and voltages in the steady state following faults is as follows:

1. Calculate currents and voltages of fundamental frequency in the system by means of symmetrical components. Since it has been assumed that fundamental-frequency currents and voltages are determined with sufficient accuracy by this method, voltages of funda-

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* α and β voltages when required can be obtained from Table A1·1 and equations A[1·20].

mental frequency generated in machines in which $x_d'' \neq x_q''$ will be neglected.

2. From the negative-sequence current flowing in machines in which x_d''/x_q'', determine the third-harmonic positive-sequence generated voltages from Table A1·1.

3. From the third-harmonic positive-sequence generated voltages, determine by symmetrical components the third-harmonic voltages and currents in the systems, assuming positive- and negative-impedance of the synchronous machines to third-harmonic currents to be $3x_2$. x_2 may differ slightly from the fundamental-frequency value.

4. From the negative-sequence third-harmonic currents flowing in machines in which $x_d'' \neq x_q''$, determine from Table A1·1 the fifth-harmonic positive-sequence generated voltages.

Proceeding in this manner, odd-harmonic currents and voltages can be determined at any point of the system.

It is also possible to calculate the transient voltage caused by odd-harmonic currents and, by using the natural frequency and even-harmonic currents in equations A[1·22], to calculate the generated transient voltages and from them in turn the resulting terminal overvoltages.

A1·4 EQUIVALENT CIRCUITS FOR VARIOUS TYPES OF FAULTS

Let i_a, i_b, and i_c be phase currents flowing into the fault; $Z_0(p)$ the zero-sequence impedance viewed from the fault; e_a, e_b, and e_c the phase-to-ground voltages at the fault, e_α, e_β, and e_0 their α-, β-, and 0-components, respectively.

Line-to-Ground Fault (Phase a). Conditions at the fault are: $i_b = i_c = 0$; $e_a = 0$.

From equations A[1·11] $i_\beta = 0$; $i_\alpha = 2i_0$; $e_\alpha = -e_0$.

It follows that

$$-e_0 = Z_0(p)i_0 = \frac{Z_0(p)}{2}\, i_\alpha = e_\alpha \qquad \text{A}[1\cdot 28]$$

From equation A[1·28] the equivalent circuit to replace the fault in α-network is $Z_0(p)/2$. When $x_d'' = x_q''$, the β-network is not involved, e_β is not changed by the fault, and e_α and e_0 have only fundamental- and natural-frequency terms. In machines in which $x_d'' \neq x_q''$, fundamental-, harmonic-, and other frequency voltages are generated (see Section A1·3) which affect e_α, e_β, and e_0. At the fault, since $e_\alpha = -e_0$, e_a, e_b, and e_c, are:

$$\left.\begin{aligned}
e_a &= e_\alpha + e_0 = 0\\
e_b &= -\frac{1}{2}e_\alpha + e_0 - \frac{\sqrt{3}}{2}e_\beta = \frac{3}{2}e_0 - \frac{\sqrt{3}}{2}e_\beta\\
&= -\frac{3}{2}e_\alpha - \frac{\sqrt{3}}{2}e_\beta\\
e_c &= -\frac{1}{2}e_\alpha + e_0 + \frac{\sqrt{3}}{2}e_\beta = \frac{3}{2}e_0 + \frac{\sqrt{3}}{2}e_\beta\\
&= -\frac{3}{2}e_\alpha + \frac{\sqrt{3}}{2}e_\beta
\end{aligned}\right\} \qquad \text{A}[1\cdot 29]$$

Line-to-Line Fault (*Phases b and c*). Conditions at the fault are: $i_b = -i_c$; $i_a = 0$; $e_b = e_c$.

From equations A[1·11] $i_\alpha = i_0 = 0$; $e_\beta = 0$. For a system symmetrical before the fault, the zero-sequence network is not involved. The β-network is shorted at the point of fault. The α-network is unaffected when $x_d'' = x_q''$, but when $x_d'' \neq x_q''$ voltages are generated in the machines which affect e_α. (See Section A1·3). At the fault,

$$\begin{aligned} e_a &= e_\alpha \\ e_b = e_c &= -\tfrac{1}{2}e_\alpha \end{aligned} \qquad \text{A}[1\cdot 30]$$

Double Line-to-Ground Fault (*Phases b and c*). Conditions at the fault are: $i_a = 0$; $e_b = e_c = 0$.

From equations A[1·11] $e_\beta = 0$; $e_\alpha = 2e_0$; $i_\alpha = -i_0$. It follows that

$$2e_0 = -2Z_0(p)i_0 = 2Z_0(p)i_\alpha = e_\alpha \qquad \text{A}[1\cdot 31]$$

The equivalent circuit to replace the fault in the α-network is therefore $2Z_0(p)$. The β-network is shorted at the point of fault. When $x_d'' = x_q''$, there is no mutual coupling between the α- and β-networks, but when $x_d'' \neq x_q''$, voltages are generated in the machines which affect e_α and e_0. (See Section A1·3.)

At the fault,

$$\begin{aligned} e_a &= e_\alpha + e_0 = 3e_0 = \tfrac{3}{2}e_\alpha \\ e_b &= e_c = 0 \end{aligned} \qquad \text{A}[1\cdot 32]$$

In Figs. 2·16, 2·17, and 2·18 are shown the equivalent circuits for line-to-ground, line-to-line, and double-line-to-ground faults at the terminals of a synchronous generator with series impedance connected to any symmetrical external system in which the positive- and negative-sequence impedances are equal. $Z_{1e}(p)$ and $Z_{0e}(p)$ are the operational expressions for the positive- and zero-sequence impedances of the external system. $r_0 + px_0$ is the zero-sequence impedance of the generator. The positive- and negative-sequence impedances of the generator to currents of frequencies other than fundamental are approximately $r + px_2$ except for very high frequencies. This is also the α and β impedance to them.

Except for voltages of fundamental frequency which are determined by the method of symmetrical components, the equivalent circuits of Figs. 2·16, 2·17, and 2·18 can be used to determine all other components of current and voltage. The principal natural-frequency terms are calculated by the application of a voltage at the point of fault equal and opposite to the initial steady-state voltage. The point of application of voltages generated when $x_d'' \neq x_q''$ is taken in the machine.

Resonance to Harmonics. Harmonic voltages are generated in machines in which $x_d'' \neq x_q''$. The external impedance which will cause resonance for various types of faults can be determined from the equivalent circuits as follows.

From equations A[1·29] and Fig. 2·16, resonance for a line-to-ground fault can occur either in the unfaulted β-circuit or in the α-circuit. For resonance to the nth harmonic in the β-circuit there is the equation:

$$nx_2 = \text{negative reactance component of } Z_{1e}(p)$$

For resonance in the α-circuit

$$nx_2 = \text{negative reactance component of}$$
$$\frac{r_0 + px_0}{2}, \frac{Z_{0e}(p)}{2}, \text{ and } Z_{1e}(p) \text{ in parallel}$$

It is thus possible to have resonance to one harmonic in the α-circuit and to another in the β-circuit.

For the line-to-line and double-line-to-ground faults resonance can occur only in the α-circuit. For the line-to-line fault it will occur when

$$nx_2 = \text{negative reactance component of } Z_{1e}(p)$$

For the double-line-to-ground fault, when

$$nx_2 = \text{negative reactance component of}$$
$$2(r_0 + px_0), 2Z_{0e}(p), \text{ and } Z_{1e}(p) \text{ in parallel}$$

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Although references are given at the end of each chapter, it is felt desirable to record here at the end of this volume a selected bibliography of the literature covering with reasonable completeness the subject of electric transients in power systems, with particular emphasis on power system overvoltages. The arrangement is in chronological order to indicate the historical development of the subject matter. An alphabetically arranged author index is included also, with the year of publication indicated, for convenience in looking up any of the references listed. An asterisk indicates that there is more than one article for that year.

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